Quench Calculations for the SHMS Q23

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1. Data

Table 1 summarizes the input data, which assumes a pressed conductor, with RRR=70 and a slightly compressed thickness for the insulation. Fig 1 shows the cross section.

Table 1: Parameters used in the Calculation

magnet self inductance	1.1H		coil B mean field at operating current	3.5T
magnet protection resistance	0.075W		coil A mean field at operating current	3.5T
coil inner radius	359.5mm		conductor width	19.2mm
coil outer radius	528.85mm		conductor thickness	3.05mm
coil B bottom angle	0		copper RRR	70
coil B top angle	21.5°		insulation thickness	0.45mm
coil A bottom angle	26.3°		cable width	11.68mm
coil A top angle	33.4°		cable mean thickness	1.16mm
number of turns in coil B	320		wire diameter	0.65mm
number of turns in coil A	107		wire matrix:superconductor ratio	1.6
mean perimeter of a turn in coil B	2955.3mm		number of wires	36
mean perimeter of a turn in coil A	3562.3mm		critical current at 5T	12333A
operating current	4250A		critical current at 6T	9875A
stored energy at operating current	9.93E+06J		critical current at 7T	7416A
coil B peak field at operating current	5T			

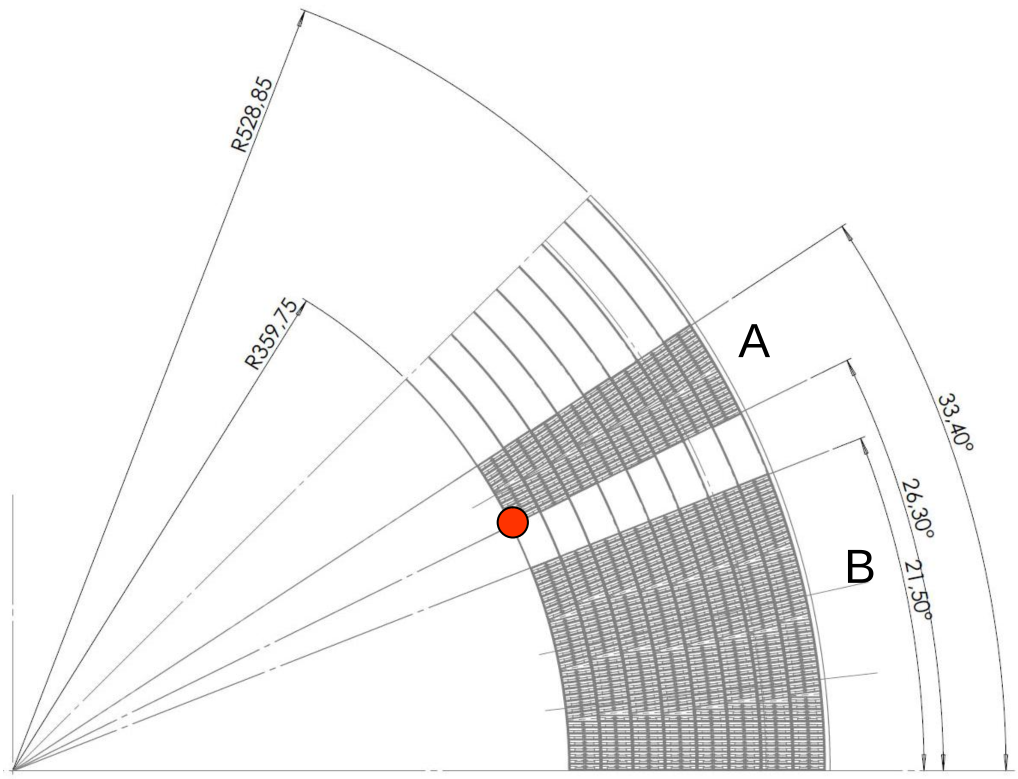


Fig 1: Cross section of the coils: quench assumed to start at red dot

2. The Calculation.

The QUENCH code is limited in assuming a constant field over the winding and I take this to be the mean field of ~ 3.5T. Extrapolating the measured data, I take the critical current to be 15860A, which gives a longitudinal quench velocity of 2.8ms^{-1} and a ratio α between transverse and longitudinal velocity of 0.026. Other details of the input data calculation are submerged in App 1.

As a worst case, I take a ‘natural quench’ with no protection resistor, assuming that the quench starts at the point shown in Fig 1 and is confined to coil A. Fig 2 plots the current decay and peak temperature and Fig 3 plots the internal voltage.

