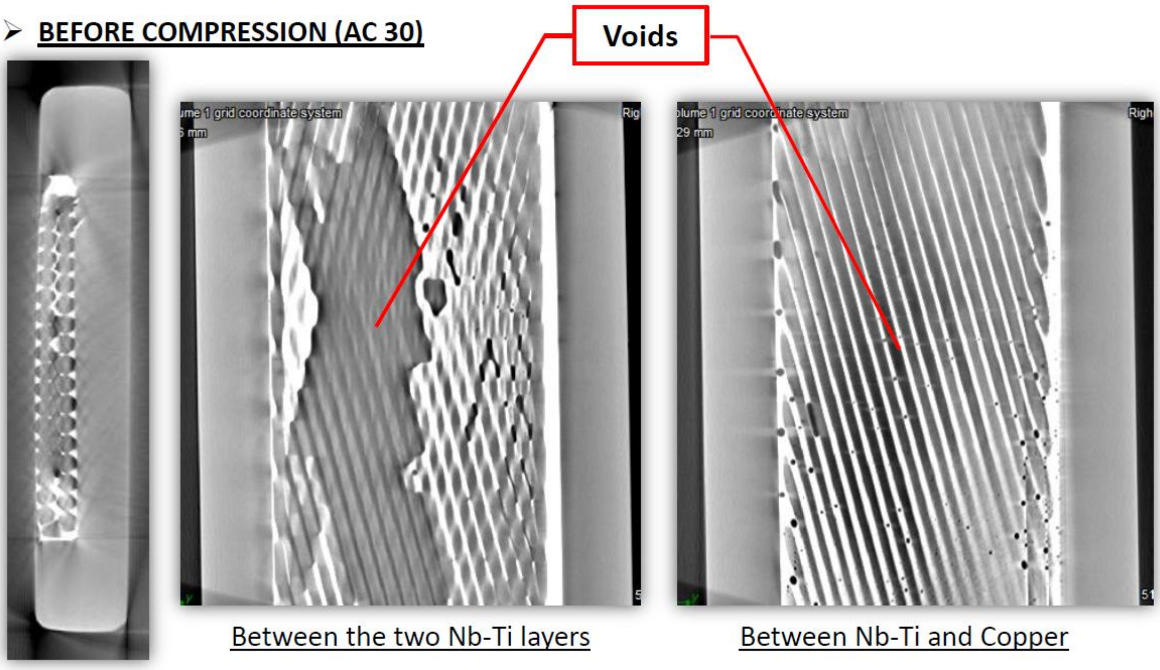
Sigmaphi SHMS Magnet Project
MNW Report SJD8 (version 1)
19th Oct 2012

Second Thoughts on MQE Calculations for the SHMS Dipole Conductor

Martin N Wilson

1. Introduction

Looking at the gaps in soldering shown on the tomography and cross section pictures of the conductor, I am feeling increasingly uncomfortable about the validity of my earlier MQE calculations (Report SJ7). The model I use is a very simple one dimensional one in which I assume that the superconductor and copper are intimately mixed, in perfect contact and sharing the same temperature and electric potential. It takes no account of any fine structure, so how to calculate the situation shown in Fig 1?



Between the two Nb-Ti layers

Between Nb-Ti and Copper

Fig 1: from 'Tomography on JLAB's conductor 23 Sept 12.

The best I can offer is to calculate a model where cable plus solder plus insulation is detached from copper, as sketched in Fig 2.

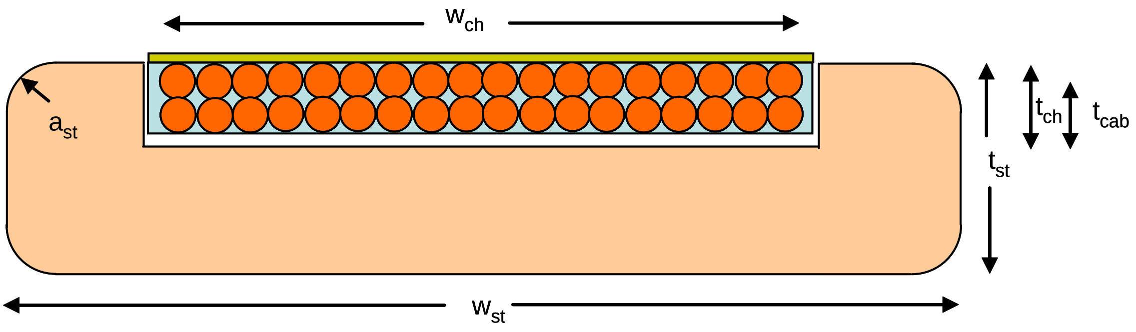


Fig 2: Calculation model with cable detached from copper.

