Jlab dⁿ₂ Collaboration Meeting Target Update

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Outline

³He Density

- RTDs
- ³He Chamber Density

2 EPR

• EPR Pumping Chamber Polarization

NMR

- Water Calibration
 - Fitting Water Signal
 - Water Calibration Constant
- ³He

What's Next

RTDs

Samantha RTDs



- There are seven RTDs in total on the target cell
- Five along the target chamber
- Two in the pumping chamber

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Figure: ³He target cell RTD locations.

³He Density

RTDs

Target Chamber RTDs





Figure: RTD temperatures along the target cell.

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³He Density

RTDs

Pumping Chamber RTDs



Figure: RTD temperatures in the pumping chamber. The blue dashed lines shows where the oven temperature was adjusted from 240 - >235 °C. The red dashed line shows period where the oven temperature was adjusted from 235 - >240 °C. The black dashed line shows where one of the oven heaters was replaced. The blue makers show the average transverse pumping chamber temperatures and the red markers show the longitudinal pumping chamber temperatures. The temperature differences between the longitudinal and transverse target directions are due to the target having a different laser alignment.



- Due to lasers on the pumping chamber, the internal temperature is always higher than the RTD reading
- This is corrected for by doing a temperature test
- Temperature test involves taking RTD readings with pumping lasers on/off

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Samantha Temperature Test Results

Temperature Summary

Date	Cell	Direction	T_Read	T_Calculated	ΔT
Feb 23th	Samantha	Longitudinal	257.21	263.39	6.18
Mar 16th	Samantha	Transverse	257.08	264.39	7.31
May 5th	Dominic	Vertical	246.55	262.41	15.86
May 19th	Moss	Longitudinal	238.75	249.14	10.39
June 9th	Moss	Transverse	247.59	264.51	16.92

Figure: Longitudinal and transverse results of Samantha target cell temperature results done by Yawei Zhang

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RTDs

Final RTD Temperatures



Figure: Corrected Average ³He pumping chamber RTD and average target chamber RTD temperatures.

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Pressure Broadening

- The D1 and D2 absorption lines of Rb are broadened by the presence of ³He
- Measuring the absorption spectrum, the ³He density was measured

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RTDs

Density Equation

$$n_0 = \frac{\Gamma - \rho_{\rm N_2} \Gamma_{\rm D1}^{\rm N_2} \left(\frac{T}{353}\right)^{0.3}}{\Gamma_{\rm D1}^{^{3}\rm He} \left(\frac{T}{353}\right)^{0.1}}$$

- Γ = Half width
- $\rho_{N_2} = N_2$ filling density
- $\Gamma_{D1}^{N_2} = N_2$ full D1 width
- Γ_{D1}^{3He} = ³He full D1 width
- T = Temperature

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RTDs

Equilibrium ³He Density, n_0

$n_0 = 8.099 + - 0.033$ amg



Figure: ³He measured from pressure broadening by Lamiaa El Fassi

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Parameter	Value	Unit	Uncertainty(%)	Source
Half width of the peak	From fit	GHz/amg	0.3	Fit
³ He D1 full width	18.7	GHz/amg	1.60	ref
N_2 D1 full width	17.8	GHz/amg	1.7	ref
N_2 density in the cell	0.1125	amg	1.8	N ₂ filling density
Temperature	353	°F	neg.	Oven
Total			1.63	

Table: List of parameters used to calculate n_0

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³He Target and Pumping Chamber Densities

- Knowledge of the target and pumping chambers is when computing the ³He polarization
- the density in the chambers is given as:

$$n_{t} = \frac{n_{0}}{1 + \frac{V_{p}}{V_{tot}} \left(\frac{T_{t}}{T_{p}} - 1\right)}, n_{p} = \frac{n_{0}}{1 + \frac{V_{t}}{V_{tot}} \left(\frac{T_{p}}{T_{t}} - 1\right)}$$