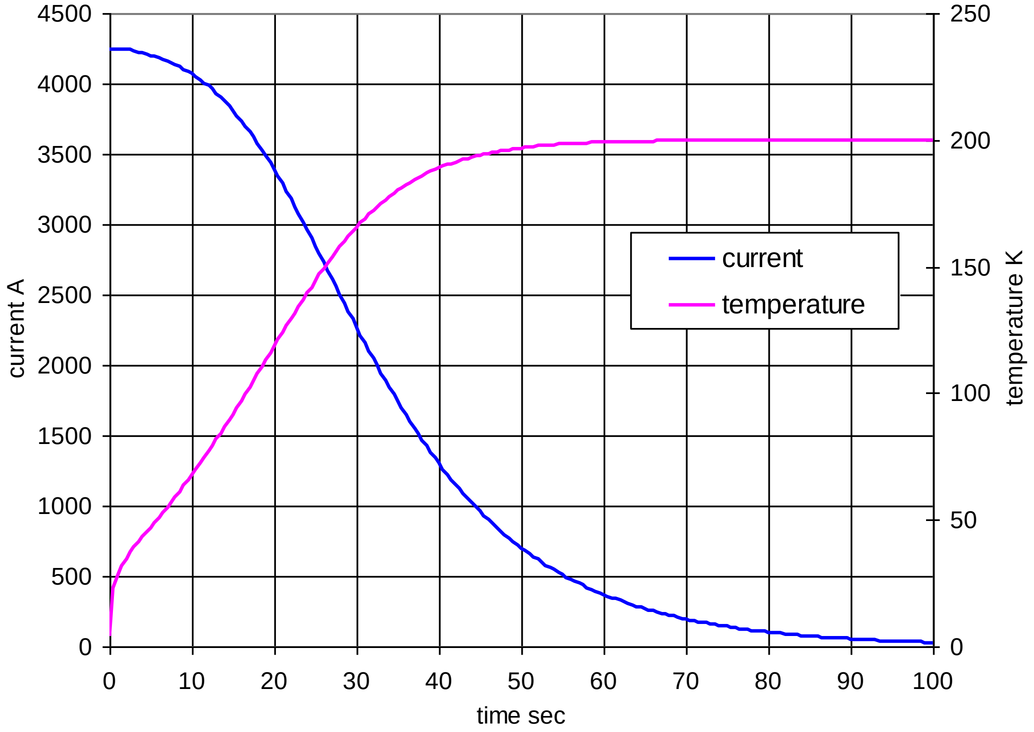


Fig 2: unprotected quench with resistive zone confined to coil A

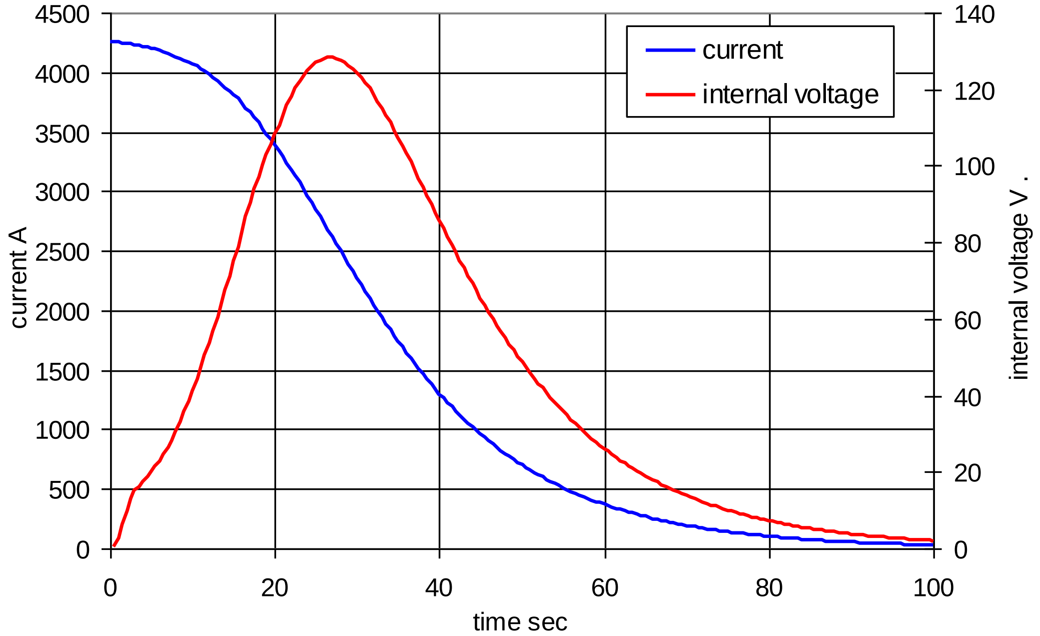


Fig 3: internal voltage of Fig 2 quench

The peak temp of 200K came as a surprise, given that the equivalent rise for the dipole was only 161K. The reason is my arbitrary decision to confine the quench to coil A. As a check on this, a simple a simple specific heat calculation with the coil energy is spread uniformly over segment A gives a temperature rise of 185K. Initially I had thought that, when the resistive zone reaches the boundary of segment A, there would be a substantial time delay as the heat diffused through the spacer before a resistive zone was triggered in segment B. QUENCH has provision for separated coils with time delays at boundaries, but only if the sections are shunted by individual resistors (not a fundamental limitation, just this version of QUENCH). However, I now realize that the assumption of time delay is wrong because the resistive zone will run along the conductor joining the two sections via the end spacer. With a longitudinal velocity of 2.8ms^{-1}, the time delay for this to happen will be negligible in comparison with the time scale of Fig 2, so we can reasonably ignore the spacer and just consider one big coil of sections A+B+B (joined along the mid plane). With this assumption, I find the decay curves of Fig 4 and 5, which should be taken as my best estimate of what will happen.