With this model, I find a big reduction in MQE. Using exactly the same data as in Report SJD7, with RRR = 100, the MQE falls from 28mJ with copper attached to 1.9mJ with the attachment broken. Details of the calculation are presented in the appendix.

The question then arises as to how large the area of void can be before things start to go wrong. Fig 3, taken from the appendix shows the temperature profile along the conductor at various times after the imposition of an energy spike.

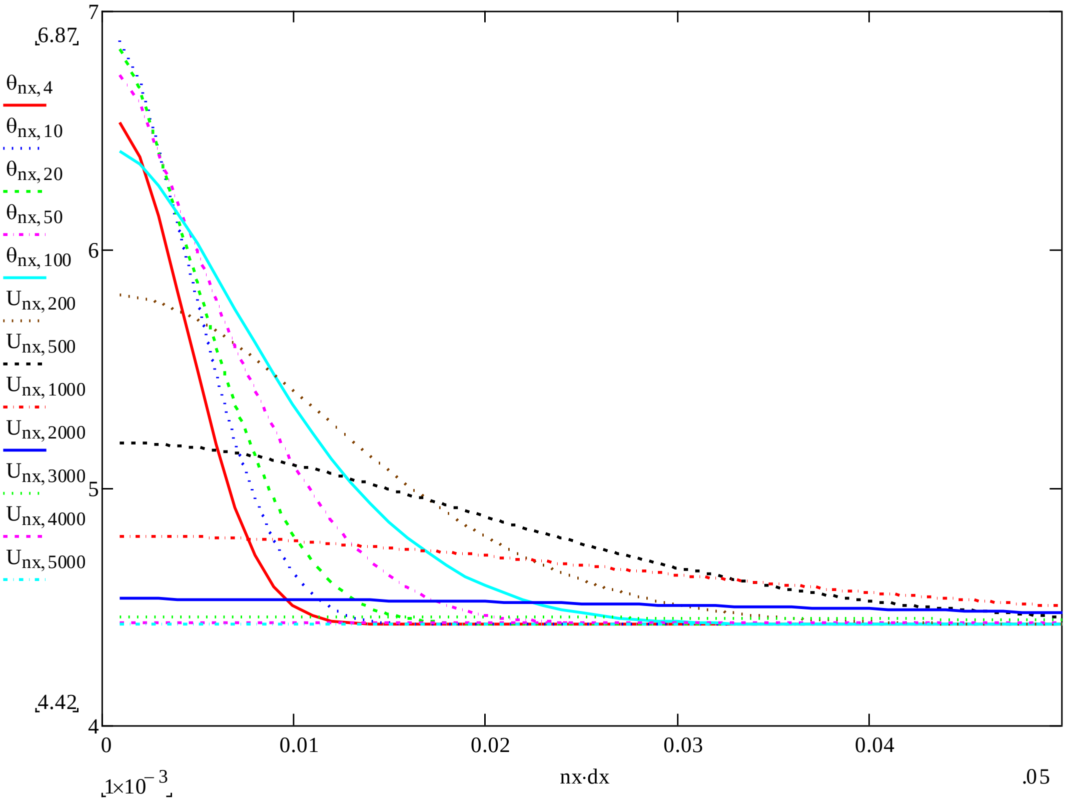


Fig 3: Temperature profiles along the conductor at different times after the initial energy spike (x axis in metres, y axis 2nd subscript in units of 10µsec, ie $U_{nx,5000}$ is at 50msec).

It may be seen that everything happens within a half width of less than 1cm; this width is often called the minimum propagating zone. Thus it would appear that, if the cable is detached from the copper over a length of more than 2cm, it's response to an energy spike will be more like a separate cable than an integrated conductor.

Concluding Remarks

These calculations are crude and approximate, but they are the best we have in trying to predict how stable the conductor will be against training and degraded performance in the magnet. The observed voids in solder bonding do appear to be larger than the predicted minimum propagating zone, so it seems likely that the MQE of those parts of the conductor will be nearer 1.9mJ than the previously calculated 28mJ. This can only mean that the magnet will suffer from much more training and may not achieve it's design field. If we add to this lack of stability the possibility of increased conductor motion coming from spongey mechanical properties, we have a double whammey.

Appendix 1: Details of the Mathcad MQE Calculation

JLab SHMS Dipole MQE series B cable not in contact with copper revisited 18 Oct 12

Revised 12 May 10 because new Mathcad will not accept fractional exponents of variables with units. Make dimensionless before raising to exponent.

