Updated Quench Calculations for the SHMS Dipole

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1. Introduction

Here I update my report SJ2 (7th Dec 2010) to incorporate the conductor with rolled stabilizer and also the revised coil length.

In this version 2, I calculate for copper with RRR=100 and RRR=50. In addition, I fix an error in the version 1 data for copper resistivity.

2. Data

QUENCH works in terms of a unit cell and all parameters are averaged over the unit cell. Appendix 1 presents the detailed calculation of parameters and Table 1 summarizes the main numbers. Some changes come from the change in conductor and coil, but others come from my change of mind, for example averaging the field over the top sector. None of the changes are large

Table 1: Parameters used in the Calculation

unit cell area	72.7mm^2		magnet inductance	2.7H
fraction of NbTi	0.063		operating current	3419A
fraction of Cu	0.706		mean field on winding	5T
fraction of G10	0.198		critical current in mean field	12333A
fraction of solder	0.033		resistivity ratio of copper	100 or 50
resistance ratio of copper	70		critical temperature in mean field	7.3K
azimuthal width of winding	95.6mm		starting temperature	4.42K
radial width of coil	118.7mm		longitudinal propagation velocity	3.26ms^{-1}
perimeter of coil	6.0m		ratio transverse / longitudinal velocity	0.038

3. Results

Fig 2 shows the current decay and temperature rise for a 'natural' quench, with no protection resistor, when the quench starts at the point shown in Fig 1. It may be seen that, although the current decays slightly quicker with RRR=50, the maximum temperature is unchanged at 161K.

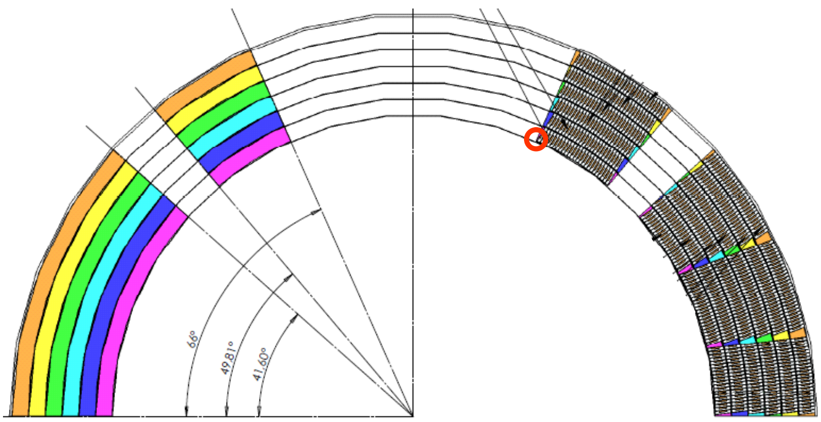


Fig 1: Quench initiation point shown by red circle.