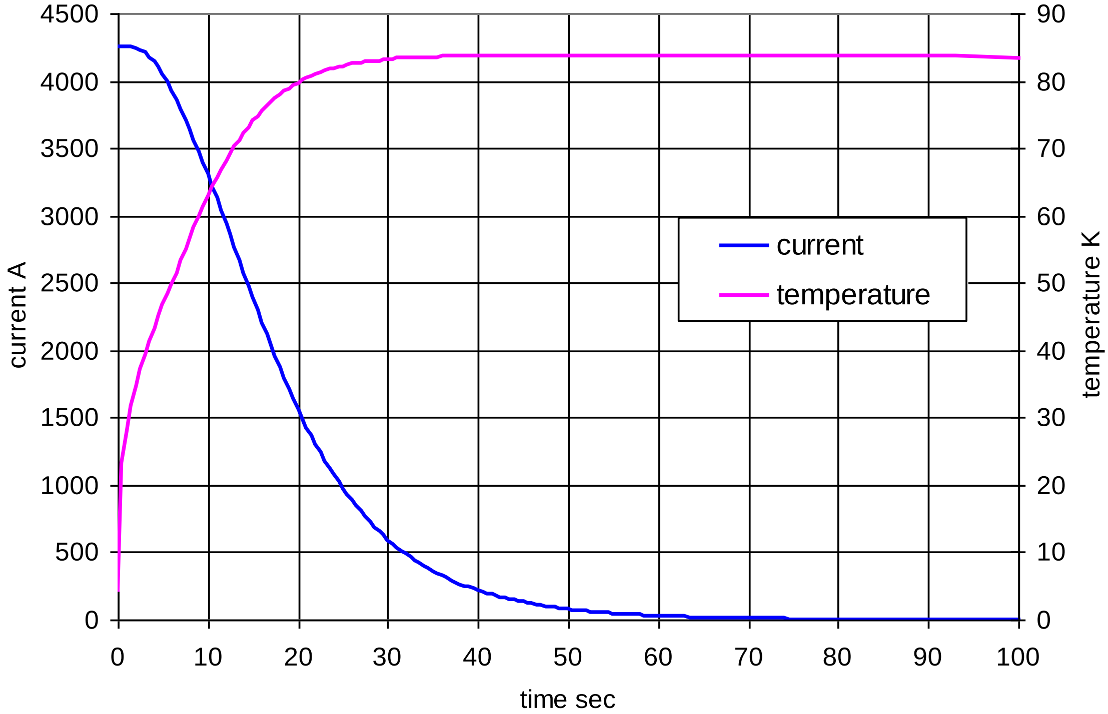


Fig 4: unprotected quench, ignoring the spacer between coils A and B

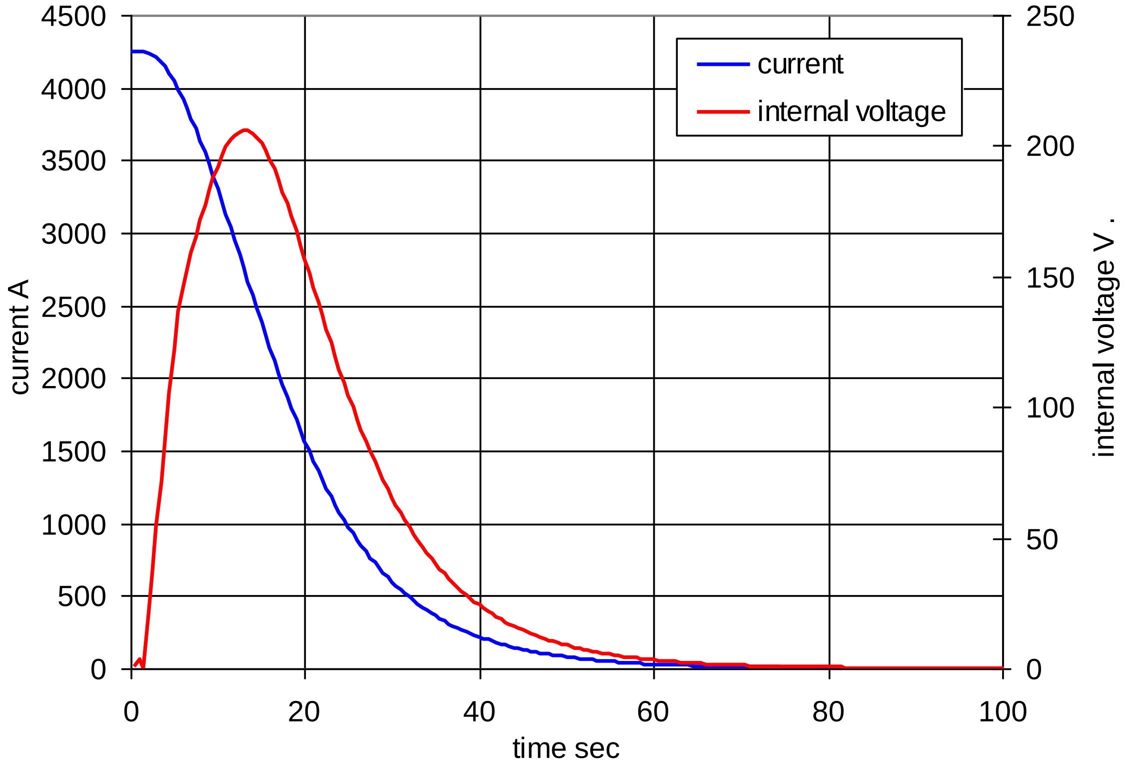


Fig 5: current and internal voltage for quench of Fig 4.

Finally, Figs 6 and 7 show the quench when a 0.075 protection is switched into circuit immediately the quench starts. In this case, the current decay is dominated by the external resistor and depends very little on assumptions about the coil boundaries. Here I show for coils A+B+B lumped together as in Fig 4.