Use generation term with resistive transition from MPZSING1 and NORMPROP6, eqs from TIME1W1 SI Uni

$$C(\theta)\cdot\frac{d\theta}{dt} = \frac{d}{dx}\cdot\left(k(\theta)\cdot\frac{d\theta}{dx}\right) + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A} \qquad C(\theta)\cdot\frac{d\theta}{dt} = k(\theta)\cdot\frac{d^2\cdot\theta}{dx^2} + \frac{d\cdot k(\theta)}{dx}\cdot\frac{d\theta}{dx} + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A}$$

solve this one $C(\theta)\cdot\frac{d\theta}{dt} = k(\theta)\cdot\frac{d^2\cdot\theta}{dx^2} + \frac{d\cdot k(\theta)}{d\theta}\cdot\left(\frac{d\theta}{dx}\right)^2 + G(\theta) + Q(x,t) - H(\theta)\cdot\frac{P}{A}$

where A is overall cross section, k is averaged over A, C and G are per unit volume and H is per unit area

boundary conditions at x large we have θ = bath temperature $\theta = \theta_o$ at x=0 have $\frac{d\theta}{dx} = 0$ for all t

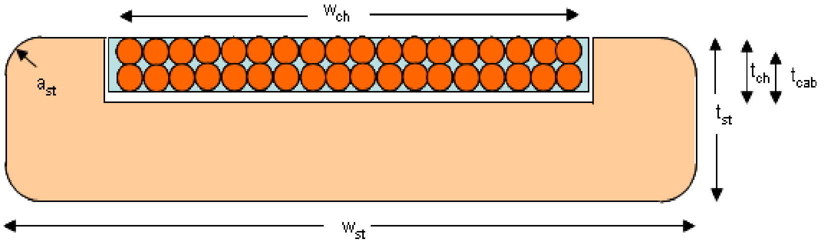
1) Winding Data $I_{op} := 3419.4\,amp$ $B_p := 5.45\,T$ $\theta_o := 4.42\,K$ $\mu_o := 4\cdot\pi\cdot 10^{-7}\cdot henry\cdot m^{-1}$

spec Ic at 5T and 6T

$I_{cd5} := 12333A$ $I_{cd6} := 9875A$

take Ic at 5.45T and 4.42K from General Jc.xls

$I_c := 10450A$



2) Conductor Geometry unit cell includes wire solder and some insulation

channel width $w_{ch} := 0.486\,in = 12.344\cdot mm$

channel thickness $t_{ch1} := 0.043\,in = 1.092\cdot mm$ $t_{ch2} := 0.051\,in = 1.295\cdot mm$ $t_{chm} := 0.5\cdot(t_{ch1} + t_{ch2}) = 1.194\cdot mm$

wire dia $d_w := 0.65\cdot mm$ number of wires $N_w := 36$ $mat := 1.6$

cable size from Paul Berindza email 3 Dec $w_{cab} := 11.68\,mm$ mean thick's $t_{cab} := \frac{(1.271\,mm + 1.0530\,mm)}{2} = 1.162\cdot mm$ channel thickness $t_{ch} := t_{cab}$

wire area $A_w := N_w\cdot\frac{\pi}{4}\cdot d_w^2 = 11.946\cdot mm^2$ $\lambda_{cab} := \frac{A_w}{w_{cab}\cdot t_{cab}} = 0.88$ channel occupied by cable $A_{ch} := w_{ch}\cdot t_{ch} = 14.344\cdot mm^2$

wire copper area $A_{wcu} := A_w\cdot\frac{mat}{1 + mat} = 7.351\cdot mm^2$ wire NbTi area $A_{nt} := A_w\cdot\frac{1}{1 + mat} = 4.595\cdot mm^2$

solder area $A_{so} := A_{ch} - A_w = 2.398\cdot mm^2$

insulation thickness $t_i := 0.28\,mm$ insulation width $w_i := w_{ch}$ insulation area $A_i := t_i\cdot w_i$

width unit cell $w_u := w_{ch}$ thickness $t_u := t_{cab} + t_i = 1.442\,mm$ unit cell area $A_u := w_u\cdot t_u = 17.801\cdot mm^2$

copper area $A_{cu} := A_{wcu} = 7.351\cdot mm^2$

$\lambda_{cu} := \frac{A_{cu}}{A_u} = 0.413$ $\lambda_{nt} := \frac{A_{nt}}{A_u} = 0.258$ $\lambda_{so} := \frac{A_{so}}{A_u} = 0.135$ $\lambda_i := \frac{A_i}{A_u} = 0.194$

check $\lambda_{cu} + \lambda_{nt} + \lambda_{so} + \lambda_i = 1$ $A_{con} := A_{ch}$ $J_{op} := \frac{I_{op}}{A_{con}} = 238.382\cdot A\cdot mm^{-2}$