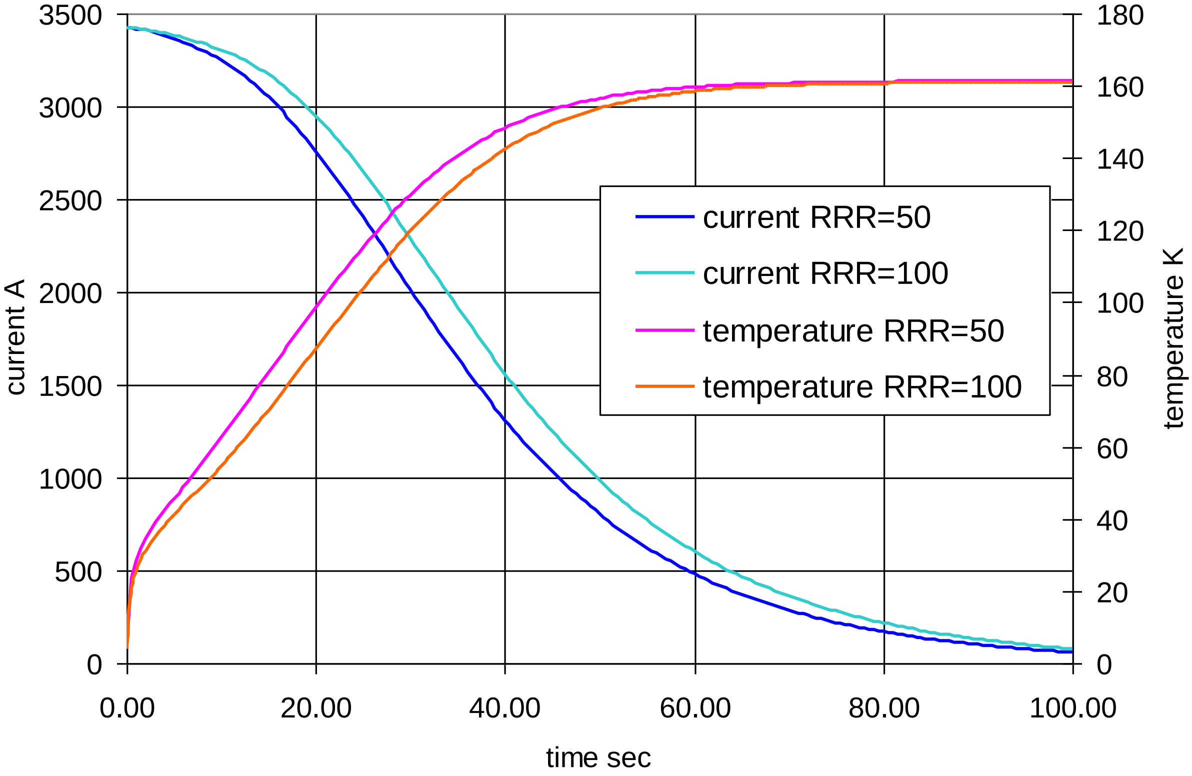


Fig 2: Current decay and temperature rise after a natural quench.

Fig 3 shows the internal resistive voltage for a natural quench – slightly greater with RRR=50.

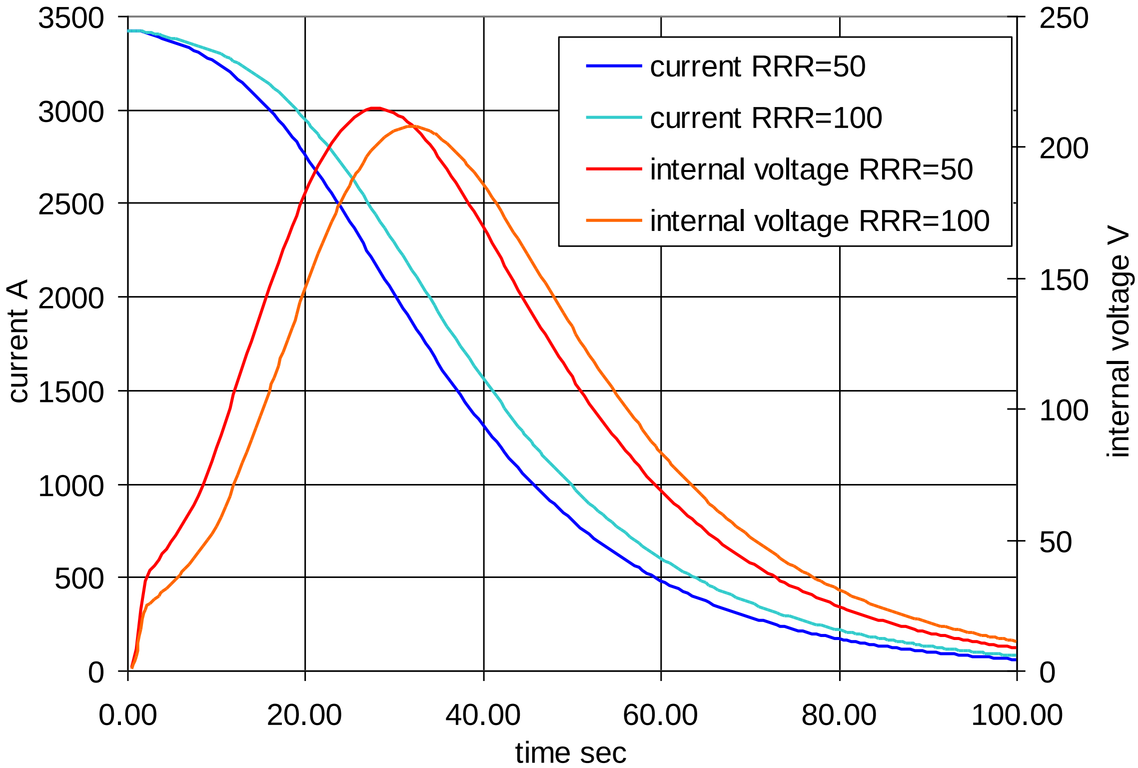


Fig 3: Current decay and internal resistive voltage after a natural quench.