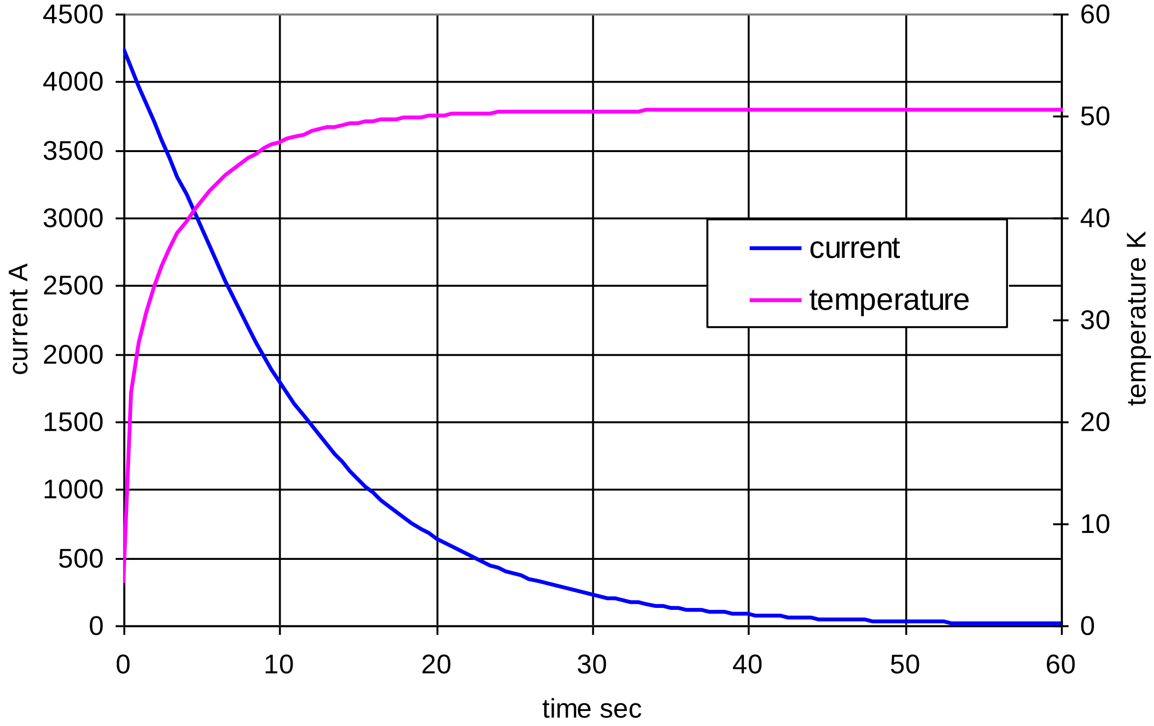


Fig 6: protected quench, ignoring the spacer between coils A and B

Fig 5: current and internal voltage for quench of Fig

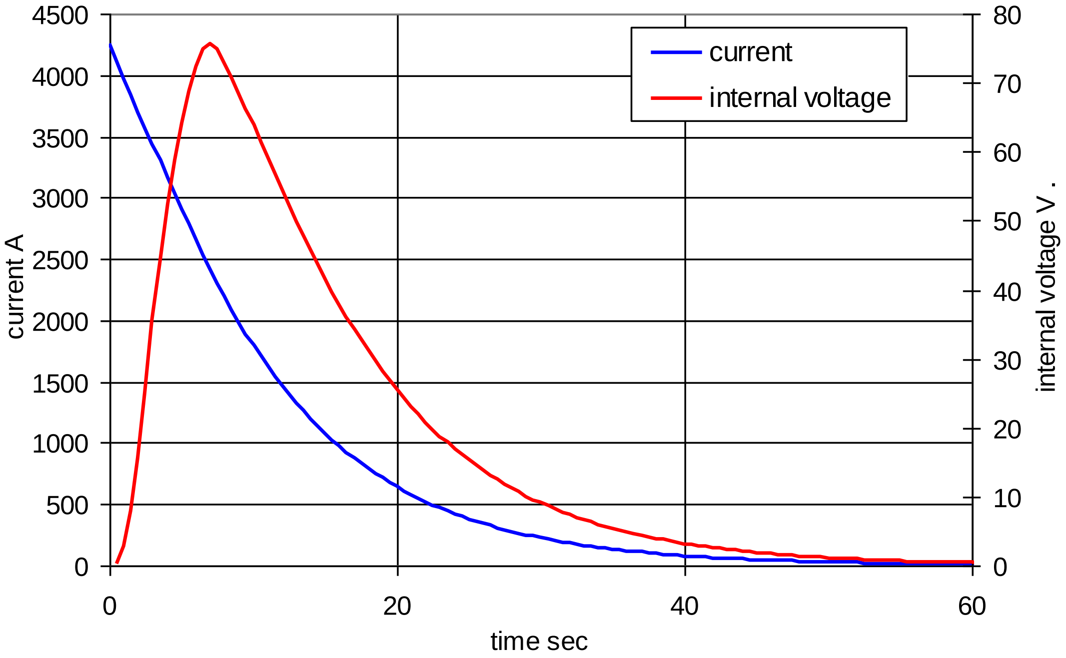


Fig 7: current and internal voltage for quench of Fig 6.

3. Conclusions.

Figs 4 to 7 represent my best estimates of the quench behaviour of the quadrupole, ie a final temperature of 84K unprotected and 51K protected. My earlier assumption of the resistive zone confined to coil A could be taken perhaps as an absolute worst case, but I feel that the assumptions involved are unrealistically pessimistic. Even so, the predicted final temperature of 200K is not dangerous – so no quench problems for this magnet.

Appendix 1: Mathcad Calculation of Input Parameters.