$C_{e1}(\theta) := \delta_e \cdot \left[A_{e1} \cdot \left(\theta \cdot K^{-1}\right)^{m_{e1}} + B_{e1} \cdot \left(\theta \cdot K^{-1}\right)^{n_{e1}}\right]$ $C_{e1}(6 \cdot K) = 9.098 \times 10^{3} K^{-1} \cdot m^{-3} \cdot joule$

$m_{e2} := 1.047$ $A_{e2} := 4.62 \cdot joule kg^{-1} \cdot K^{-1}$ $n_{e2} := 0.052$ $B_{e2} := -21.4 joule kg^{-1} \cdot K^{-1}$

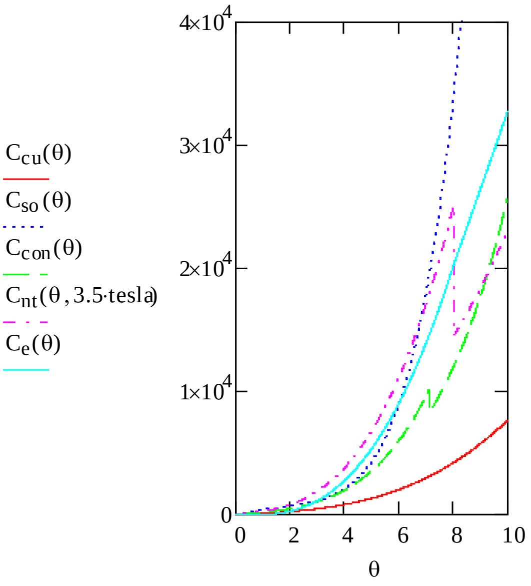
$C_{e2}(\theta) := \delta_e \cdot \left[A_{e2} \cdot \left(\theta \cdot K^{-1}\right)^{m_{e2}} + B_{e2} \cdot \left(\theta \cdot K^{-1}\right)^{n_{e2}}\right]$ $C_{e2}(10 \cdot K) = 3.283 \times 10^{4} K^{-1} \cdot m^{-3} \cdot joule$

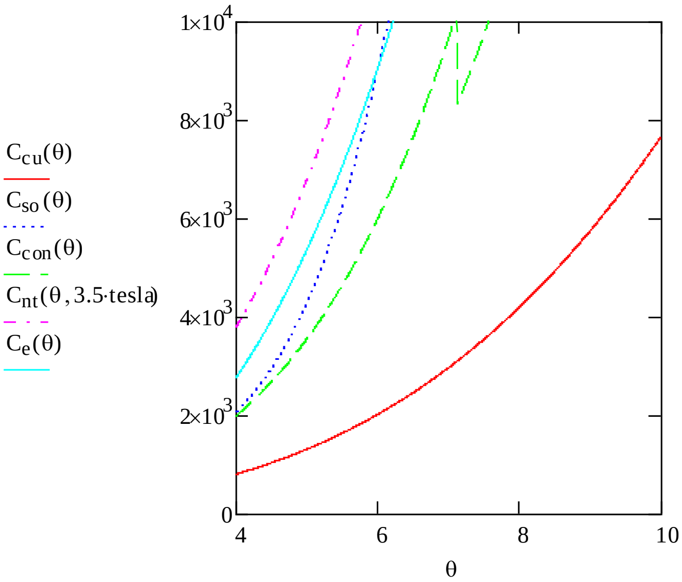
$\theta_{e12} := 8 \cdot K$

$C_e(\theta) := \mathrm{if}\left(\theta < \theta_{e12}, C_{e1}(\theta), C_{e2}(\theta)\right)$

e) Unit cell take only half the epoxy to allow for heat diffusion $\lambda_{id} := \lambda_i \cdot 0.5$

$C_{con}(\theta) := \lambda_{nt} \cdot C_{nt}\left(\theta, B_p\right) + \lambda_{cu} \cdot C_{cu}(\theta) + \lambda_{so} \cdot C_{so}(\theta) + \lambda_{id} \cdot C_e(\theta)$ $C_{con}(6K) = 6.035 \times 10^{3} K^{-1} \cdot m^{-3} \cdot J$