- n_t/n_p : Target/pumping chamber density
- n₀: Equilibrium ³He density
- V_t/V_p: Target/pumping chamber volumes
- V_{tot}: Total target cell (pumping + target + transfer tube) volumes
- T_t/T_p : Target/pumping chamber temperatures

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Samantha Target Cell Properties

UVa d2n Cell Properties from Gas System & Buoyancy

Longitudinal 42-deg cells in blue

Cell	$ ho_{ m He}$	ρ_{N_2}	$\rm V_{PC}$	V_{TT}	$\rm V_{\rm TC}$
	(amg)	(amg)	(cc)	(cc)	(cc)
Alex	$7.932{\pm}0.072$	$0.1133 {\pm} 0.002$	193.85	6.92	77.29
Boris	$7.993 {\pm} 0.072$	$0.1126 {\pm} 0.002$	166.13	5.83	73.91
Moss	$7.808 {\pm} 0.071$	$0.1132 {\pm} 0.002$	184.13	6.54	78.23
Samantha	$7.847 {\pm} 0.070$	$0.1125 {\pm} 0.002$	176.90	6.51	75.47
Tigger	$7.807 {\pm} 0.071$	$0.1124 {\pm} 0.002$	186.94	6.35	78.36
	PRELIMINA	RY: Al Tobias, M	ay 5, 2009)	

Figure: Cell properties measured at UVa.

Following errors were propagated through n_t and n_p density equations:

Parameter	Value	Unit	Uncertainty(%)	Source
V _{tt}	6.51	СС	1.0	-
V_t	75.47	CC	1.0	-
V_p	176.90	сс	1.0	-
V_{tot}	258.88	СС	1.73	-
T_t (trans.)	349.87	k	1.43	Fit and +/-5k variation
T_p (tran.)	543.3	k	0.92	Fit and +/- 5k variation
T_t (long.)	348.98	k	1.44	Fit and +/-5k variation
T_p (long.)	536.0	k	0.93	Fit and +/- 5k variation
n_0	8.099	amg	1.68	n_0 Fit and pressure broadening

Table: List of parameters used to calculate n_t and n_p

n_t and n_p Results

My values:

Parameter	Direction	Value	Unit	Uncertainty(%)
n_t	Long.	10.63	amg	2.07
n_p	Long.	7.005	amg	1.85
n_t	Trans.	10.72	amg	2.07
n_p	Trans.	6.975	amg	1.85

Table: Target and pumping chamber density results.

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³He Density

³He Chamber Density

Target and Pumping Chamber Densities



Figure: Target and pumping chamber densities with error applied.

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EPR Polarization

- Polarized ³He nuclei in the pumping chamber create a shift of the EPR
- EPR shift is due to two sources:
 - Small effective magnetic field due to Rb-³He spin exchange interactions (B_{SE})
 - Magnetic field created by polarized ³He (B_M)
- EPR shift is then given by $B_0 \pm \delta B_{3He}$
- Where $\delta B_{3He} = B_{SE} + B_M$

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EPR Spectrum



Figure: EPR measurement spectrum.

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Extracting EPR Polarization

$$P_{^{3}\mathrm{He}} = \frac{\Delta\nu_{EPR}}{\frac{2}{3}\mu_{0}\frac{d\nu_{EPR}}{dB}\kappa_{0}\mu_{^{3}\mathrm{He}}n_{po}}$$

• $\Delta \nu_{EPR}$: EPR frequency shift

- μ_0 : Vacuum permeability
- $\frac{d\nu_{EPR}}{dB}$: Derivative frequency with respect to field, from atomic physics experiments
- κ₀: Constant from atomic physics experiments
- n_{pc} : ³He pumping chamber density
- $\mu_{^{3}\mathrm{He}}$: Magnetic moment of ³He

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Calculating $\frac{d\nu_{EPR}}{dB}$: Parameters Used