Quench Input Data for JLab Quadrupole - mean field properties 20 Aug 13

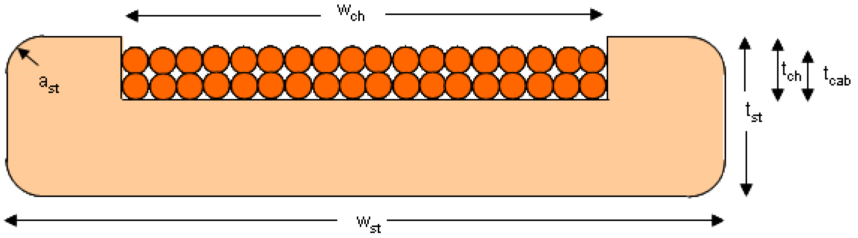
1) Winding Data $I_{op} := 4250 \cdot amp$ $B_m := 3.5T$ $\theta_o := 4.42K$ $\mu_o := 4 \cdot \pi \cdot 10^{-7} \cdot henry \cdot m^{-1}$

$I_{cd5} := 12333A$

$I_{cd6} := 9875A$

Ic from General Jc.xls

Ic at 3.5T and 4.42K $I_c := 15863A$



2) Conductor Geometry

ignore corner radii to make quench prog fit

stabilizer width $w_{st} := 19.2mm$ stabilizer thickness $t_{st} := 3.05mm$

channel width = cable width $w_{cab} := 11.68mm$ $w_{ch} := w_{cab} = 11.68mm$ $mat := 1.6$

channel thickness = cable thickness $t_{cab} := 1.16mm$ $t_{ch} := t_{cab} = 1.16mm$

wire dia $d_w := 0.65mm$ number of wires $N_w := 36$

wire area $A_w := N_w \cdot \frac{\pi}{4} \cdot d_w^2 = 11.946mm^2$ $\lambda_{cab} := \frac{A_w}{w_{cab} \cdot t_{cab}} = 0.882$ channel occupied by cable $A_{ch} := w_{ch} \cdot t_{ch} = 13.549mm^2$

wire copper area $A_{wcu} := A_w \cdot \frac{mat}{1 + mat} = 7.351mm^2$ wire NbTi area $A_{nt} := A_w \cdot \frac{1}{1 + mat} = 4.595mm^2$

solder area $A_{so} := A_{ch} - A_w = 1.603mm^2$ conductor area $A_{co} := w_{st} \cdot t_{st} = 58.56mm^2$

insulation thickness $t_i := 0.45mm$ interlayer insulation $t_{il} := 0.5mm$

width unit cell $w_u := w_{st} + 2 \cdot t_i + t_{il} = 20.6mm$ thickness unit cell $t_u := t_{st} + 2 \cdot t_i = 3.95mm$

unit cell area $A_u := w_u \cdot t_u = 81.37mm^2$ insulation area $A_i := A_u - A_{co} = 22.81mm^2$

stabilizer area $A_{st} := A_{co} - w_{ch} \cdot t_{ch} = 45.011mm^2$ mat for quench $mat_q := \frac{A_{st} + A_{wcu}}{A_{nt}} = 11.397$

over unit cell $\lambda_{st} := \frac{A_{st}}{A_u} = 0.553$ $\lambda_{wcu} := \frac{A_{wcu}}{A_u} = 0.09$ $\lambda_{Cu} := \lambda_{st} + \lambda_{wcu} = 0.644$ $\lambda_{so} := \frac{A_{so}}{A_u} = 0.02$

$\lambda_{nt} := \frac{A_{nt}}{A_u} = 0.056$ $\lambda_i := \frac{A_i}{A_u} = 0.28$ check $\lambda_{Cu} + \lambda_{nt} + \lambda_{so} + \lambda_i = 1$ $J_{op} := \frac{I_{op}}{A_u} = 52.231A \cdot mm^{-2}$

for quench solder thickness $t_{sq} := \frac{A_{so}}{w_{st}} = 8.348 \times 10^{-3} \cdot cm$ conductor thickness $t_{cq} := t_{st} - t_{sq} = 0.297cm$

check $\lambda_{cq} := \frac{t_{cq} \cdot w_{st}}{A_u} = 0.7$ $\lambda_{Cu} + \lambda_{nt} = 0.7$ $\lambda_{soq} := t_{sq} \cdot \frac{w_{st}}{A_u} = 0.02$

2) Geometry for Quench Data sheath is under insulation, so inner and outer widths are stabilizer and unit c

put on the solder as a thin strip 'sheath' $A_{so} = 1.603mm^2$ $t_{so} := \frac{A_{so}}{w_{st}} = 8.348 \times 10^{-3} \cdot cm$

thicknes of condr without solder $t_{con} := t_{st} - t_{so} = 0.297cm$ $t_{con} \cdot w_{st} + A_{so} = 58.56mm^2$ check

inner width of sheath radial $w_{isr} := t_{con} = 0.297cm$ inner width of sheath axial $w_{isa} := w_{st} = 1.92cm$

outer width of sheath radial $w_{osr} := t_{st} = 0.305cm$ outer width of sheath axial $w_{osa} := w_{st} = 1.92cm$

3) Superconductor critical temperature from General Jc.xls, mean field $B_m = 3.5T$

$C_o := 21.77K$ $C_1 := -0.278K\cdot tesla^{-1}$ $C_2 := -0.015K\cdot tesla^{-2}$ $n := 0.032$ $B_{c2} := 14.05tesla$

$C_3 := -13.60K$ $\theta_c(B) := C_o + C_1\cdot B + C_2\cdot B^2 + \dfrac{C_3}{\left[\left(B_{c2} - B\right)\cdot T^{-1}\right]^n}$ $\theta_c(B_m) = 8.001K$