Fig 4 shows the current decay and temperature when a 0.075Ω protection resistor is switched into the circuit immediately after a quench. The peak temperature is reduced to 74K, with RRR=100, or 83K with RRR=50K.

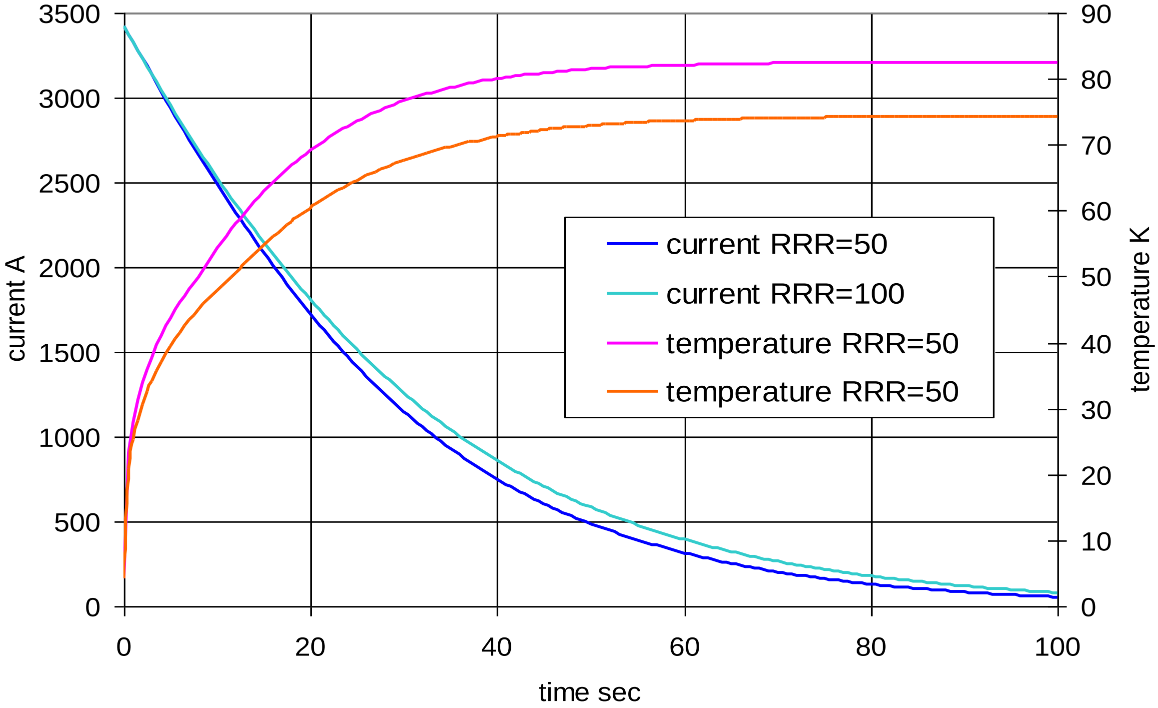


Fig 4: Current decay and temperature rise after a quench with 0.075Ω protection resistor