6) Generation using resistive transition model from RESTRAN3.mcd $J_{op} = 238.382 \cdot A \cdot mm^{-2}$

define ρ, G and Jt over whole conductor section, break at Js

define rho over metal area $\rho_{em} := \rho_{cB} \cdot \frac{\left(A_{con}\right)}{A_{cu}} = 7.528 \times 10^{-10} \cdot ohm \cdot m$ $J_{cBp} := \frac{I_c}{A_{nt}} = 2.274 \times 10^{3} \cdot A \cdot mm^{-2}$

$J_s = J_o(\theta) \cdot \left[\frac{\rho_{ecu}}{\rho_o \cdot (n+1)}\right]^{\frac{1}{n}}$ $J_0 := J_{cBp} \cdot \frac{A_{nt}}{A_{con}}$ $\rho_o := 10^{-14} \cdot ohm \cdot m$ $n := 30$

$J_{o1}(\theta) := J_0 \cdot \frac{\theta_c\left(B_p\right) - \theta}{\left(\theta_c\left(B_p\right) - \theta_o\right)}$ $J_o(\theta) := \mathrm{if}\left(\theta < \theta_c\left(B_p\right), J_{o1}(\theta), 0.1 \cdot amp \cdot m^{-2}\right)$ $J_s(\theta) := J_o(\theta) \cdot \left[\frac{\rho_{em}}{\rho_o \cdot (n+1)}\right]^{\frac{1}{n}}$

above J.s   below Js $G1 = V \cdot J_t = J_{op}^{2} \cdot \rho_o \cdot \left(\frac{J_t}{J_o}\right)^{n}$ $G1(\theta) := J_{op}^{2} \cdot \rho_o \cdot \left(\frac{J_{op}}{J_o(\theta)}\right)^{n}$

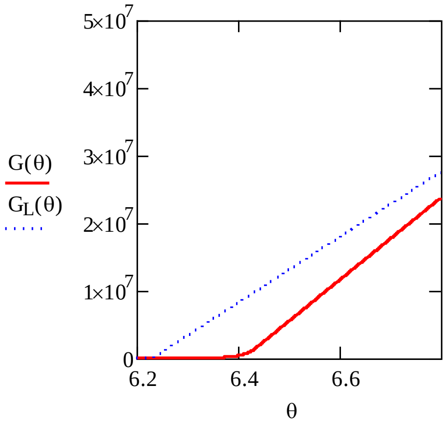
$G2 = J_t \cdot V = J_{op} \cdot J_o \cdot \rho_o \cdot \left[\frac{\rho_{ecu}}{\rho_o \cdot (n+1)}\right]^{\frac{n+1}{n}} + J_{op} \cdot \left(J_t - J_s\right) \cdot \rho_{ecu}$ $G2(\theta) := J_{op} \cdot J_o(\theta) \cdot \rho_o \cdot \left[\frac{\rho_{em}}{\rho_o \cdot (n+1)}\right]^{\frac{n+1}{n}} + J_{op} \cdot \left(J_{op} - J_s(\theta)\right) \cdot \rho_{em}$

Global G function $G_t(\theta) := \mathrm{if}\left(J_{op} < J_s(\theta), G1(\theta), G2(\theta)\right)$ $G(\theta) := \mathrm{if}\left(\theta < \theta_c\left(B_p\right), G_t(\theta), G_t\left(\theta_c\left(B_p\right)\right)\right)$

linear G for comparison

$G_c := J_{op}^{\;2}\cdot\rho_{em}$ $G_c = 4.278\times 10^{7}\,m^{-3}\cdot watt$ $\theta_g := \theta_c(B_p) - \left(\theta_c(B_p)-\theta_o\right)\cdot\frac{I_{op}}{J_{cBp}\cdot A_{nt}}$ $\theta_g = 6.233K$

$Rl(\theta) := if\left[\theta<\theta_g, 0, \frac{\theta-\theta_g}{\left(\theta_c(B_p)-\theta_g\right)}\right]$ $Rf(\theta) := if\left(\theta<\theta_c(B_p), Rl(\theta), 1\right)$ $G_L(\theta) := G_c\cdot Rf(\theta)$



7) Heat Transfer through insulation

$n_i := 0.43$ $B_i := 0.0425\cdot watt\cdot m^{-1}\cdot K^{-1}$ $k_i(\theta) := B_i\cdot\left(\theta\cdot K^{-1}\right)^{n_i}$

$k_i(4\cdot K) = 0.077K^{-1}\cdot watt\cdot m^{-1}$ $k_i(10K) = 0.114\cdot watt\cdot m^{-1}\cdot K^{-1}$

$dQ' = k(\theta)\cdot A\cdot\frac{d}{dx}\theta$ $Q'\cdot x = A\cdot\int_{\theta_o}^{\theta} k(\theta)\,d\theta$

