Thoughts about Current Redistribution and MQE

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After my last note on MQE, Paul quite rightly pointed out that the critical current of the conductor is ~ 3 x the operating current. So if the outer layer of cable quenches, the current could simply redistribute to the inner layer. I agreed with this but worried about how long the redistribution would take. Here I make a simple estimate.

Fig 1 shows a typical Mathcad calculation for a cable with solder a gap. The energy spike finishes at 50μs, shown by the solid red curve in Fig 1(a), after which the heat diffuses along the conductor and temperature falls until it reaches a minimum (blue line) when the conductor 'decides' whether to quench or not. In this case it quenches, but with a smaller pulse it would have continued to cool.

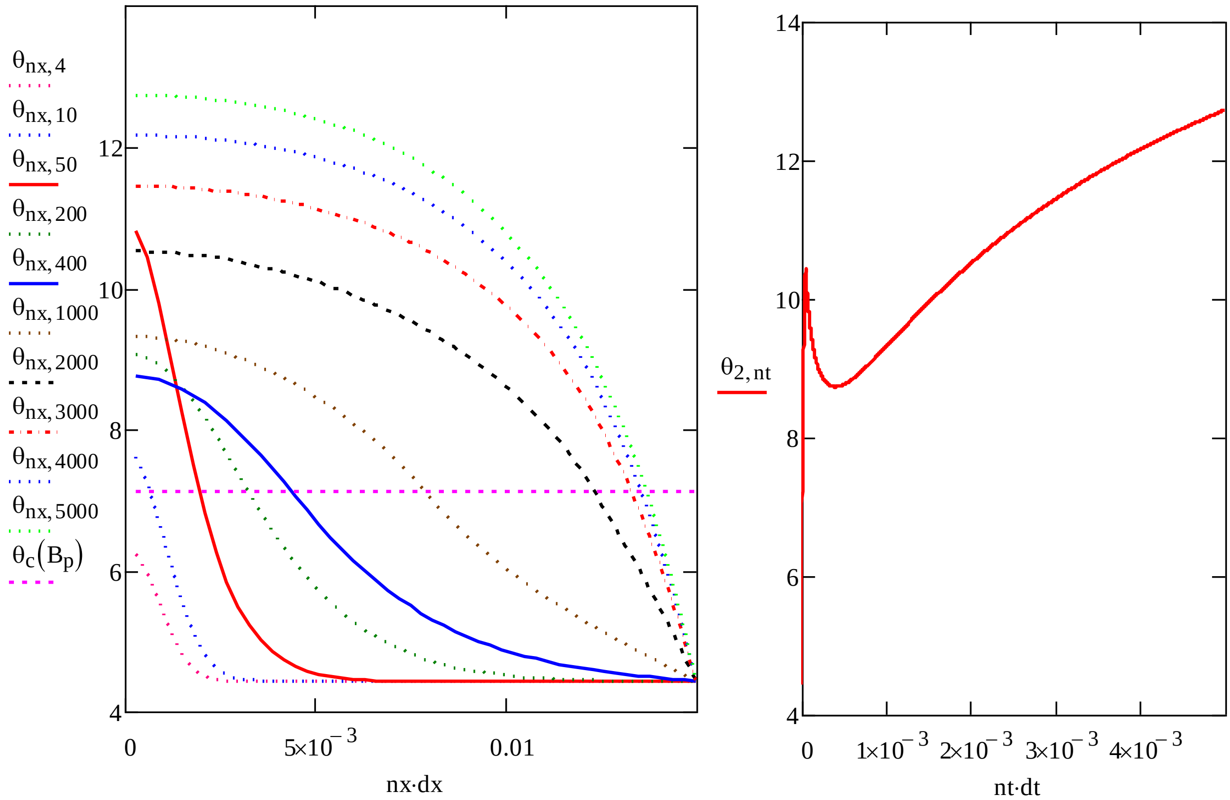


Fig 1: Temperature evolution after an energy spike of 1.6mJ, (a) as a function of distance (m) along conductor (nx.dx) at different times and (b) temperature at centre as function of time.

The redistribution of current between outer and inner faces of the cable may be thought of as the decay of a circulating current superposed on a steady state current. To calculate the decay time of this circulating current, in the Appendix I estimate the self inductance between the inner and outer faces of the cable to be 3.8×10^{-9}H. For the blue curve in Fig 1(a), the resistance is 1μΩ, giving a time constant of 3.8msec. This time is comparable with the thermal times shown in Fig 1.

