MQE Calculations for the SHMS Dipole Conductor

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1. Introduction

The minimum quench energy MQE is defined as that pulse of energy, applied instantaneously at a point in a conductor, which is just enough to trigger a quench. For slightly smaller pulses, the conductor will recover. So the MQE may be taken to give some indication of the stability of the conductor against energy releases by motion etc in the magnet.

In my report SJ1 'Minimum Quench Energy of the SHMS Dipole Conductor', I calculated MQEs for the original conductor specification. Here I repeat those calculations for the conductor with thinner copper strip and look at two different hardnesses with RRR=50 and RRR=100.

2. Data

Table 1 lists the key parameters used, more detail in Appendix 1. The calculation assumes that the disturbances hits just one conductor and that the conductor is cooled by conduction along its length and by conduction through the insulation to neighbouring turns.

Table 1: Parameters used in the Calculation

stabilizer width	18.7mm		fraction of Cu	0.706
stabilizer thickness	3.12mm		fraction of NbTi	0.063
channel width	12.34mm		fraction of solder	0.033
mean channel thickness	1.194mm		fraction of G10	0.198
fraction of solder	0.033		resistivity ratio of copper	100 or 50
unit cell width	19.8mm		peak field	5.44T
unit cell thickness	3.68mm		starting temperature	4.42K
insulation thickness on broad face	2 × 0.28mm		critical temperature in peak field	7.11K
insulation thickness on edge	1.06mm			

3. Results

Table 2 lists the calculated MQEs.

Table 2: MQE for Different Conductors

conductor	MQE
original (report SJ1)	40mJ
rolled with RRR=100	28mJ
rolled with RRR=50	21mJ

4. Conclusion

I know of no way of saying whether any given MQE is OK or not. The best we can do is compare with other magnets whose performance is known. For a fair comparison on energy released by conductor motion, we need to scale each MQE by the magnetic force ($B \wedge I$) acting on that conductor. Table 3 presents a few numbers.

Table 3: Calculated MQEs for Some Existing Magnets.

magnet	JLD RRR=50	Grenoble Hybrid	MRI magnet	CLAS Torus	LHC dipole
peak field	5.45T	8.5T	6.09T	3.5T	8.4T
operating current	3419A	1330A	461A	3790A	11500A
MQE (mJ)	21mJ	1.735mJ	0.2515mJ	44.65mJ	1.5mJ
MQE scaled to JLD current and field (mJ)	21mJ	2.9mJ	1.7mJ	63mJ	0.29mJ

While not as stable as the 'gold standard' CLAS Torus, it seems that the dipole conductor, even with hardened copper, is substantially more stable than the rest. LHC dipoles have significant training and need very careful attention to preventing conductor motion if they are to achieve their operating fields – so their MQE is really too small. MRI magnets work pretty well, but with some training. In general however solenoids have less training problems than dipoles because they can support the electromagnetic forces without needing to react them against an external clamping structure. The Grenoble Hybrid is still a painful memory, but the killer problem here was not training but a shorted turn caused by conductor motion – so perhaps we should also learn something from that also.

In conclusion, my feeling is that, even with RRR=50, the MQE is probably high enough and that, for avoiding training, it is more important to worry about conductor motion.

Appendix 1: Details of the Calculation: showing a disturbance greater than MQE

JLab SHMS Dipole MQE revisited 3 May 12

Revised 12 May 10 because new Mathcad will not accept fractional exponents of variables with units. Make th dimensionless before raising to exponent.

Use generation term with resistive transition from MPZSING1 and NORMPROP6, eqs from TIME1W1 SI Units

$$C(\theta)\cdot\frac{d\theta}{dt} = \frac{d}{dx}\cdot\left(k(\theta)\cdot\frac{d\theta}{dx}\right) + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A} \qquad C(\theta)\cdot\frac{d\theta}{dt} = k(\theta)\cdot\frac{d^2\cdot\theta}{dx^2} + \frac{d\cdot k(\theta)}{dx}\cdot\frac{d\theta}{dx} + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A}$$

solve this one $$C(\theta)\cdot\frac{d\theta}{dt} = k(\theta)\cdot\frac{d^2\cdot\theta}{dx^2} + \frac{d\cdot k(\theta)}{d\theta}\cdot\left(\frac{d\theta}{dx}\right)^2 + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A}$$

where A is overall cross section, k is averaged over A, C and G are per unit volume and H is per unit area

boundary conditions — at x large we have θ = bath temperature $\theta = \theta_o$ — at x=0 have $\frac{d\theta}{dx} = 0$ for all t

