Drift chamber orientation in hcana

Jure Bericic

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Abstract

A closer look is taken at the parameters that describe the spatial orientation of the drift chambers within the hcana framework.

1 Introduction

The tracking algorithm used in the hcana is based on the same procedure as is described in [1]. This paper, however, focuses only on the transformation from the local coordinate system to the transport coordinate system, where the local system is defined by the orientation of the wires within the drift chambers and the transport system follows the ideal particle track through the spectrometer. Ignoring offsets, this transformation is described by [1]:

$$z = \psi\,[\sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta)\sin(\gamma)] - \chi\,[\cos(\alpha)\sin(\beta) - \sin(\alpha)\cos(\beta)\sin(\gamma)] \tag{1a}$$

$$x = \psi\,[\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)\sin(\gamma)] - \chi\,[\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\sin(\gamma)] \tag{1b}$$

$$y = \psi\cos(\alpha)\cos(\gamma) + \chi\sin(\alpha)\cos(\gamma) \tag{1c}$$

The x, y, and z are coordinates in the transport system, and ψ and χ are in the local system of each wire-plane. ψ is also called the measuring axis, which is perpendicular to the wire directions, while χ is oriented parallel to the wires. In the transport system, the $+z$ points along the ideal particle track, that is from dipole towards the shower. The $+y$ is pointing beam-left, and $+x$ such that it forms a positive coordinate system, i.e., towards the hall floor.

The transformation can be broken into three distinct steps, one for each angle:

$$\begin{bmatrix}\psi\\ \chi\end{bmatrix} \Leftrightarrow \begin{bmatrix}x'''\\ y'''\\ z'''\end{bmatrix} \overset{\alpha}{\leftrightarrow} \begin{bmatrix}x''\\ y''\\ z''\end{bmatrix} \overset{\gamma}{\leftrightarrow} \begin{bmatrix}x'\\ y'\\ z'\end{bmatrix} \overset{\beta}{\leftrightarrow} \begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{2}$$

The next sections describe each of these steps in more detail.

2 Alpha angle: $\beta = \gamma = 0$

The transformation from the Equation 1 now becomes:

$$z = 0 \tag{3a}$$

$$x = \psi\sin(\alpha) - \chi\cos(\alpha) \tag{3b}$$

$$y = \psi\cos(\alpha) + \chi\sin(\alpha) \tag{3c}$$

and is shown in Figure 1. This transformation describes a rotation of the local coordinate system about the z-axis, or in other words, it is connected to the orientation of the wires, since the ψ and χ are defined by their orientation.

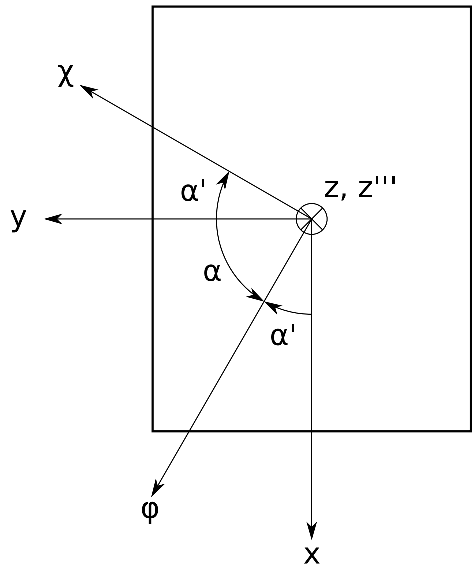

Figure 1: The depiction of transformations from Equations 3 and 4 - front view. They describe a rotation about the local z'''-axis and with that the orientation of the wires within the plane. As such, angles α and α' are connected to the roll of the chambers. The wires are parallel to the χ-axis and perpendicular to the ψ-axis. The z-axis is oriented into the plane.

However, the angle α has an unfortunate definition. When $\alpha = 0$, the ψ-axis coincides with the y-axis, and the χ-axis is anti-parallel to the x-axis. This describes the case, where wires are strung along the x-direction and measure the y-coordinate. The α measures the negative rotation of the ψ-axis from the y-axis about the z'''-axis.

I find it easier to think in terms of the $\alpha' = 90° - \alpha$, which describes the positive rotation of the ψ-axis from the x-axis about the local z'''-axis:

$$\alpha' = 90° - \alpha \quad \Rightarrow \quad \Delta\alpha' = -\Delta\alpha \tag{4a}$$
$$z = 0 \tag{4b}$$
$$x = \psi\cos(\alpha) - \chi\sin(\alpha) \tag{4c}$$
$$y = \psi\sin(\alpha) + \chi\cos(\alpha) \tag{4d}$$

3 Gamma angle: $\alpha = \beta = 0$

The transformation from the Equation 1 considering only γ-angle can be written as:

$$z = \psi\sin(\gamma) \tag{5a}$$
$$x = -\chi \tag{5b}$$
$$y = \psi\cos(\gamma) \tag{5c}$$
$$\tag{5d}$$

These equations describe a rotation about the x-axis. For $\alpha = 0$, the ψ- and y-axis align, and the χ-axis is anti-parallel to the x-axis.