Parameter	Description	Value	Unit
g_I	K g-factor	0.2601	-
	Rb g-factor	0.5412	-
μ_N	nuclear magnetron	5.051 x 10^{-27}	J/T
g_S	electron g-factor	2.0023	-
μ_B	Bohr magnetron	9.275 x 10^{-24}	J/T
h	Plank's constant	6.626 x 10^{-34}	Js
I	K nuclear spin	1.5	\hbar
	Rb nuclear spin	2.5	\hbar
ν_{hfs}	K	461.719	MHz
	Rb	3035.732	MHz

Table: List of parameters used to calculate $\frac{d\nu_{EPR}}{dB}$

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Calculating $\frac{d\nu_{EPR}}{dB}$

$$\frac{d\nu_{\pm}}{dB} = \frac{g_I \mu_N - g_S \mu_B}{h[I]} \sum_{n=0}^5 b_n \frac{x^n}{[I]^n}$$

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$$x: (g_I \mu_N - g_S \mu_B) \frac{B}{h \nu_{hfs}}$$

- [I]: 2I + 1
- b_0 : 1
- b_1 : $\mp 4I$
- b_2 : 6I(2I-1)
- b_3 : $\mp 8I \left(4I^2 6I + 1 \right)$

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$$b_4$$
: $10I(2I-1)(4I^2-10I+1)$
• b_5 : $\mp 12I(16I^4-80I^3+80I^2-20I+1)$

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Calculating κ_0

New Value:

$$\kappa_0^{39K} = (5.99 \pm 0.11) + (0.0086 \pm 0.002) \left(T_0 - 200\right) \tag{5}$$

Old Value:

$$\kappa_0^{39K} = 4.52 + (0.0093T_0) \, 0.95 \tag{6}$$

- [5]: N.J. Stone, Atomic Data and Nuclear Data Tables 90 (2005) 75-176
- κ₀ is temperature dependent
- Uses reference temperature of 200°C
- T₀: pumping chamber temperature

Uncertainty on EPR Polarization

Parameter	Uncertainty (%)	Source
$n_p c$	1.88	pumping chamber
$\Delta \nu_{EPR}$	0.5	Fit
κ_0	2.79	See paper
Total	3.40	

Table: Longitudinal EPR Polarization

Parameter	Uncertainty (%)	Source
$n_p c$	1.85	pumping chamber
$\Delta \nu_{EPR}$	0.5	Fit
κ_0	2.97	See paper
Total	3.54	

Table: Transverse EPR Polarization

EPR Polarization Results (Modern κ_0 Value)

Date	Direction	Alkali	Polarization (%)	Constant	Datafile
02/07/2009	Longitudinal	K	48.71	-	EPR_AFP_20090207_2015
02/07/2009	Longitudinal	K	41.32	-	EPR_AFP_20090207_2056
02/09/2009	Transverse	K	47.64	-	EPR_AFP_20090209_1652
02/17/2009	Transverse	K	60.64	-	EPR_AFP_20090217_0940
02/17/2009	Transverse	K	55.10	-	EPR_AFP_20090217_0947
02/23/2009	Longitudinal	K	53.00	-	EPR_AFP_20090223_1022
03/11/2009	Transverse	K	53.39	-	EPR_AFP_20090311_1907
03/11/2009	Transverse	K	51.26	-	EPR_AFP_20090311_1912
03/16/2009	Transverse	K	57.79	-	EPR_AFP_20090316_1007
03/16/2009	Transverse	К	52.77	-	EPR_AFP_20090316_1042

Table: Summary of EPR measurements taken during E06-014, using modern κ_0 .