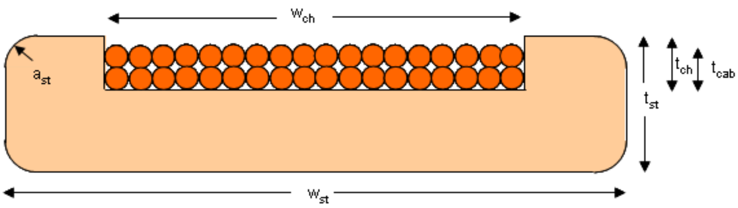
1) Winding Data $I_{op} := 3419.4\,amp$ $B_p := 5.45T$ $\theta_o := 4.42K$ $\mu_o := 4\cdot\pi\cdot10^{-7}\cdot henry\cdot m^{-1}$

spec Ic at 5T and 6T

$I_{cd5} := 12333A$ $I_{cd6} := 9875A$

take Ic at 5.45T and 4.42K from General Jc.xls

$I_c := 10450A$



2) Conductor Geometry same as quench input

ignore corner radii to make quench prog fit

stabilizer width $w_{st} := 18.73\cdot mm$ stabilizer thickness $t_{st} := 3.118mm$

channel width $w_{ch} := 0.486in = 12.344\cdot mm$ conductor area $A_{con} := w_{st}\cdot t_{st} = 58.4\cdot mm^2$

channel thickness $t_{ch1} := 0.043in = 1.092\cdot mm$ $t_{ch2} := 0.051in = 1.295\cdot mm$ $t_{ch} := 0.5\cdot(t_{ch1} + t_{ch2}) = 1.194\cdot mm$

wire dia $d_w := 0.65\cdot mm$ number of wires $N_w := 36$ $mat := 1.6$

cable size from Paul Berindza email 3 Dec $w_{cab} := 11.68mm$ mean thick's $t_{cab} := \frac{(1.271mm + 1.0530mm)}{2} = 1.162\cdot mm$ channel thickness $t_{ch} := t_{cab}$

wire area $A_w := N_w\cdot\frac{\pi}{4}\cdot d_w^2 = 11.946\cdot mm^2$ $\lambda_{cab} := \frac{A_w}{w_{cab}\cdot t_{cab}} = 0.88$ channel occupied by cable $A_{ch} := w_{ch}\cdot t_{ch} = 14.344\cdot mm^2$

wire copper area $A_{wcu} := A_w\cdot\frac{mat}{1 + mat} = 7.351\cdot mm^2$ wire NbTi area $A_{nt} := A_w\cdot\frac{1}{1 + mat} = 4.595\cdot mm^2$

solder area $A_{so} := A_{ch} - A_w = 2.398\cdot mm^2$

interlayer insulation $t_{il} := 0.5mm$ insulated conductor width $w_{ic} := 19.29mm$

width unit cell $w_u := w_{ic} + t_{il} = 19.79\cdot mm$ thickness $t_u := 3.678mm$ unit cell area $A_u := w_u\cdot t_u = 72.788\cdot mm^2$

insulation thickness $t_i := 0.5\cdot(t_u - t_{st}) = 0.28\cdot mm$ insulation area $A_i := A_u - A_{con}$

stabilizer area $A_{st} := A_{con} - w_{ch}\cdot t_{ch} = 44.056\cdot mm^2$ copper area $A_{cu} := A_{st} + A_{wcu} = 51.407\cdot mm^2$

over unit cell $\lambda_{st} := \frac{A_{st}}{A_u} = 0.605$ $\lambda_{wcu} := \frac{A_{wcu}}{A_u} = 0.101$ $\lambda_{cu} := \lambda_{st} + \lambda_{wcu} = 0.706$ $\lambda_{so} := \frac{A_{so}}{A_u} = 0.033$

$\lambda_{nt} := \frac{A_{nt}}{A_u} = 0.063$ $\lambda_i := \frac{A_i}{A_u} = 0.198$ check $\lambda_{cu} + \lambda_{nt} + \lambda_{so} + \lambda_i = 1$ $J_{op} := \frac{I_{op}}{A_{con}} = 58.551\cdot A\cdot mm^{-2}$

2) Superconductor critical temperature from General Jc.xls,

$C_o := 21.77 \cdot K$ $\quad C_1 := -0.278 \cdot K \cdot tesla^{-1}$ $\quad C_2 := -0.015 \cdot K \cdot tesla^{-2}$ $\quad n := 0.032$ $\quad B_{c2} := 14.05 \cdot tesla$

$C_3 := -13.60 \cdot K$ $\quad \theta_c(B) := C_o + C_1 \cdot B + C_2 \cdot B^2 + \dfrac{C_3}{\left[(B_{c2} - B) \cdot T^{-1}\right]^n}$ $\quad \theta_c(B_p) = 7.114K$

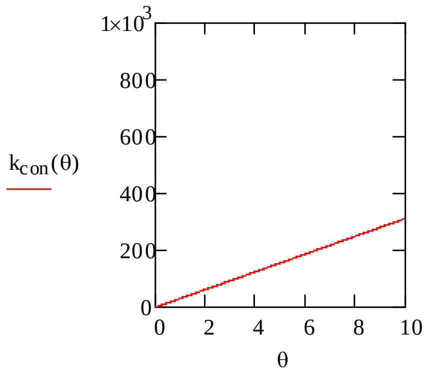
3) Copper resistivity magnetoresistance from copper magres Fickett.xls $\quad m_B := 4 \cdot 10^{-11} ohm \cdot m \cdot T^{-1}$ $\quad RRR := 50$