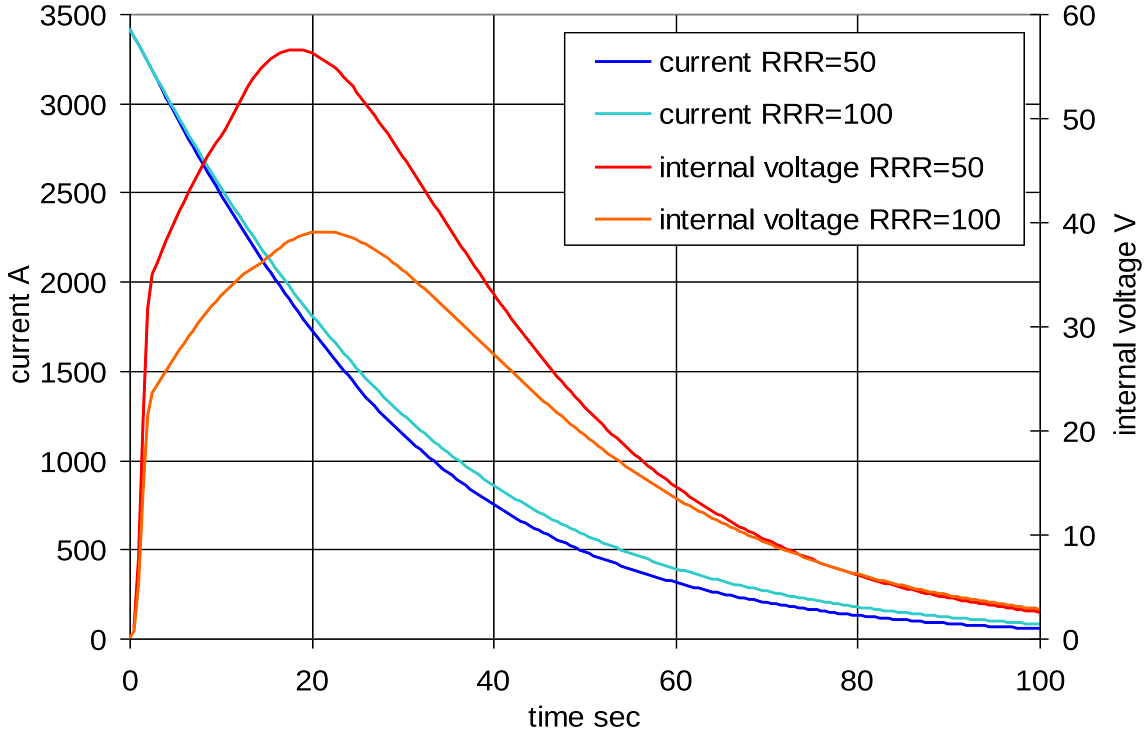


Fig 5: Current decay and internal voltage after a quench with 0.075Ω protection resistor

4. Conclusion

Hardening the conductor will not make a significant difference to the quench behaviour of the dipole.

Appendix 1: Mathcad calculation of input data.

Quench Input Data for JLab Dipole with rolled conductor change Bm, RRR 3 May 12

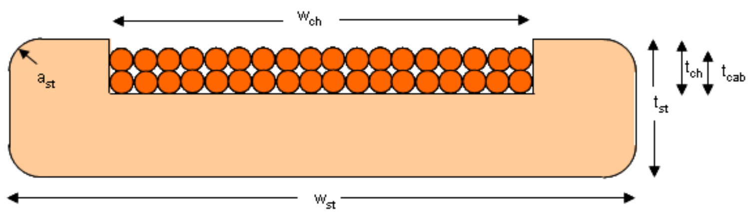
1) Winding Data $I_{op} := 3419.4amp$ $B_m := 4.5T$ $\theta_o := 4.42K$ $\mu_o := 4\cdot\pi\cdot10^{-7}\cdot henry\cdot m^{-1}$

spec Ic at 5T and 6T

$I_{cd5} := 12333A$ $I_{cd6} := 9875A$

take Ic at 4.5T and 4.42K from General Jc.xls

$I_c := 12801A$



2) Conductor Geometry

ignore corner radii to make quench prog fit

stabilizer width $w_{st} := 18.73\cdot mm$ stabilizer thickness $t_{st} := 3.118mm$

channel width $w_{ch} := 0.486in = 12.344\cdot mm$ $mat := 1.6$

channel thickness $t_{ch1} := 0.043in = 1.092\cdot mm$ $t_{ch2} := 0.051in = 1.295\cdot mm$ $t_{ch} := 0.5\cdot(t_{ch1} + t_{ch2}) = 1.194\cdot mm$

wire dia $d_w := 0.65\cdot mm$ number of wires $N_w := 36$

cable size from Paul Berindza email 3 Dec $w_{cab} := 11.68mm$ mean thick's $t_{cab} := \frac{(1.271mm + 1.0530mm)}{2} = 1.162\cdot mm$ channel thickness $t_{ch} := t_{cab}$