Date	Direction	Alkali	Polarization (%)	Constant	Datafile
02/07/2009	Longitudinal	K	46.49	-	EPR_AFP_20090207_2015
02/07/2009	Longitudinal	K	39.43	-	EPR_AFP_20090207_2056
02/09/2009	Transverse	K	45.47	-	EPR_AFP_20090209_1652
02/17/2009	Transverse	K	57.88	-	EPR_AFP_20090217_0940
02/17/2009	Transverse	K	52.60	-	EPR_AFP_20090217_0947
02/23/2009	Longitudinal	K	50.59	-	EPR_AFP_20090223_1022
03/11/2009	Transverse	K	50.97	-	EPR_AFP_20090311_1907
03/11/2009	Transverse	K	48.94	-	EPR_AFP_20090311_1912
03/16/2009	Transverse	K	55.17	-	EPR_AFP_20090316_1007
03/16/2009	Transverse	К	50.37	-	EPR_AFP_20090316_1042

Table: Summary of EPR measurements taken during E06-014, Using older $\kappa_0 = 4$

EPR Polarization Results: κ_0 Comparisons



Figure: EPR polarizations using the more recent and older value of κ_0^{39K}

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- Using the modern κ_0^{39K} gives a polarization that is higher than the older value.
- I will use the more recent value of κ_0 for the rest of the analysis.

Bloch Equations (1)

- Water polarization can be described by solutions to the Bloch equations
- Solution is used to fit the water NMR signal

$$\frac{dP_x(t)}{dt} = -\frac{1}{T_2} P_x(t) + \gamma \left(H(t) - H_0\right) P_y(t) + \frac{1}{T_2} \chi H_1 \tag{7}$$

$$\frac{dP_y(t)}{dt} = -\gamma \left(H(t) - H_0\right) P_x(t) - \frac{1}{T_2} P_y(t) + \gamma H_1 P_z(t)$$
(8)

$$\frac{dP_z(t)}{dt} = -\gamma H_1 P_y(t) - \frac{1}{T_1} P_z(t) + \frac{1}{T_1} \chi H(t)$$
(9)

- T1: Longitudinal relax time
- T₂: Transverse relax time
- H₀: Resonance field
- *H*₁: Transverse field component
- *H*(*t*) = *H*₀ + α*t*: field component along z axis (α = 1.2 G/s)

- γ : gyro-magnetic ratio of the proton
- $\chi: \frac{\mu_{p,H_2O}}{kT}$
- μ_{p,H2O}: magnetic moment of proton in water
- k: Boltzmann constant
- T: Temperature of target chamber

Bloch Equation Solutions

- Solutions to Bloch equations from Mathematica
- Water polarizations in 3 directions



 P_y



 P_x



Effective Polarization

We can express the polarization as an effective polarization

$$P_{eff} = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

with the solution

$$alertP_{eff}(t) = e^{-(t-t_i)/T_1} \left[P_{eq}(t_i) + \frac{1}{T_1} \int_{t_i}^t e^{(u-t_i)/T_1} P_{eq}(u) du \right]$$
(10)

where

$$P_{eq}(t) = \chi \frac{H_1^2 + \alpha t (H_0 + \alpha t)}{\sqrt{H_1^2 + \alpha^2 t^2}}$$

 P_{eff} does not have an analytic solution

But we can for an analytic solution by expanding P_{eff} in three regions

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Analytic Solution: Case 1

• $t_i \leq t < t_a, \alpha |t| >> H_1$

Expand square root

$$\frac{H_1^2 + H_0 \alpha u + \alpha^2 u^2}{\alpha u \sqrt{1 + \frac{H_1^2}{\alpha^2 u^2}}} \simeq -\left(H_0 + \alpha u + \frac{H_1^2}{2\alpha u}\right)$$
(11)

Solution in this region:

$$P_{eff}(t) \simeq e^{-(t-t_i)/T_1} \left(P_{eq}(t_i) - \frac{\chi}{T_1} \int_{t_i}^t e^{(u-t_i)/T_1} \left(H_0 + \alpha u + \frac{H_1^2}{2\alpha u} \right) du \right)$$
(12)

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Analytic Solution: Case 2

- $t_a \leq t < t_b, |u| \ll T_1$
- Expand exponential

$$e^{(u-t_i)/T_1} \simeq e^{-t_i/T_1} \left(1 + \frac{u}{T_1} + \frac{u^2}{2T_1^2} \right)$$
 (13)