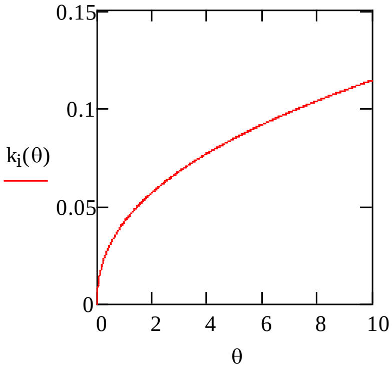
$Q'\cdot x = A\cdot\int_{\theta_o}^{\theta} B_i\cdot\theta^{n}\,d\theta$ $Q' = \frac{A}{x}\cdot\frac{B_i}{n+1}\cdot\left(\theta^{n+1}-\theta_o^{\;n+1}\right)$

$h_i(\theta) = \frac{1}{x}\cdot\frac{B_i}{n_i+1}\cdot\left[\left(\theta\cdot K^{-1}\right)^{n_i}\cdot\theta - \left(\theta_o\cdot K^{-1}\right)^{n_i}\cdot\theta_o\right]$

per unit length let heat transfer = Hi

insulation wrap thickness $t_i = 0.28\cdot mm$

interturn perimeter $P_{i1} := w_{ch} = 12.344\cdot mm$



cooling per unit length

$H_i(\theta) = \left(\sum_{\blacksquare}\frac{P}{x}\right)\cdot\frac{B_i}{n_i+1}\cdot\left[\left(\theta\cdot K^{-1}\right)^{n_i}\cdot\theta - \left(\theta_o\cdot K^{-1}\right)^{n_i}\cdot\theta_o\right]$

$H_{ip}(\theta) := \left(\frac{P_{i1}}{t_i}\right)\cdot\frac{B_i}{n_i+1}\cdot\left[\left(\theta\cdot K^{-1}\right)^{n_i}\cdot\theta - \left(\theta_o\cdot K^{-1}\right)^{n_i}\cdot\theta_o\right]$

avoid negative cooling $H_i(\theta) := if\left(\theta<\theta_o, 10^{-9}\cdot watt\cdot m^{-1}, H_{ip}(\theta)\right)$ spot value check $H_i(6\cdot K) = 6.014m^{-1}\cdot watt$

rough check $H_{i2}(\theta) := k_i\left[\frac{\left(\theta+\theta_o\right)}{2}\right]\cdot\left(\theta-\theta_o\right)\cdot\left(\frac{P_{i1}}{t_i}\right)$ $H_{i2}(6K) = 6.02m^{-1}\cdot watt$

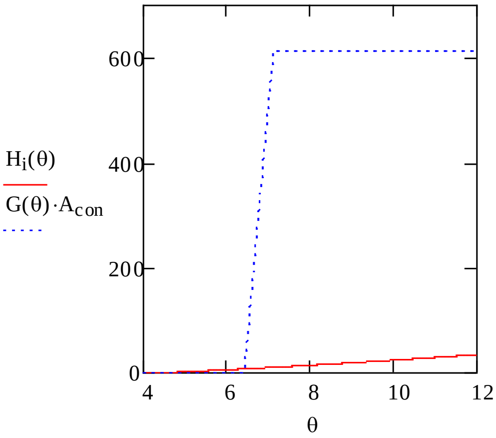
spot value check

$G_c\cdot A_{con} = 613.615m^{-1}\cdot watt$

$G(7.32K)\cdot A_{con} = 613.615m^{-1}\cdot watt$

$H_i(7.32K) = 11.602m^{-1}\cdot watt$

$\theta_{cryo} := 7.32$



9 Parameters for solution

define x interval $dx := 1\cdot mm$ time interval $dt := 10\cdot 10^{-6}\cdot sec$

cf MPZ radius $\theta_{sh} := \dfrac{\theta_c(B_p) + \theta_g}{2}$ $X_g := \sqrt{\dfrac{2\cdot k_{con}(\theta_{sh})\cdot(\theta_c(B_p) - \theta_g)}{G_c}}$ $X_g = 2.686\,mm$ $\dfrac{dx}{X_g} = 0.372$

for stability James p661 $\kappa := \dfrac{k_{con}(\theta_o)}{C_{con}(\theta_o)}$ $\lambda := \kappa\cdot\dfrac{dt}{dx^2}$ $\lambda = 0.449$ for stability should be < 0.5