$\rho_{RT} := 1.678 \cdot 10^{-8} ohm \cdot m$ $\quad \rho_o := \dfrac{\rho_{RT}}{RRR} = 3.356 \times 10^{-10} \cdot ohm \cdot m$ $\quad \rho_{cB} := \rho_o + m_B \cdot B_p = 5.536 \times 10^{-10} \cdot ohm \cdot m$

4) Thermal conductivity (over whole conductor section) $\quad k \cdot \rho = L_o \cdot \theta$

$L_o := 2.45 \cdot 10^{-8} \cdot watt \cdot ohm \cdot K^{-2}$ $\quad k_c(\theta) = \kappa_c \cdot \theta$ $\quad \kappa_c := \dfrac{L_o}{\rho_{cB}}$ $\quad \kappa_{con} := \lambda_{cu} \cdot \dfrac{L_o}{\rho_{cB}}$ $\quad k_{con}(\theta) := \kappa_{con} \cdot \theta$

$\kappa_{con} = 31.256 m^{-1} \cdot K^{-2} \cdot watt$



5 Specific Heat NB make ? dimensionless in polynomial fits

a) NbTi Dresner p22 use Dresner Bc20

$B_{c2o} := 14 \cdot tesla$ density $\delta_{nt} := 6.2 \cdot 10^3 \cdot kg \cdot m^{-3}$

NbTi above crit $\quad C = \beta \cdot \theta^3 + \gamma \cdot \theta$ $\quad \beta_{ntr} := 2.3 \cdot 10^{-3} \cdot joule \cdot kg^{-1} \cdot K^{-1}$

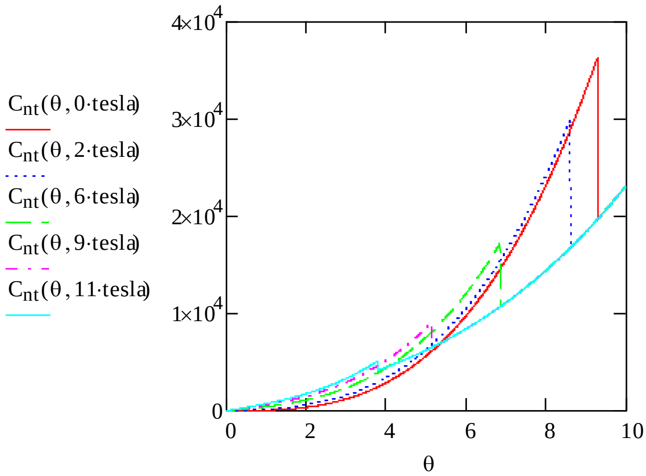
$\gamma_{ntr} := 0.145 \cdot joule \cdot kg^{-1} \cdot K^{-1}$ $\quad C_{ntr}(\theta) := \delta_{nt} \cdot \left[\beta_{ntr} \cdot \left(\theta \cdot K^{-1}\right)^3 + \gamma_{ntr} \cdot \theta \cdot K^{-1}\right]$

NbT below crit $\quad \gamma_{nts}(B) := \gamma_{ntr} \cdot \dfrac{B}{B_{c2o}}$ $\quad \beta_{nts} := \beta_{ntr} + 3 \cdot \dfrac{\gamma_{ntr}}{\left(\theta_c(0 \cdot tesla) \cdot K^{-1}\right)^2}$

$C_{nts}(\theta, B) := \delta_{nt} \cdot \left[\beta_{nts} \cdot \left(\theta \cdot K^{-1}\right)^3 + \gamma_{nts}(B) \cdot \theta \cdot K^{-1}\right]$ $\quad C_{nt}(\theta, B) := if\left(\theta < \theta_c(B), C_{nts}(\theta, B), C_{ntr}(\theta)\right)$

check $\quad C_{nt}(6K, 6T) = 1.217 \times 10^4 m^{-3} \cdot K^{-1} \cdot joule$

b) copper from copper.xls



$\gamma_{cu} := 10.9 \cdot 10^{-3} \cdot joule \cdot kg^{-1} \cdot K^{-1}$

$\beta_{cu} := 7.5 \cdot 10^{-4} \cdot joule \cdot kg^{-1} \cdot K^{-1}$

density $\quad \delta_{cu} := 8.96 \cdot 10^3 \cdot kg \cdot m^{-3}$

$C_{cu}(\theta) := \delta_{cu} \cdot \left[\beta_{cu} \cdot \left(\theta \cdot K^{-1}\right)^3 + \gamma_{cu} \cdot \theta \cdot K^{-1}\right]$