Solution in this region:

$$\begin{split} P_{eff}(t) &\simeq e^{-(t-t_i)/T_1} [P_{eq}(t_i) - \frac{\chi}{T_1} \int_{t_i}^{t_a} e^{(u-t_i)/T_1} \left(H_0 + \alpha u\right) du \\ &+ \frac{\chi}{T_1} e^{-t_i/T_1} \int_{t_a}^t \left(1 + \frac{u}{T_1} + \frac{u^2}{2T_1^2}\right) \frac{H_1^2 + H_0 \alpha u + \alpha^2 u^2}{\sqrt{H_1^2 + \alpha^2 u^2}} du] \end{split}$$

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Analytic Solution: Case 3

• $t_b \leq t < t_f, \alpha |t| >> H_1$

Expand square root

$$\frac{H_1^2 + H_0 \alpha u + \alpha^2 u^2}{\sqrt{H_1^2 + \alpha^2 u^2}} \simeq \frac{H_0 \alpha u + \alpha^2 u^2}{\alpha u} \frac{1}{sqrt1 + \frac{H_1^2}{\alpha^2 u^2}} \simeq (H_0 + \alpha u)$$
(14)

Solution in this region:

$$\begin{split} P_{eff}(t) &\simeq e^{-(t-t_i)/T_1} [P_{eq}(t_i) - \frac{\chi}{T_1} \int_{t_i}^{t_a} e^{(u-t_i)/T_1} \left(H_0 + \alpha u\right) du \\ &+ \frac{\chi}{T_1} e^{-t_i/T_1} \int_{t_a}^{t_b} \left(1 + \frac{u}{T_1} + \frac{u^2}{2T_1^2}\right) \frac{H_1^2 + H_0 \alpha u + \alpha^2 u^2}{\sqrt{H_1^2 + \alpha^2 u^2}} du \\ &+ \frac{\chi}{T_1} \int_{t_b}^{t} e^{(u-t_i)/T_1} \left(H_0 + \alpha u\right) du] \end{split}$$

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Analytic Water NMR Fit Function

$$f(H) = a \frac{g(H - H_{res})}{g(0)} \frac{H_1}{\sqrt{[H - H_{res}]^2 + H_1^2}} + b [H - H_{res}] + c$$

with:

 $\begin{array}{l} \bullet \ t_i \leq t < t_a, \ H_{min} \leq H < H_a \\ \bullet \ g(x) = F_1(x) \\ \\ \bullet \ g(x) = F_2(x) \ \text{and} \ H_a \leq H_{res} < H_b \\ \bullet \ g(x) = F_2(x) \ \text{and} \ g(0) = F_2(0) \\ \\ \bullet \ t_b \leq t < t_f, \ H_b \leq H < H_max \\ \bullet \ g(x) = F_3(x) \end{array}$

where

•
$$H_a = (H_{res} + \alpha t_a)$$

- $H_b = (H_{res} + \alpha t_b)$
- F₁, F₂ and F₃ are analytic function of the Bloch equations in each expansion region

NMR Water X-Y Signals

6,189 sweeps



Figure: Presented are the sweep up and sweep down signals for the downstream and upstream coils. The Y lock in channel is 2 4 Matthew Posik (Temple University) 35 / 46

NMR Water Fit Residuals

6,189 sweeps



Figure: Presented are the sweep up and sweep down fit residuals for the downstream and upstream coils. The fit residual is defined as 100(data - fit)/peak.