wire area $A_w := N_w\cdot\frac{\pi}{4}\cdot d_w^2 = 11.946\cdot mm^2$ $\lambda_{cab} := \frac{A_w}{w_{cab}\cdot t_{cab}} = 0.88$ channel occupied by cable $A_{ch} := w_{ch}\cdot t_{ch} = 14.344\cdot mm^2$

wire copper area $A_{wcu} := A_w\cdot\frac{mat}{1 + mat} = 7.351\cdot mm^2$ wire NbTi area $A_{nt} := A_w\cdot\frac{1}{1 + mat} = 4.595\cdot mm^2$

solder area $A_{so} := A_{ch} - A_w = 2.398\cdot mm^2$ conductor area $A_{co} := w_{st}\cdot t_{st} = 58.4\cdot mm^2$

interlayer insulation $t_{il} := 0.5mm$ insulated conductor width $w_{ic} := 19.29mm$

width unit cell $w_u := w_{ic} + t_{il} = 19.79\cdot mm$ thickness $t_u := 3.678mm$ unit cell area $A_u := w_u\cdot t_u = 72.788\cdot mm^2$

insulation thickness $t_i := 0.5\cdot(t_u - t_{st}) = 0.28\cdot mm$ insulation area $A_i := A_u - A_{co}$

stabilizer area $A_{st} := A_{co} - w_{ch}\cdot t_{ch} = 44.056\cdot mm^2$ mat for quench $mat_q := \frac{A_{st} + A_{wcu}}{A_{nt}} = 11.189$

over unit cell $\lambda_{st} := \frac{A_{st}}{A_u} = 0.605$ $\lambda_{wcu} := \frac{A_{wcu}}{A_u} = 0.101$ $\lambda_{Cu} := \lambda_{st} + \lambda_{wcu} = 0.706$ $\lambda_{so} := \frac{A_{so}}{A_u} = 0.033$

$\lambda_{nt} := \frac{A_{nt}}{A_u} = 0.063$ $\lambda_i := \frac{A_i}{A_u} = 0.198$ check $\lambda_{Cu} + \lambda_{nt} + \lambda_{so} + \lambda_i = 1$ $J_{op} := \frac{I_{op}}{A_u} = 46.978\cdot A\cdot mm^{-2}$

2) Geometry for Quench Data sheath is under insulation, so inner and outer widths are stabilizer and unit

put on the solder as a thin strip 'sheath' $A_{so} = 2.398\cdot mm^2$ $t_{so} := \frac{A_{so}}{w_{st}} = 0.128\cdot mm$

thicknes of condr without solder $t_{con} := t_{st} - t_{so} = 0.299\cdot cm$ $t_{con}\cdot w_{st} + A_{so} = 58.4\cdot mm^2$ check

inner width of sheath radial $w_{isr} := t_{con} = 0.299\cdot cm$ inner width of sheath axial $w_{isa} := w_{st} = 1.873\cdot cm$

outer width of sheath radial $w_{osr} := t_{st} = 0.312\cdot cm$ outer width of sheath axial $w_{osa} := w_{st} = 1.873\cdot cm$

3) Superconductor critical temperature from General Jc.xls, mean field $B_m := 5T$

$C_o := 21.77\cdot K$ $C_1 := -0.278\cdot K\cdot tesla^{-1}$ $C_2 := -0.015\cdot K\cdot tesla^{-2}$ $n := 0.032$ $B_{c2} := 14.05\cdot tesla$

$C_3 := -13.60 \cdot K$ $\theta_c(B) := C_o + C_1 \cdot B + C_2 \cdot B^2 + \dfrac{C_3}{\left[\left(B_{c2} - B\right)\cdot T^{-1}\right]^n}$ $\theta_c(B_m) = 7.331K$

4) Current Sharing $\theta_g := \theta_c(B_m) - \left(\theta_c(B_m) - \theta_o\right)\cdot\dfrac{I_{op}}{I_c}$ $\theta_g = 6.553K$ $\theta_s := \dfrac{\theta_c(B_m) + \theta_g}{2}$ $\theta_s = 6.942K$

$\theta_t := \dfrac{\theta_s + \theta_c(B_m)}{2}$ $\theta_t = 7.136K$

5) Resistivity magnetoresistance from Copper magres Fickett.xls resistivity NbTi $\rho_{nt} := 6\cdot 10^{-7} \cdot ohm \cdot m$