$C_{cu}(6 \cdot K) = 2.038 \times 10^3 m^{-3} \cdot K^{-1} \cdot joule$

c) Solder from solder.xls

$\gamma_{so} := 3.9 \cdot 10^{-2} \cdot joule \cdot kg^{-1} \cdot K^{-1}$

$\beta_{so} := 9.38 \cdot 10^{-5} \cdot joule \cdot kg^{-1} \cdot K^{-1}$

density $\quad \delta_{so} := 10.49 \cdot 10^3 \cdot kg \cdot m^{-3}$ $\quad C_{so}(\theta) := \delta_{so} \cdot \left[\beta_{so} \cdot \left(\theta \cdot K^{-1}\right)^5 + \gamma_{so} \cdot \left(\theta \cdot K^{-1}\right)^{0.7}\right]$ $\quad C_{so}(8 \cdot K) = 3.4 \times 10^4 m^{-3} \cdot K^{-1} \cdot joule$

d) Epoxy from epoxy spec ht.xls Johnson data density $\delta_e := 1.2\cdot10^3\cdot kg\cdot m^{-3}$

$m_{e1} := 2.302$ $A_{e1} := 0.612\cdot joulekg^{-1}\cdot K^{-1}$ $n_{e1} := 2.17$ $B_{e1} := -0.62\cdot joulekg^{-1}\cdot K^{-1}$

$C_{e1}(\theta) := \delta_e\cdot\left[A_{e1}\cdot(\theta\cdot K^{-1})^{m_{e1}} + B_{e1}\cdot(\theta\cdot K^{-1})^{n_{e1}}\right]$ $C_{e1}(6\cdot K) = 9.098\times10^3\,m^{-3}\cdot K^{-1}\cdot joule$

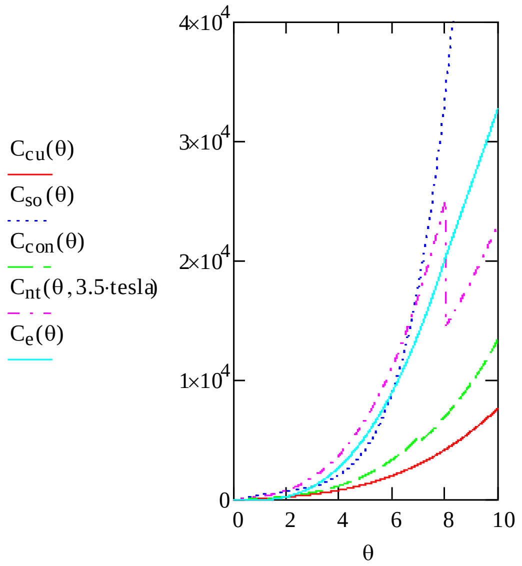
$m_{e2} := 1.047$ $A_{e2} := 4.62\cdot joulekg^{-1}\cdot K^{-1}$ $n_{e2} := 0.052$ $B_{e2} := -21.4\cdot joulekg^{-1}\cdot K^{-1}$

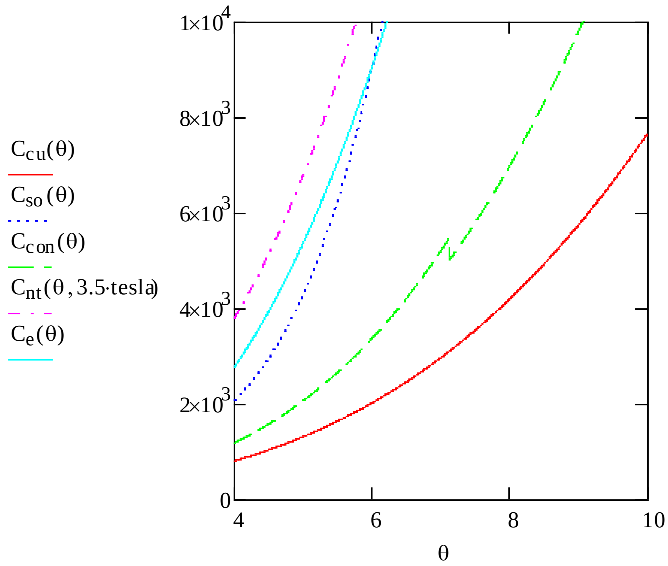
$C_{e2}(\theta) := \delta_e\cdot\left[A_{e2}\cdot(\theta\cdot K^{-1})^{m_{e2}} + B_{e2}\cdot(\theta\cdot K^{-1})^{n_{e2}}\right]$ $C_{e2}(10\cdot K) = 3.283\times10^4\,m^{-3}\cdot K^{-1}\cdot joule$

$C_e(\theta) := if(\theta < \theta_{e12}, C_{e1}(\theta), C_{e2}(\theta))$ $\theta_{e12} := 8\cdot K$

e) Unit cell take only half the epoxy to allow for heat diffusion $\lambda_{id} := \lambda_i\cdot0.5$

$C_{con}(\theta) := \lambda_{nt}\cdot C_{nt}(\theta, B_p) + \lambda_{cu}\cdot C_{cu}(\theta) + \lambda_{so}\cdot C_{so}(\theta) + \lambda_{id}\cdot C_e(\theta)$ $C_{con}(6K) = 3.392\times10^3\,m^{-3}\cdot K^{-1}\cdot J$