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Water Calibration Constant

Extract absolute ³He polarization by calibrating ³He NMR to known proton polarization in water

$$P_{He} = c_w \frac{S_{He}}{n_{He} \Phi_{He}}$$

$$c_w = \frac{1}{S_w} \frac{G_w}{G_{He}} \frac{\mu_p}{\mu_{He}} n_p \Phi_w P_w$$

- $P_{He,W}$: Polarization for ³He and water
- $S_{He,W}$: NMR signal heights for ³He and water
- μ_{He,p}: Magnetic moment for ³He and protons
- $G_{He,W}$: Pre-amp gains for ³He and water
- $\Phi_{He,W}$: Magnetic Flux through pick up coils for ³He and water cells

NMR

Water Calibration

Water Calibration Constant: P_w

Water Polarization

$$c_w = \frac{1}{S_w} \frac{G_w}{G_{He}} \frac{\mu_p}{\mu_{He}} n_p \Phi_w \mathbf{P}_w$$

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Use Bloch equations to model water polarization

- Bloch equations sensitive
 - integration limits
 - water temperature

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P_w: Temperature Dependence

Table: Results of varying temperature on water polarization.

- Water cell RTD read outs show a spread of 2°C
- Use Block Equations to calculate P_w with a 2°C change

$T_w [^{\circ}C]$	Up Stream P_w [10 ⁻⁹]	Down Stream P_w [10 ⁻⁹]
21.25	6.58391	7.77057
23.25	6.53996	7.71805
19.25	6.6306	7.82387

Pw uncertainty from temperature: 1.4%



Figure: Top plot shows the water cell RTD read outs. The bottom plot shows the average of the water cell RTD read outs.

P_w Bloch Integration Dependence

Table: Results of varying integration limits on water polarization.

H_i [G]	H_f [G]	Up Sweep Value $ imes 10^{-9}$	Down Sweep Value $ imes 10^{-9}$
18	24	6.58391	7.77057
18	26	6.58391	7.92428
16	24	6.4652	7.777057

Table: Uncertainty results from Bloch equation.

Parameter	Value	Uncertainty Source [%]		
Up sweep P_w	6.58391	1.80		
Down sweep P_w	7.77057	1.98		

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Water Calibration Constant: Error Budget

Table: Parameters used to compute the water constant. Highlighted parameters still need to be calculated.

Parameter	Description	Value	Units	Uncertainty [%]	Source
S_w	Sweep Up	1.849	μV	0.349	Fit
	Sweep Down	2.280	μV	0.290	Fit
G_w	Gain of pick-up coil pre-amp. for water cell	20	-	-	-
G_{He}	Gain of pick-up coil pre-amp. for ³ He cell	1	-	-	-
μ_p/μ_{He}	-	1.3127	-	neg.	-
n_p	at 22°C	2482	amg	0.1	see M. Romalis thesis
Φ_w	Upstream Magnetic flux	-	cm ²	-	-
Φ_w	Downstream Magnetic flux	-	cm ²	-	-
P_w	Sweep Up	6.58391	$\times 10^{-9}$	2.28	Model of Bloch Eqs + T
	Sweep Down	7.77057	$\times 10^{-9}$	2.43	Model of Bloch Eqs + 7

Total Uncertainty:

- Up Sweep: 2.31%
- Down Sweep: 2.45%

³He NMR measurements were done about every 4 hours
 ³He signal:

$$S_{He} = \sqrt{S_x^2 + S_y^2}$$

³He

NMR

Signal Fit Function:

$$f(H) = a \frac{H_1}{\sqrt{[H - H_{res}]^2 + H_1^2}} + b \left[H - H_{res}\right]^2 + c \left[H - H_{res}\right] + d$$
(15)

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NMR ³He

3He NMR Signal Fits



Figure: Example plot of ³He NMR signals and fits.

NMR

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3He NMR Signal Heights Over Elapsed Time



Figure: ³He heights as a function of elapsed time.

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What's Next

• EPR

- Apply Diffusion model to EPR polarizations
- Calibrate NMR to EPR polarizations
- Interpolate between NMR measurements

• NMR

- Refine NMR measurement list
- Compute flux in ³He and water cells
- Calibrate NMR using water constant
- Interpolate between NMR measurements