$\rho_{RT} := 1.69 \cdot 10^{-8} \cdot ohm \cdot m$ $m_{\rho c} := 4 \cdot 10^{-11} ohm \cdot m \cdot T^{-1}$

stabilizer $RRR_{st} := 70$ $\rho_{st}(B) := \dfrac{\rho_{RT}}{RRR_{st}} + m_{\rho c} \cdot B$ $\rho_{st}(B_m) = 4.414 \times 10^{-10} \cdot ohm \cdot m$

wire $RRR_w := 150$ $\rho_{wcu}(B) := \dfrac{\rho_{RT}}{RRR_w} + m_{\rho c} \cdot B$ $\rho_{wcu}(B_m) = 3.127 \times 10^{-10} \cdot ohm \cdot m$

$$\frac{1}{R_u} = \frac{1}{R_{NbTi}} + \frac{1}{R_{ch}} + \frac{1}{R_w} = \frac{A_{NbTi}}{\rho_{NbTi} \cdot L} + \frac{A_{ch}}{\rho_{ch} \cdot L} + \frac{A_{wcu}}{\rho_{wcu} \cdot L}$$

$$\rho_u = R_u \cdot \frac{A_u}{L} = \frac{1}{L} \cdot \frac{A_u}{\frac{A_{NbTi}}{\rho_{NbTi}} \cdot L + \frac{A_{ch}}{\rho_{ch}} \cdot L + \frac{A_{wcu}}{\rho_{wcu}} \cdot L}$$

$$\rho_u := \frac{1}{\frac{\lambda_{nt}}{\rho_{nt}} + \frac{\lambda_{st}}{\rho_{st}(B_m)} + \frac{\lambda_{wcu}}{\rho_{wcu}(B_m)}}$$ $\rho_u = 5.902 \times 10^{-10} \cdot ohm \cdot m$

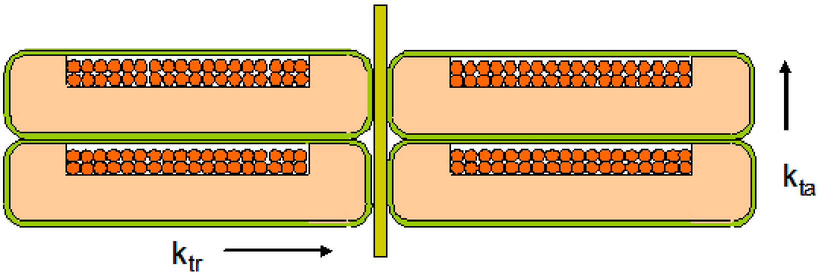
6) Longitudinal Thermal conductivity (over unit cell)

$L_o := 2.45 \cdot 10^{-8} \cdot watt \cdot ohm \cdot K^{-2}$ $k(\theta)\cdot\rho(\theta) = L_o \cdot \theta$ $k(\theta) = \dfrac{L_o \cdot \theta}{\rho(\theta)}$

over the unit cell $k_u(\theta) := \dfrac{L_o \cdot \theta}{\rho_u}$ $k_u(\theta_o) = 183.473 m^{-1} \cdot K^{-1} \cdot watt$ $k_u(\theta_t) = 296.225 m \cdot kg \cdot K^{-1} \cdot s^{-3}$

7) Transverse Thermal Conductivity

for insulation take John Ross G10 at transition temperature



$k_i(\theta) = \kappa_i \cdot \theta^{n_i}$

$n_i := 0.8$ $\kappa_i := 0.017 \cdot watt \cdot m^{-1} \cdot K^{-1}$

$k_i := \kappa_i \cdot \left(\theta_t \cdot K^{-1}\right)^{n_i} = 0.082 \cdot watt \cdot m^{-1} \cdot K^{-1}$

corner radius of strip $a_{st} := 0.25 mm$

a) Radial conduction per unit length of unit cell $Q_{r'} = k_i \cdot \left(t_{st} - 2 \cdot a_{st}\right) \cdot \dfrac{\Delta\theta}{2 \cdot t_i + t_{il}}$ $Q'_r = k_{tr} \cdot t_u \cdot \dfrac{\Delta\theta}{w_u}$