4) Current Sharing $\theta_g := \theta_c(B_m) - \left(\theta_c(B_m) - \theta_o\right)\cdot\dfrac{I_{op}}{I_c}$ $\theta_g = 7.042K$ $\theta_s := \dfrac{\theta_c(B_m) + \theta_g}{2}$ $\theta_s = 7.521K$

$\theta_t := \dfrac{\theta_s + \theta_c(B_m)}{2}$ $\theta_t = 7.761K$

5) Resistivity magnetoresistance from Copper magres Fickett.xls resistivity NbTi $\rho_{nt} := 6\cdot10^{-7}\cdot ohm\cdot m$

$\rho_{RT} := 1.69\,10^{-8}\cdot ohm\cdot m$ $m_{\rho c} := 4\cdot10^{-11} ohm\cdot m\cdot T^{-1}$

stabilizer $RRR_{st} := 70$ $\rho_{st}(B) := \dfrac{\rho_{RT}}{RRR_{st}} + m_{\rho c}\cdot B$ $\rho_{st}(B_m) = 3.814\times 10^{-10}\cdot ohm\cdot m$

wire $RRR_w := 150$ $\rho_{wcu}(B) := \dfrac{\rho_{RT}}{RRR_w} + m_{\rho c}\cdot B$ $\rho_{wcu}(B_m) = 2.527\times 10^{-10}\cdot ohm\cdot m$

$$\frac{1}{R_u} = \frac{1}{R_{NbTi}} + \frac{1}{R_{ch}} + \frac{1}{R_w} = \frac{A_{NbTi}}{\rho_{NbTi}\cdot L} + \frac{A_{ch}}{\rho_{ch}\cdot L} + \frac{A_{wcu}}{\rho_{wcu}\cdot L} \qquad \rho_u = R_u\cdot\frac{A_u}{L} = \frac{1}{L}\cdot\frac{A_u}{\dfrac{A_{NbTi}}{\rho_{NbTi}}\cdot L + \dfrac{A_{ch}}{\rho_{ch}}\cdot L + \dfrac{A_{wcu}}{\rho_{wcu}}\cdot L}$$

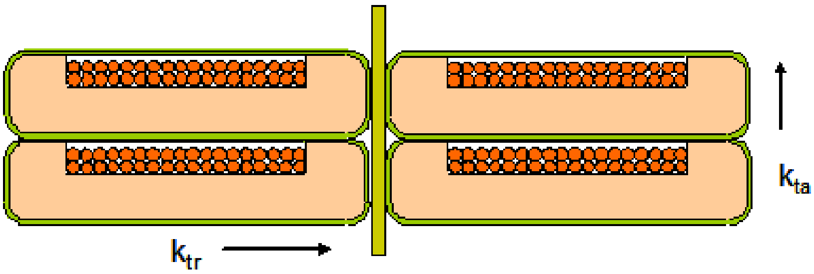
$$\rho_u := \frac{1}{\dfrac{\lambda_{nt}}{\rho_{nt}} + \dfrac{\lambda_{st}}{\rho_{st}(B_m)} + \dfrac{\lambda_{wcu}}{\rho_{wcu}(B_m)}} \qquad \rho_u = 5.531\times 10^{-10}\cdot ohm\cdot m$$

6) Longitudinal Thermal conductivity (over unit cell)

$L_o := 2.45\,10^{-8}\cdot watt\cdot ohm\cdot K^{-2}$ $k(\theta)\cdot\rho(\theta) = L_o\cdot\theta$ $k(\theta) = \dfrac{L_o\cdot\theta}{\rho(\theta)}$

over the unit cell $k_u(\theta) := \dfrac{L_o\cdot\theta}{\rho_u}$ $k_u(\theta_o) = 195.778m^{-1}\cdot K^{-1}\cdot watt$ $k_u(\theta_t) = 343.768m\cdot kg\cdot K^{-1}\cdot s^{-3}$

7) Transverse Thermal Conductivity



for insulation take John Ross G10 at transition temperature

$k_i(\theta) = \kappa_i\cdot\theta^{n_i}$

$n_i := 0.8$ $\kappa_i := 0.017watt\cdot m^{-1}\cdot K^{-1}$

$k_i := \kappa_i\cdot\left(\theta_t\cdot K^{-1}\right)^{n_i} = 0.088watt\cdot m^{-1}\cdot K^{-1}$

corner radius of strip $a_{st} := 0.25mm$

a) Radial conduction per unit length of unit cell $Q_{r'} = k_i\cdot\left(t_{st} - 2\cdot a_{st}\right)\cdot\dfrac{\Delta\theta}{2\cdot t_i + t_{il}}$ $Q'_r = k_{tr}\cdot t_u\cdot\dfrac{\Delta\theta}{w_u}$

$$k_{tr} = k_i\cdot\left(t_{st} - 2\cdot a_{st}\right)\cdot\frac{\Delta\theta}{2\cdot t_i + t_{il}}\cdot\frac{1}{t_u}\cdot\frac{w_u}{\Delta\theta} = k_i\cdot\frac{\left(t_{st} - 2\cdot a_{st}\right)}{\left(2\cdot t_i + t_{il}\right)}\cdot\frac{w_u}{t_u} \qquad k_{tr} := k_i\cdot\frac{\left(t_{st} - 2\cdot a_{st}\right)}{\left(2\cdot t_i + t_{il}\right)}\cdot\frac{w_u}{t_u} = 0.832watt\cdot m^{-1}\cdot K^{-1}$$