Initiating heat pulse, energy / unit volume $\theta_i := 7.086K$ $\Delta H := \displaystyle\int_{\theta_o}^{\theta_i} C_{con}(\theta)\, d\theta$ $\Delta H = 1.51\times 10^4\cdot joule\, m^{-3}$

time of pulse τ $\tau_i := 50\cdot 10^{-6}\cdot sec$ $x_i := 5\cdot mm$ $Q_i(x) := \dfrac{\Delta H}{\tau_i}\cdot e^{-\left(\frac{x}{x_i}\right)^2}$

total heat $Q_{it} := 2\cdot A_{con}\cdot\tau_i\cdot\displaystyle\int_0^{10\cdot x_i} Q_i(x)\, dx$ $Q_{it} = 1.92\times 10^{-3}\cdot joule$ $Q(x,t) := \text{if}(t < \tau_i, Q_i(x), 0)$

ntm := 5000 nxm := 100 solving $C(\theta)\cdot\dfrac{d\theta}{dt} = k(\theta)\cdot\dfrac{d^2\cdot\theta}{dx^2} + \dfrac{d\cdot k(\theta)}{d\theta}\cdot\left(\dfrac{d\theta}{dx}\right)^2 + G(\theta) + Q(x,t) - H(\theta)\cdot\dfrac{P}{A}$

```
$u_s(nx, nt) :=$
    for $nx \in 1..nxm$
        $u_{nx,1} \leftarrow \theta_o$
    for $nt \in 1..ntm$
        $u_{1,nt+1} \leftarrow u_{1,nt} + \frac{dt}{C_{con}(u_{1,nt})}\cdot\left[k_{con}(u_{1,nt})\cdot\left(\frac{u_{2,nt} - u_{1,nt}}{dx^2}\right) + G(u_{1,nt}) + Q(dx, nt\cdot dt) - H_i(u_{1,nt})\cdot\frac{1}{A_{con}}\right]$
        $u_{nxm,nt+1} \leftarrow u_{nxm,nt} + \frac{dt}{C_{con}(u_{nxm,nt})}\cdot\left[k_{con}(u_{nxm,nt})\cdot\left(\frac{u_{nxm-1,nt} - u_{nxm,nt}}{dx^2}\right) + G(u_{nxm,nt}) + Q(nxm\,dx, nt\cdot dt) - H_i(u_{nxm,nt})\cdot\frac{1}{A_{con}}\right]$
        for $nx \in 2..(nxm - 1)$
            $u_{nx,nt+1} \leftarrow u_{nx,nt} + \frac{dt}{C_{con}(u_{nx,nt})}\cdot\left[k_{con}(u_{nx,nt})\cdot\left(\frac{u_{nx+1,nt} - 2\cdot u_{nx,nt} + u_{nx-1,nt}}{dx^2}\right) \ldots + \kappa_{con}\cdot\left(\frac{u_{nx+1,nt} - u_{nx-1,nt}}{2\cdot dx}\right)^2 + G(u_{nx,nt}) + Q(nx\cdot dx, nt\cdot dt) - H_i(u_{nx,nt})\cdot\frac{1}{A_{con}}\right]$
    $u$
```