Thus we cannot say that the current redistributes instantly to match the resistance, but neither can we say that it remains fixed as assumed in my calculation. The truth lies somewhere between the two. So, although I still believe that the MQE will be significantly reduced by gaps in the soldering, I must admit that my estimate of this reduction in SJD9 is too strong.

Appendix 1: Calculating the Inductance and Time Constant

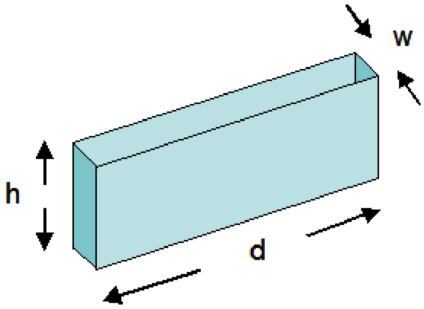
JLab SHMS Dipole MQE: Estimate Magnetic time constant revisited 18 Oct 12

series C cable with gap on centre line

1) Self Inductance of Loop formed by layers of cable with gap in centre

assume h >>w so field in gap $B = \mu_o \cdot \frac{I}{h}$

flux linkage $\varphi = B \cdot w \cdot d = \mu_o \cdot \frac{I}{h} \cdot w \cdot d = L \cdot I$ $L = \mu_o \cdot \frac{w \cdot d}{h}$



2) Winding Data $I_{op} := 3419.4 amp$ $B_p := 5.45T$ $\theta_o := 4.42K$ $\mu_o := 4 \cdot \pi \cdot 10^{-7} \cdot henry \cdot m^{-1}$

spec Ic at 5T and 6T $I_{cd5} := 12333A$ $I_{cd6} := 9875A$ take Ic at 5.45T and 4.42K from General Jc.xls $I_c := 10450A$

channel width $w_{ch} := 0.486in = 12.344 \cdot mm$ wire dia $d_w := 0.65 \cdot mm$ number of wires $N_w := 36$

$mat := 1.6$ wire area $A_w := N_w \cdot \frac{\pi}{4} \cdot d_w^2 = 11.946 \cdot mm^2$ wire copper area $A_{wcu} := A_w \cdot \frac{mat}{1 + mat} = 7.351 \cdot mm^2$

$A_{cu} := \frac{A_{wcu}}{2} = 3.676 \cdot mm^2$

3) Superconductor critical temperature from General Jc.xls,

$C_o := 21.77 \cdot K$ $C_1 := -0.278 \cdot K \cdot tesla^{-1}$ $C_2 := -0.015 \cdot K \cdot tesla^{-2}$ $n := 0.032$ $B_{c2} := 14.05 \cdot tesla$

$C_3 := -13.60 \cdot K$ $\theta_c(B) := C_o + C_1 \cdot B + C_2 \cdot B^2 + \frac{C_3}{\left[\left(B_{c2} - B\right) \cdot T^{-1}\right]^n}$ $\theta_c\left(B_p\right) = 7.114K$

4) Copper resistivity magnetoresistance from copper magres Fickett.xls $m_B := 4 \cdot 10^{-11} ohm \cdot m \cdot T^{-1}$ $RRR := 100$

$\rho_{RT} := 1.678 \cdot 10^{-8} ohm \cdot m$ $\rho_o := \frac{\rho_{RT}}{RRR} = 1.678 \times 10^{-10} \cdot ohm \cdot m$ $\rho_{cB} := \rho_o + m_B \cdot B_p = 3.858 \times 10^{-10} \cdot ohm \cdot m$

5 Time Constant

assume a 6mm section of cable goes completely resistive $l_r := 2 \cdot 4.8mm$ $R_g := \rho_{cB} \cdot \frac{l_r}{A_{cu}} = 1.008 \times 10^{-6} \cdot ohm$

unsoldered gap $w_g := 0.1mm$ $w := d_w + w_g$ $h := w_{ch}$ $d := 50mm$

$L_g := \frac{\mu_o}{h} \cdot w \cdot d = 3.817 \times 10^{-9} \cdot H$ $\tau_g := \frac{L_g}{R_g} = 3.789 \times 10^{-3} s$