b) Axial conduction

$$Q_{a'} = k_i \cdot (w_{st} - 2 \cdot a_{st}) \cdot \frac{\Delta\theta}{2 \cdot t_i} \qquad Q'_a = k_{ta} \cdot w_u \cdot \frac{\Delta\theta}{t_u}$$

$$k_{ta} = k_i \cdot (w_{st} - 2 \cdot a_{st}) \cdot \frac{\Delta\theta}{2 \cdot t_i} \cdot \frac{1}{w_u} \cdot \frac{t_u}{\Delta\theta} = k_i \cdot \frac{(w_{st} - 2 \cdot a_{st})}{2 \cdot t_i} \cdot \frac{t_u}{w_u} \qquad k_{ta} := k_i \cdot \frac{(w_{st} - 2 \cdot a_{st})}{2 \cdot t_i} \cdot \frac{t_u}{w_u} = 0.349\,\mathrm{watt \cdot m^{-1} \cdot K^{-1}}$$

take the mean $\quad k_t := \dfrac{k_{tr} + k_{ta}}{2} = 0.59\,\mathrm{watt \cdot m^{-1} \cdot K^{-1}}$

8) Specific Heat use standard quench form $C = Cf \cdot \theta^{c} + D \cdot \theta^{d}$ no units

a) Copper from Cu spec heat.xls $\quad Cf_c := 0.01149 \quad c_c := 1.07 \quad D_c := 0.00046 \quad d_c := 3.21$

$C_{Cu}(\theta) := Cf_c \cdot \theta^{c_c} + D_c \cdot \theta^{d_c} \qquad C_{Cu}(4.) = 0.09 \qquad \gamma_{Cu} := 8.89 \cdot 10^{3} \qquad \gamma_{Cu} \cdot C_{Cu}(7) = 2.93 \times 10^{3}$

b) NbTi below ?c from NbTi spec heat.xls $\quad Cf_n := -0.0976 \quad c_n := 1.1 \quad D_n := 0.0540 \quad d_n := 2.17$

$C_{nt}(\theta) := Cf_n \cdot \theta^{c_n} + D_n \cdot \theta^{d_n} \qquad C_{nt}(4.) = 0.645 \qquad \gamma_{nt} := 6.2 \cdot 10^{3} \qquad \gamma_{nt} \cdot C_{nt}(7) = 1.769 \times 10^{4}$

c) Solder $\quad Cf_{so} := 3.91 \cdot 10^{-2} \quad c_{so} := 0.7 \quad D_{so} := 9.38 \cdot 10^{-5} \quad d_{so} := 5 \quad \gamma_{so} := 10.49 \cdot 10^{3}$

$C_{so}(\theta) := Cf_{so} \cdot \theta^{c_{so}} + D_{so} \cdot \theta^{d_{so}} \qquad C_{so}(4) = 0.199 \qquad \gamma_{so} \cdot C_{so}(7) = 1.814 \times 10^{4}$

d) G10 from G10 spec heat.xls $\quad C_g := 0.0225 \quad c_g := 3 \quad D_g := -.00646 \quad d_g := 3.263$

$C_{G10}(\theta) := C_g \cdot \theta^{c_g} + D_g \cdot \theta^{d_g} \qquad C_{G10}(4) = 0.845 \qquad \gamma_{G10} := 1.8 \cdot 10^{3} \qquad \gamma_{G10} \cdot C_{G10}(7) = 7.238 \times 10^{3}$

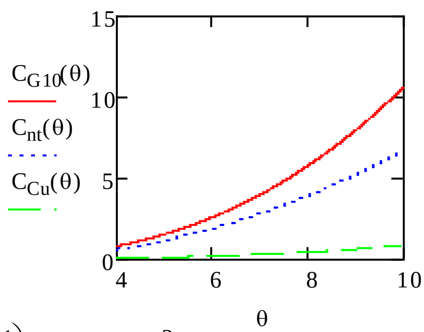
overall spec heat / volume of metal only $\quad \gamma C_{um}(\theta) := (\lambda_{Cu}) \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + \lambda_{nt} \cdot \gamma_{nt} \cdot C_{nt}(\theta) + \lambda_{so} \cdot \gamma_{so} \cdot C_{so}(\theta)$

overall spec heat / volume $\quad \gamma C_u(\theta) := (\lambda_{Cu}) \cdot \gamma_{Cu} \cdot C_{Cu}(\theta) + \lambda_{nt} \cdot \gamma_{nt} \cdot C_{nt}(\theta) + \lambda_{so} \cdot \gamma_{so} \cdot C_{so}(\theta) + \lambda_i \cdot \gamma_{G10} \cdot C_{G10}(\theta)$

enthalpy change metal only $\quad \gamma H_{um} := \displaystyle\int_{\theta_s}^{\theta_c(B_m)} \gamma C_{um}(\theta \cdot K^{-1})\, d\theta \qquad \gamma H_{um} = 2.088 \times 10^{3}\,K \qquad \theta_s = 7.521\,K$

averaged spec heat metal only $\quad \gamma C_{aum} := \dfrac{\gamma H_{um}}{\theta_c(B_m) - \theta_s} \qquad \gamma C_{aum} = 4.353 \times 10^{3}$

$\gamma C_{um}(\theta_s \cdot K^{-1}) = 3.972 \times 10^{3}$



enthalpy change unit cell $\quad \gamma H_u := \displaystyle\int_{\theta_s}^{\theta_c(B_m)} \gamma C_u(\theta \cdot K^{-1})\, d\theta \qquad \gamma H_u = 3.382 \times 10^{3}\,K$

averaged spec heat unit cell $\quad \gamma C_{au} := \dfrac{\gamma H_u}{\theta_c(B_m) - \theta_s} \qquad \gamma C_{au} = 7.05 \times 10^{3} \qquad \gamma C_u(\theta_s \cdot K^{-1}) = 6.445 \times 10^{3}$