6) Generation using resistive transition model from RESTRAN3.mcd $J_{op} = 58.551\cdot A\cdot mm^{-2}$

define ρ, G and Jt over whole conductor section, break at Js

define rho over metal area $\rho_{em} := \rho_{cB}\cdot\frac{(A_{con})}{A_{cu}} = 6.289\times10^{-10}\cdot ohm\cdot m$ $J_{cBp} := \frac{I_c}{A_{nt}} = 2.274\times10^3\cdot A\cdot mm^{-2}$

$J_s = J_o(\theta)\cdot\left[\frac{\rho_{ecu}}{\rho_o\cdot(n+1)}\right]^{\frac{1}{n}}$ $J_0 := J_{cBp}\cdot\frac{A_{nt}}{A_{con}}$ $\rho_o := 10^{-14}\cdot ohm\cdot m$ $n := 30$

$J_{o1}(\theta) := J_0\cdot\frac{\theta_c(B_p) - \theta}{(\theta_c(B_p) - \theta_o)}$ $J_o(\theta) := if(\theta < \theta_c(B_p), J_{o1}(\theta), 0.1\cdot amp\cdot m^{-2})$ $J_s(\theta) := J_o(\theta)\cdot\left[\frac{\rho_{em}}{\rho_o\cdot(n+1)}\right]^{\frac{1}{n}}$

above J.s below Js $G1 = V\cdot J_t = J_{op}^{\;2}\cdot\rho_o\cdot\left(\frac{J_t}{J_o}\right)^n$ $G1(\theta) := J_{op}^{\;2}\cdot\rho_o\cdot\left(\frac{J_{op}}{J_o(\theta)}\right)^n$

$G2 = J_t\cdot V = J_{op}\cdot J_o\cdot\rho_o\cdot\left[\frac{\rho_{ecu}}{\rho_o\cdot(n+1)}\right]^{\frac{n+1}{n}} + J_{op}\cdot(J_t - J_s)\cdot\rho_{ecu}$ $G2(\theta) := J_{op}\cdot J_o(\theta)\cdot\rho_o\cdot\left[\frac{\rho_{em}}{\rho_o\cdot(n+1)}\right]^{\frac{n+1}{n}} + J_{op}\cdot(J_{op} - J_s(\theta))\cdot\rho_{em}$

above Js below Js $G1 = V \cdot J_t = J_{op} \cdot \rho_o \cdot \left(\frac{}{J_o}\right)$ $G1(\theta) := J_{op} \cdot \rho_o \cdot \left(\frac{}{J_o(\theta)}\right)$

$G2 = J_t \cdot V = J_{op} \cdot J_o \cdot \rho_o \cdot \left[\frac{\rho_{ecu}}{\rho_o \cdot (n+1)}\right]^{\frac{n+1}{n}} + J_{op} \cdot (J_t - J_s) \cdot \rho_{ecu}$ $G2(\theta) := J_{op} \cdot J_o(\theta) \cdot \rho_o \cdot \left[\frac{\rho_{em}}{\rho_o \cdot (n+1)}\right]^{\frac{n+1}{n}} + J_{op} \cdot (J_{op} - J_s(\theta)) \cdot \rho_{em}$

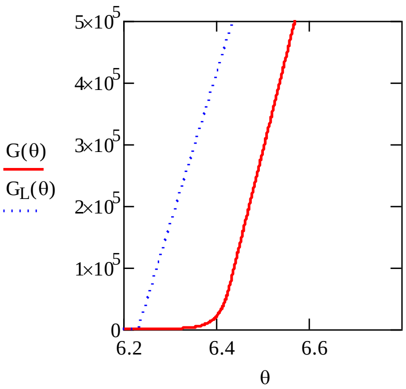
Global G function $G_t(\theta) := \text{if}(J_{op} < J_s(\theta), G1(\theta), G2(\theta))$ $G(\theta) := \text{if}(\theta < \theta_c(B_p), G_t(\theta), G_t(\theta_c(B_p)))$

linear G for comparison

$G_c := J_{op}^{\,2} \cdot \rho_{em}$ $G_c = 2.156 \times 10^{6}\, m^{-3} \cdot watt$ $\theta_g := \theta_c(B_p) - (\theta_c(B_p) - \theta_o) \cdot \frac{I_{op}}{J_{cBp} \cdot A_{nt}}$ $\theta_g = 6.233K$

$Rl(\theta) := \text{if}\left[\theta < \theta_g, 0, \frac{\theta - \theta_g}{(\theta_c(B_p) - \theta_g)}\right]$ $Rf(\theta) := \text{if}(\theta < \theta_c(B_p), Rl(\theta), 1)$ $G_L(\theta) := G_c \cdot Rf(\theta)$