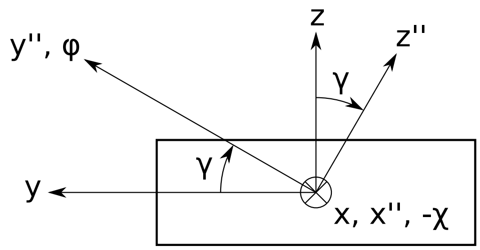

Figure 2: The depiction of transformations from Equations 5 and 6 - top view. They describe a positive rotation about the local x''-axis for angle γ, or yaw of the chambers. The x-axis is oriented into the plane and the χ-axis is pointing out of the plane.

To make things a bit easier to see, the ψ- and χ-axis can be relabeled like:

$$x'' = -\chi \tag{6a}$$
$$y'' = \psi \tag{6b}$$
$$z = y'' \sin(\gamma) \tag{6c}$$
$$x = x'' \tag{6d}$$
$$y = y'' \cos(\gamma) \tag{6e}$$

From here it can be seen, that γ describes a positive rotation of the z''-axis about the x''-axis or yaw of the chambers.

4 Beta angle: $\gamma = \alpha = 0$

The transformation from the Equation 1 for this combination of angles are:

$$z = -\chi \sin(\beta) \tag{7a}$$
$$x = -\chi \cos(\beta) \tag{7b}$$
$$y = \psi \tag{7c}$$

which describe a rotation about the y-axis.

By making a relabeling similar to 6, the transformations can be written more clearly as:

$$x' = -\chi \tag{8a}$$
$$y' = \psi \tag{8b}$$
$$z = x' \sin(\beta) \tag{8c}$$
$$x = x' \cos(\beta) \tag{8d}$$
$$y = \psi \tag{8e}$$

These equations describe a negative rotation for angle β about the local y'-axis, or pitch of the drift chambers.

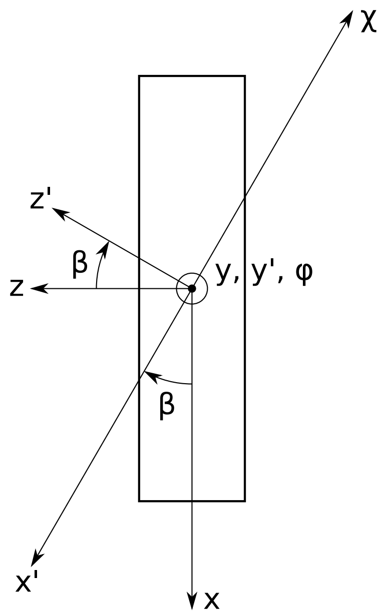

Figure 3: The depiction of transformations from Equations 7 and 8 - side view. They describe a negative rotation for angle β about the local y'-axis, or pitch of the drift chambers. The y'- and ψ-axis are pointing out of the plane.

5 Conclusion

As shown in previous sections, the angles α, β, and γ can be seen as a roll, pitch, and yaw of the drift chambers, respectively. The α-angle describes a negative rotation about the local z'''-axis. Unfortunately, its definition is a bit couter-intuitive, since for $\alpha = 0$ the wires are parallel to the x-axis and the local coordinate system does not coincide with the global one, but is rotated for $90°$. However, its complementary angle α' describes a positive rotation about the local z'''-axis, with $\alpha' = 0$ meaning the wires are parallel to the y-axis. The γ-angle describes a positive rotation about the local x''-axis. And the β-angle describes the negative rotation about the local y'-axis.

With this, the overview of the transformation from Equation 2 can be rewritten as:

$$\begin{bmatrix}\psi\\ \chi\end{bmatrix} \Leftrightarrow \begin{bmatrix}x'''\\ y'''\\ z'''=z''\end{bmatrix} \underset{-}{\overset{\alpha}{\longleftrightarrow}} \begin{bmatrix}x''=x'\\ y''\\ z''\end{bmatrix} \underset{+}{\overset{\gamma}{\longleftrightarrow}} \begin{bmatrix}x'\\ y'=y\\ z'\end{bmatrix} \underset{-}{\overset{\beta}{\longleftrightarrow}} \begin{bmatrix}x\\ y\\ z\end{bmatrix} \tag{9}$$

References

[1] D. F. Geesaman, *Tracking in the SOS Spectrometer*, 14 September 1993. https://hallcweb.jlab.org/DocDB/0008/000812/001/SOS_Tracking.pdf (23 March 2017)