9) Quench velocity

longitudinal velocity $v_{ad} = \frac{J}{\gamma \cdot C(\theta_s)} \cdot \sqrt{\frac{\rho \cdot k}{\theta_s - \theta_o}}$ $v_{ad} := \frac{J_{op}}{\gamma C_{aum} \cdot J \cdot m^{-3} \cdot K^{-1}} \cdot \sqrt{\frac{\rho_u \cdot k_u(\theta_t)}{\theta_t - \theta_o}}$ $v_{ad} = 286.272 cm \cdot s^{-1}$

transverse velocity $\alpha := \frac{\gamma C_{aum}}{\gamma C_{au}} \cdot \sqrt{\frac{k_t}{k_u(\theta_t)}}$ $\alpha = 0.026$

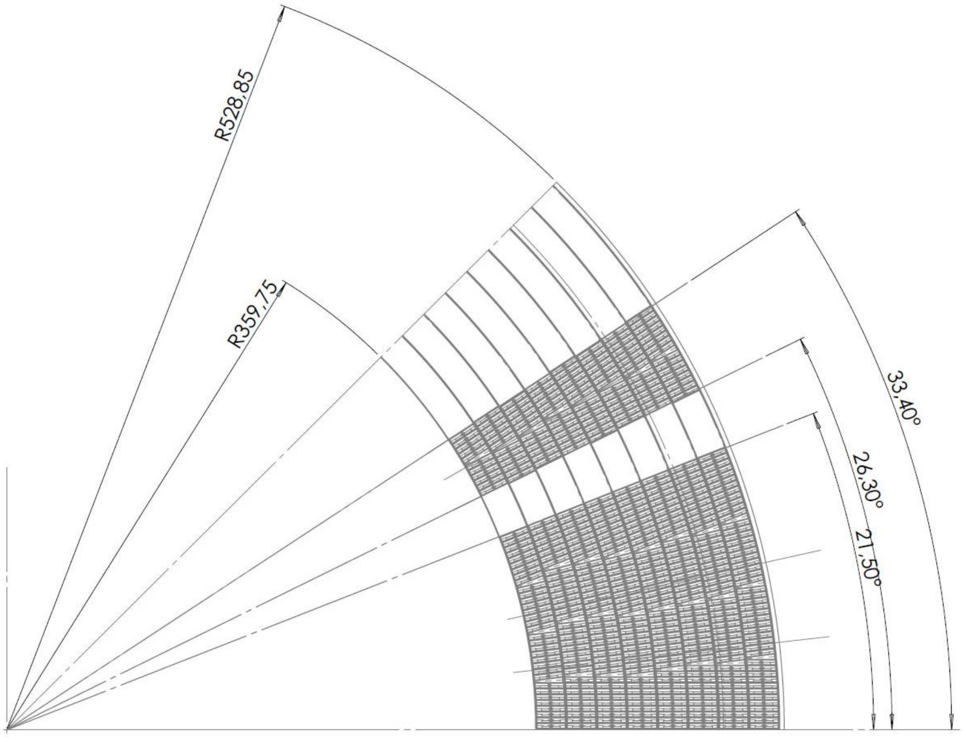
$J_{op} = 52.231 A \cdot mm^{-2}$ $\gamma C_{aum} = 4.353 \times 10^3$ $\rho_u = 5.531 \times 10^{-10} \cdot ohm \cdot m$ $k_u(\theta_t) = 343.768 m^{-2} \cdot watt \cdot m \cdot K^{-1}$

$\theta_t = 7.761 K$

10) Coil geometry

take dimensions from 318711-squelette-Q POLES-VIII.pdf

Assume quench starts in the upper sector A



inner radius $r_i := 359.75 mm$ outer radius $r_o := 528.85 mm$ radial thickness $t_{rA} := r_o - r_i = 16.91 cm$

number of turns $N_{tA} := 11 + 12 + 12 + 13 + 14 + 14 + 15 + 16 = 107$

area of coil $A_{cA} := N_{tA} \cdot A_u = 8.707 \times 10^3 \cdot mm^2$ mean width of coil $w_{cA} := \frac{A_{cA}}{t_{rA}} = 5.149 cm$

check mean radius $r_{mA} := \frac{r_i + r_o}{2} = 444.3 mm$ sector spread $ss := 33.4 - 26.3 = 7.1$

mean width check $w_{cAc} := r_{mA} \cdot \frac{ss \cdot 2 \cdot \pi}{360} = 5.506 cm$ difference must be triangular spacer

mean turns perimeter $p_u := 3.562 m$ equivalent solenoid inner radius $r_{us} := \frac{p_u}{2 \cdot \pi} = 56.691 cm$

For Series C add coil B + B

$N_{tB} := 33 + 35 + 37 + 39 + 41 + 43 + 45 + 47 = 320$

area of coil $A_{cB} := N_{tB} \cdot A_u = 2.604 \times 10^4 \cdot mm^2$ mean width of coil $w_{cB} := \frac{A_{cB}}{t_{rA}} = 15.398 cm$

effective width for quench $w_{ceQ} := w_{cA} + 2 \cdot w_{cB} = 35.945 cm$ number of turns $N_{tA} + 2 \cdot N_{tB} = 747$