7) Heat Transfer through insulation



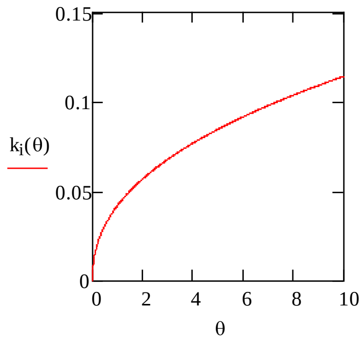
$n_i := 0.43$ $B_i := 0.0425 \cdot watt \cdot m^{-1} \cdot K^{-1}$ $k_i(\theta) := B_i \cdot (\theta \cdot K^{-1})^{n_i}$

$k_i(4 \cdot K) = 0.077 K^{-1} \cdot watt \cdot m^{-1}$ $k_i(10K) = 0.114 \cdot watt \cdot m^{-1} \cdot K^{-1}$

$dQ' = k(\theta) \cdot A \cdot \frac{d}{dx}\theta$ $Q' \cdot x = A \cdot \int_{\theta_o}^{\theta} k(\theta)\, d\theta$

$Q' \cdot x = A \cdot \int_{\theta_o}^{\theta} B_i \cdot \theta^{n}\, d\theta$ $Q' = \frac{A}{x} \cdot \frac{B_i}{n+1} \cdot (\theta^{n+1} - \theta_o^{\,n+1})$

$h_i(\theta) = \frac{1}{x} \cdot \frac{B_i}{n_i + 1} \cdot \left[(\theta \cdot K^{-1})^{n_i} \cdot \theta - (\theta_o \cdot K^{-1})^{n_i} \cdot \theta_o\right]$



per unit length let heat transfer = Hi

insulation wrap thickness $t_i = 0.28 \cdot mm$

inter-pancake insulation thickness $t_{ip} := 0.5mm$

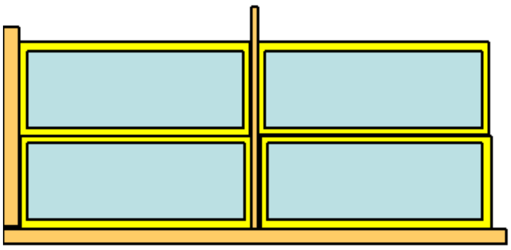
ground plane insulation thickness $t_{ig} := 0.5mm$

interturn conduction thickness $x_{i1} := 2 \cdot t_i$

interturn perimeter $P_{i1} := 2 \cdot w_{st} = 37.46 \cdot mm$

inter-pancake conduction thickness $x_{i2} := 2 \cdot t_i + t_{ip} = 1.06 \cdot mm$

inter-pancake perimeter $P_{i2} := 2 \cdot t_{st} = 6.236 \cdot mm$



conduction thickness to ground $x_{i3} := t_i + t_{ig} = 0.78 \cdot mm$

perimeter to ground $P_{i3} := t_{st} + w_{st} = 21.848 \cdot mm$

cooling per unit length

$H_i(\theta) = \left(\sum \frac{P}{x}\right) \cdot \frac{B_i}{n_i + 1} \cdot \left[(\theta \cdot K^{-1})^{n_i} \cdot \theta - (\theta_o \cdot K^{-1})^{n_i} \cdot \theta_o\right]$

$H_{ip}(\theta) := \left(\frac{P_{i1}}{x_{i1}} + \frac{P_{i2}}{x_{i2}}\right) \cdot \frac{B_i}{n_i + 1} \cdot \left[(\theta \cdot K^{-1})^{n_i} \cdot \theta - (\theta_o \cdot K^{-1})^{n_i} \cdot \theta_o\right]$

avoid negative cooling $H_i(\theta) := \text{if}(\theta < \theta_o, 10^{-9} \cdot watt \cdot m^{-1}, H_{ip}(\theta))$ spot value check $H_i(6 \cdot K) = 9.928 m^{-1} \cdot watt$

rough check $H_{i2}(\theta) := k_i\left[\frac{(\theta + \theta_o)}{2}\right] \cdot (\theta - \theta_o) \cdot \left(\frac{P_{i1}}{x_{i1}} + \frac{P_{i2}}{x_{i2}}\right)$ $H_{i2}(6K) = 9.937 m^{-1} \cdot watt$

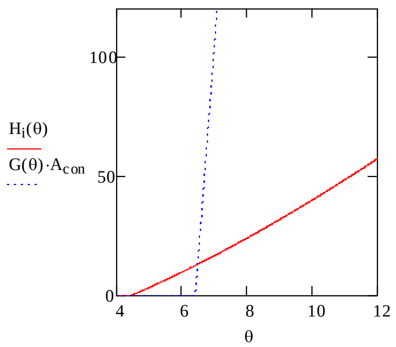
spot value check

$G_c \cdot A_{con} = 125.913 m^{-1} \cdot watt$

$G(7.32K) \cdot A_{con} = 125.913 m^{-1} \cdot watt$

$H_i(7.32K) = 19.151 m^{-1} \cdot watt$

$\theta_{cryo} := 7.32$



9 Parameters for solution define x interval $dx := 5 \cdot mm$ time interval $dt := 70 \cdot 10^{-6} \cdot sec$

cf MPZ radius $\theta_{sh} := \dfrac{\theta_c(B_p) + \theta_g}{2}$ $X_g := \sqrt{\dfrac{2 \cdot k_{con}(\theta_{sh}) \cdot (\theta_c(B_p) - \theta_g)}{G_c}}$ $X_g = 13.061 \cdot mm$ $\dfrac{dx}{X_g} = 0.383$

for stability James p661 $\kappa := \dfrac{k_{con}(\theta_o)}{C_{con}(\theta_o)}$ $\lambda := \kappa \cdot \dfrac{dt}{dx^2}$ $\lambda = 0.254$ for stability should be < 0.5