$$k_{tr} = k_i \cdot \left(t_{st} - 2 \cdot a_{st}\right) \cdot \frac{\Delta\theta}{2 \cdot t_i + t_{il}} \cdot \frac{1}{t_u} \cdot \frac{w_u}{\Delta\theta} = k_i \cdot \frac{\left(t_{st} - 2 \cdot a_{st}\right)}{\left(2 \cdot t_i + t_{il}\right)} \cdot \frac{w_u}{t_u}$$

$$k_{tr} := k_i \cdot \frac{\left(t_{st} - 2 \cdot a_{st}\right)}{\left(2 \cdot t_i + t_{il}\right)} \cdot \frac{w_u}{t_u} = 1.088 \cdot watt \cdot m^{-1} \cdot K^{-1}$$

b) Axial conduction $Q_{a'} = k_i \cdot \left(w_{st} - 2 \cdot a_{st}\right) \cdot \dfrac{\Delta\theta}{2 \cdot t_i}$ $Q'_a = k_{ta} \cdot w_u \cdot \dfrac{\Delta\theta}{t_u}$

$$k_{ta} = k_i \cdot \left(w_{st} - 2 \cdot a_{st}\right) \cdot \frac{\Delta\theta}{2 \cdot t_i} \cdot \frac{1}{w_u} \cdot \frac{t_u}{\Delta\theta} = k_i \cdot \frac{\left(w_{st} - 2 \cdot a_{st}\right)}{2 \cdot t_i} \cdot \frac{t_u}{w_u}$$

$$k_{ta} := k_i \cdot \frac{\left(w_{st} - 2 \cdot a_{st}\right)}{2 \cdot t_i} \cdot \frac{t_u}{w_u} = 0.495 \cdot watt \cdot m^{-1} \cdot K^{-1}$$

take the mean $k_t := \dfrac{k_{tr} + k_{ta}}{2} = 0.792 \cdot watt \cdot m^{-1} \cdot K^{-1}$

8) Specific Heat use standard quench form $C = Cf\cdot\theta^{c} + D\cdot\theta^{d}$ no units

a) Copper from Cu spec heat.xls $Cf_c := 0.01149$ $c_c := 1.07$ $D_c := 0.00046$ $d_c := 3.21$

$C_{Cu}(\theta) := Cf_c\cdot\theta^{c_c} + D_c\cdot\theta^{d_c}$ $C_{Cu}(4.) = 0.09$ $\gamma_{Cu} := 8.89\cdot10^{3}$ $\gamma_{Cu}\cdot C_{Cu}(7) = 2.93\times10^{3}$

b) NbTi below ?c from NbTi spec heat.xls $Cf_n := -0.0976$ $c_n := 1.1$ $D_n := 0.0540$ $d_n := 2.17$

$C_{nt}(\theta) := Cf_n\cdot\theta^{c_n} + D_n\cdot\theta^{d_n}$ $C_{nt}(4.) = 0.645$ $\gamma_{nt} := 6.2\cdot10^{3}$ $\gamma_{nt}\cdot C_{nt}(7) = 1.769\times10^{4}$

c) Solder $Cf_{so} := 3.91\cdot10^{-2}$ $c_{so} := 0.7$ $D_{so} := 9.38\cdot10^{-5}$ $d_{so} := 5$ $\gamma_{so} := 10.49\cdot10^{3}$

$C_{so}(\theta) := Cf_{so}\cdot\theta^{c_{so}} + D_{so}\cdot\theta^{d_{so}}$ $C_{so}(4) = 0.199$ $\gamma_{so}\cdot C_{so}(7) = 1.814\times10^{4}$

d) G10 from G10 spec heat.xls $C_g := 0.0225$ $c_g := 3$ $D_g := -.00646$ $d_g := 3.263$

$C_{G10}(\theta) := C_g\cdot\theta^{c_g} + D_g\cdot\theta^{d_g}$ $C_{G10}(4) = 0.845$ $\gamma_{G10} := 1.8\cdot10^{3}$ $\gamma_{G10}C_{G10}(7) = 7.238\times10^{3}$