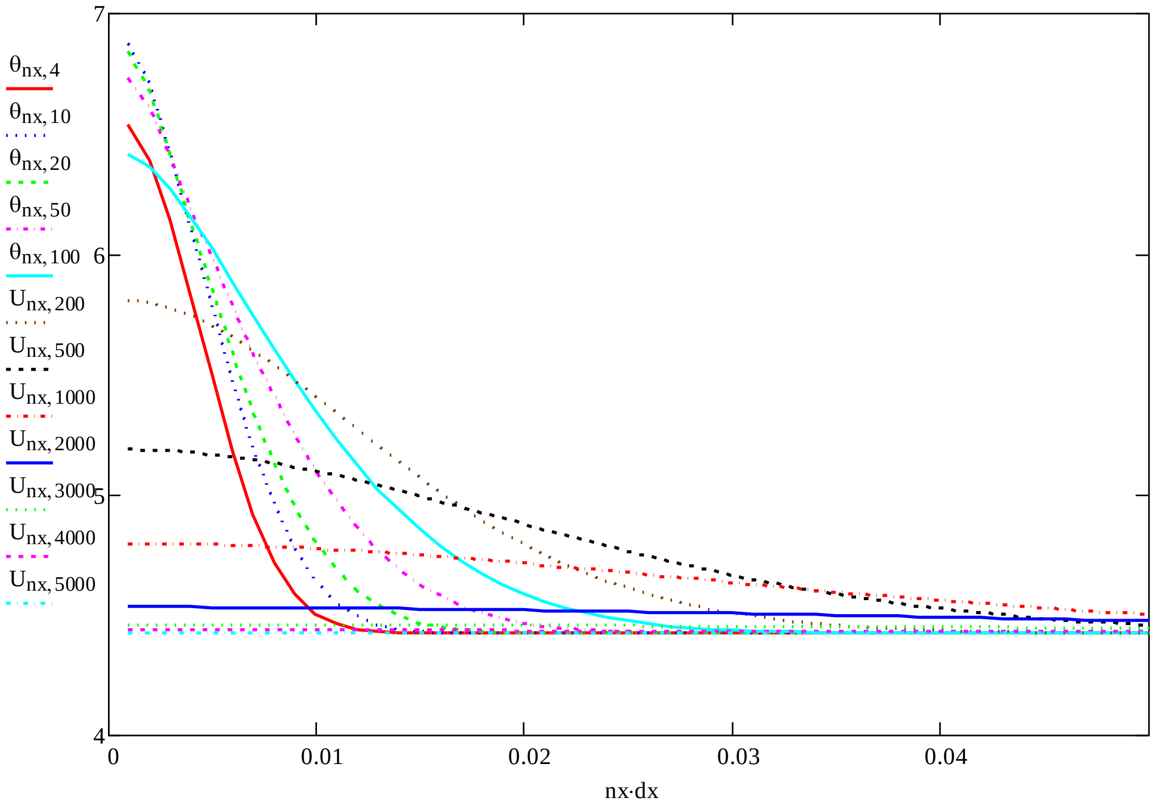
$\theta := u_s(nxm, ntm)$ $nx := 1..nxm$ $nt := 1..ntm$ $U := u_s(nxm, ntm)\cdot K^{-1}$

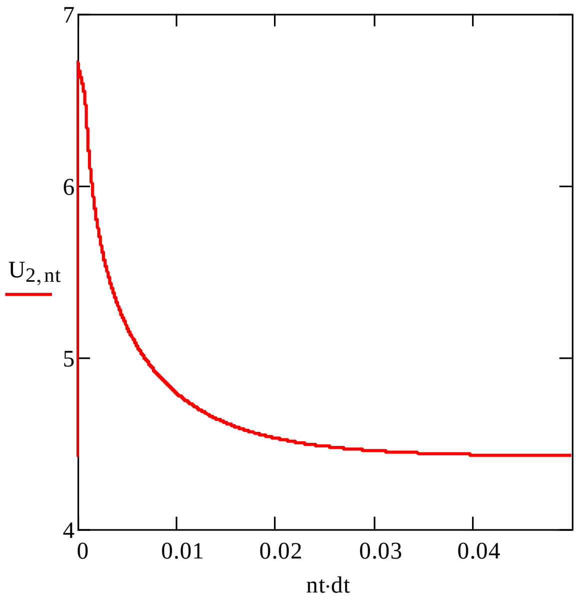
$par_0 := dx\cdot m^{-1}$ $par_1 := nxm$ $par_2 := dt\cdot sec^{-1}$ $par_3 := ntm$ $par_4 := \theta_i\cdot K^{-1}$ $par_5 := x_i\cdot m^{-1}$

$par_6 := \tau_i\cdot sec^{-1}$ $par_7 := Q_{it}\cdot joule^{-1}$

$dx = 1\times 10^{-3}m$ $nxm = 100$ $dx\cdot nxm = 0.1m$ $dt = 1\times 10^{-5}s$ $ntm = 5\times 10^3$ $dt\cdot ntm = 0.05s$

$\theta_i = 7.086K$ $x_i = 5\cdot mm$ $\tau_i = 5\times 10^{-5}s$ $Q_{it} = 1.92\times 10^{-3}\cdot joule$ $\theta_g = 6.233K$





1,8

$5 \cdot dt = 5 \times 10^{-5} s$

$10 \cdot dt = 1 \times 10^{-4} s$

$20 \cdot dt = 2 \times 10^{-4} s$

$50 \cdot dt = 5 \times 10^{-4} s$

$100 \cdot dt = 1 \times 10^{-3} s$

$200 \cdot dt = 2 \times 10^{-3} s$

$300 \cdot dt = 3 \times 10^{-3} s$

$500 \cdot dt = 5 \times 10^{-3} s$

$1000 \cdot dt = 0.01 s$

$2000 \cdot dt = 0.02 s$

$3000 \cdot dt = 0.03 s$

WRITEPRN("parCLAS7a5.prn") := par

WRITEPRN("tempCLAS7a5.prn") := U

$\theta_g = 6.233K$

$U_{3,2000} = 4.528$

$U_{3,2500} = 4.481$

$U_{3,3000} = 4.454$