Initiating heat pulse, energy / unit volume $\theta_i := 7.4K$ $\Delta H := \int_{\theta_o}^{\theta_i} C_{con}(\theta)\, d\theta$ $\Delta H = 1.013 \times 10^{4} \cdot joule m^{-3}$

time of pulse = $\tau_i := 400 \cdot 10^{-6} \cdot sec$ $x_i := 20 \cdot mm$ $Q_i(x) := \dfrac{\Delta H}{\tau_i} \cdot e^{-\left(\frac{x}{x_i}\right)^2}$

total heat $Q_{it} := 2 \cdot A_{con} \cdot \tau_i \cdot \int_0^{10 \cdot x_i} Q_i(x)\, dx$ $Q_{it} = 2.1 \times 10^{-2} \cdot joule$ $Q(x,t) := if(t < \tau_i, Q_i(x), 0)$

ntm := 5000 nxm := 100 solving $C(\theta) \cdot \dfrac{d\theta}{dt} = k(\theta) \cdot \dfrac{d^2\theta}{dx^2} + \dfrac{d \cdot k(\theta)}{d\theta} \cdot \left(\dfrac{d\theta}{dx}\right)^2 + G(\theta) + Q(x,t) - H(\theta) \cdot \dfrac{P}{A}$

$u_s(nx, nt) :=$
- for $nx \in 1..nxm$
  - $u_{nx,1} \leftarrow \theta_o$
- for $nt \in 1..ntm$
  - $u_{1,nt+1} \leftarrow u_{1,nt} + \dfrac{dt}{C_{con}(u_{1,nt})} \cdot \left[k_{con}(u_{1,nt}) \cdot \left(\dfrac{u_{2,nt} - u_{1,nt}}{dx^2}\right) + G(u_{1,nt}) + Q(dx, nt \cdot dt) - H_i(u_{1,nt}) \cdot \dfrac{1}{A_{con}}\right]$
  - $u_{nxm,nt+1} \leftarrow u_{nxm,nt} + \dfrac{dt}{C_{con}(u_{nxm,nt})} \cdot \left[k_{con}(u_{nxm,nt}) \cdot \left(\dfrac{u_{nxm-1,nt} - u_{nxm,nt}}{dx^2}\right) + G(u_{nxm,nt}) + Q(nxm dx, nt \cdot dt) - H_i(u_{nxm,nt}) \cdot \dfrac{1}{A_{con}}\right]$
  - for $nx \in 2..(nxm - 1)$
    - $u_{nx,nt+1} \leftarrow u_{nx,nt} + \dfrac{dt}{C_{con}(u_{nx,nt})} \cdot \left[k_{con}(u_{nx,nt}) \cdot \left(\dfrac{u_{nx+1,nt} - 2 \cdot u_{nx,nt} + u_{nx-1,nt}}{dx^2}\right) \ldots + \kappa_{con} \cdot \left(\dfrac{u_{nx+1,nt} - u_{nx-1,nt}}{2 \cdot dx}\right)^2 + G(u_{nx,nt}) + Q(nx \cdot dx, nt \cdot dt) - H_i(u_{nx,nt}) \cdot \dfrac{1}{A_{con}}\right]$
- u