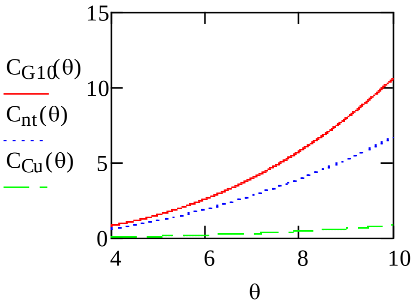
overall spec heat / volume of metal only $\gamma C_{um}(\theta) := (\lambda_{Cu})\cdot\gamma_{Cu}\cdot C_{Cu}(\theta) + \lambda_{nt}\cdot\gamma_{nt}\cdot C_{nt}(\theta) + \lambda_{so}\cdot\gamma_{so}\cdot C_{so}(\theta)$

overall spec heat / volume $\gamma C_u(\theta) := (\lambda_{Cu})\cdot\gamma_{Cu}\cdot C_{Cu}(\theta) + \lambda_{nt}\cdot\gamma_{nt}\cdot C_{nt}(\theta) + \lambda_{so}\cdot\gamma_{so}\cdot C_{so}(\theta) + \lambda_i\cdot\gamma_{G10}C_{G10}(\theta)$

enthalpy change metal only $\gamma H_{um} := \int_{\theta_s}^{\theta_c(B_m)} \gamma C_{um}(\theta\cdot K^{-1})\,d\theta$ $\gamma H_{um} = 1.557\times10^{3}K$ $\theta_s = 6.942K$

averaged spec heat metal only $\gamma C_{aum} := \frac{\gamma H_{um}}{\theta_c(B_m) - \theta_s}$ $\gamma C_{aum} = 4.005\times10^{3}$

$\gamma C_{um}(\theta_s\cdot K^{-1}) = 3.694\times10^{3}$



enthalpy change unit cell $\gamma H_u := \int_{\theta_s}^{\theta_c(B_m)} \gamma C_u(\theta\cdot K^{-1})\,d\theta$ $\gamma H_u = 2.144\times10^{3}K$

averaged spec heat unit cell $\gamma C_{au} := \frac{\gamma H_u}{\theta_c(B_m) - \theta_s}$ $\gamma C_{au} = 5.515\times10^{3}$ $\gamma C_u(\theta_s\cdot K^{-1}) = 5.092\times10^{3}$

9) Quench velocity

longitudinal velocity $v_{ad} = \frac{J}{\gamma\cdot C(\theta_s)}\cdot\sqrt{\frac{\rho\cdot k}{\theta_s - \theta_o}}$ $v_{ad} := \frac{J_{op}}{\gamma C_{aum}\cdot J\cdot m^{-3}\cdot K^{-1}}\cdot\sqrt{\frac{\rho_u\cdot k_u(\theta_t)}{\theta_t - \theta_o}}$ $v_{ad} = 2.976\cdot m\cdot s^{-1}$

transverse velocity $\alpha := \frac{\gamma C_{aum}}{\gamma C_{au}}\cdot\sqrt{\frac{k_t}{k_u(\theta_t)}}$ $\alpha = 0.038$

$J_{op} = 46.978\cdot A\cdot mm^{-2}$ $\gamma C_{aum} = 4.005\times10^{3}$ $\rho_u = 5.902\times10^{-10}\cdot ohm\cdot m$ $k_u(\theta_t) = 296.225m^{-2}\cdot watt\cdot m\cdot K^{-1}$

$\theta_t = 7.136K$

10) Coil geometry take dimensioons from 317111-spec-coil-Dipole JLab RevB.pdf

a) Assume quench starts in the upper sector

sector azimuthal width $w_{as} := 26 \cdot t_u = 95.628 \cdot mm$

sector radial height $h_{as} := 6 \cdot w_u = 118.74 \cdot mm$

total turns in sector $N_t := 6 \cdot 26 = 156$

area sector $A_s := w_{as} \cdot h_{as} = 1.135 \times 10^4 \cdot mm^2$

turns area $A_{s2} := N_t \cdot A_u = 1.135 \times 10^4 \cdot mm^2$

mean turns perimeter $p_u := \frac{(5.95 + 6.045)m}{2} = 5.998m$

equivalent solenoid inner radius $r_{us} := \frac{p_u}{2 \cdot \pi} = 95.453 \cdot cm$

area per turn $A_{tq} := \frac{w_{as} \cdot h_{as}}{N_t} = 72.788 \cdot mm^2$ $A_u = 72.788 \cdot mm^2$ check