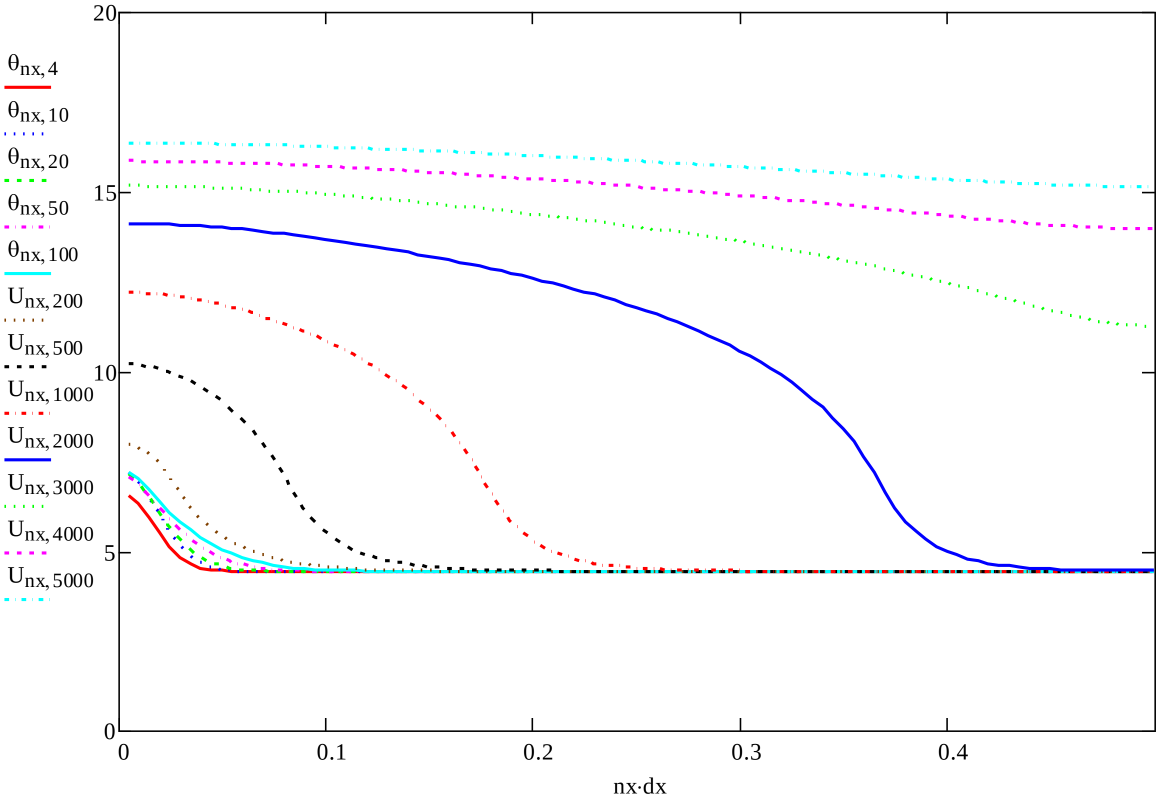
$\theta := u_s(nxm, ntm)$ $nx := 1..nxm$ $nt := 1..ntm$ $U := u_s(nxm, ntm) \cdot K^{-1}$

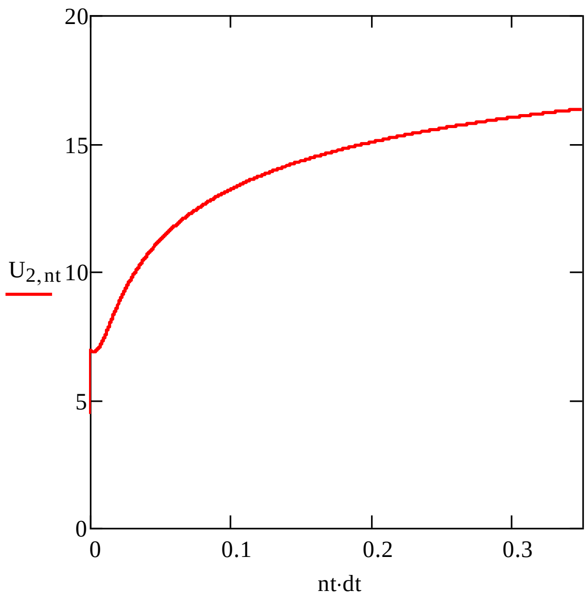
$par_0 := dx \cdot m^{-1}$ $par_1 := nxm$ $par_2 := dt \cdot sec^{-1}$ $par_3 := ntm$ $par_4 := \theta_i \cdot K^{-1}$ $par_5 := x_i \cdot m^{-1}$

$par_6 := \tau_i \cdot sec^{-1}$ $par_7 := Q_{it} \cdot joule^{-1}$

$dx = 5 \times 10^{-3} m$ $nxm = 100$ $dx \cdot nxm = 0.5m$ $dt = 7 \times 10^{-5} s$ $ntm = 5 \times 10^{3}$ $dt \cdot ntm = 0.35s$

$\theta_i = 7.4K$ $x_i = 20 \cdot mm$ $\tau_i = 4 \times 10^{-4} s$ $Q_{it} = 2.10 \times 10^{-2} \cdot joule$ $\theta_g = 6.233K$





$5 \cdot dt = 3.5 \times 10^{-4} s$

$10 \cdot dt = 7 \times 10^{-4} s$

$20 \cdot dt = 1.4 \times 10^{-3} s$

$50 \cdot dt = 3.5 \times 10^{-3} s$

$100 \cdot dt = 7 \times 10^{-3} s$

$200 \cdot dt = 0.014 s$

$300 \cdot dt = 0.021 s$

$500 \cdot dt = 0.035 s$

$1000 \cdot dt = 0.07 s$

$2000 \cdot dt = 0.14 s$

$3000 \cdot dt = 0.21 s$

WRITEPRN("parCLAS7a5.prn") := par   WRITEPRN("tempCLAS7a5.prn") := U

$\theta_g = 6.233 K$ $U_{3,2000} = 14.092$ $U_{3,2500} = 14.677$ $U_{3,3000} = 15.138$