3 First moment of g_2^p : $\Gamma_2^p(Q^2)$

The Burkhardt-Cottingham sum rule:

$$\Gamma_2^p(Q^2) = \int_0^1 g_2(x, Q^2)dx = 0 \tag{5}$$

3.1 DIS contribution to $\Gamma_2(Q^2)$

One way to estimate the unmeasured contribution to Γ_2 as $x \to 0$ is to assume that the Wandzura-Wilczek [16] relation holds:

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy \tag{6}$$

and that

$$\begin{aligned} \int_0^x g_2(y, Q^2)dy &\simeq \int_0^x g_2^{WW}(y, Q^2)dy && (7) \\ &= x \int_x^1 \frac{g_1(y, Q^2)}{y} dy && (8) \\ &= x \left[g_2^{WW}(x, Q^2) + g_1(x, Q^2) \right] \end{aligned}$$

where Eq. 8 follows from integration by parts (See for example Ref. [19]). This was the method used in E155x [17], and E94010 [18]. E94010 assumed an (arbitrary) 20% error on this contribution, while E155x made no estimate of the uncertainty. We have two choices as to how to proceed:

1. Form g_2^{WW} over the full range $0 \leq x \leq 1$ and then numerically integrate. This requires a model of g_1 for $0 \leq x \leq x_0$
2. Use Eq. 8, which depends only on the experimentally measured g_1.

In both cases we use the RSS fit to g_1 in the measured region. For the numerical integration, we chose the Bianchi and Thomas fit.

Γ_2^{DIS}	method
0.0262	Eq. 7 with g1 from BT
0.0254	Eq. 7 with g1 from AAC-1
0.0281	Eq. 7 with g1 from AAC-2
0.0275	Eq. 7 with g1 from GRSV-1
0.0276	Eq. 7 with g1 from GRSV-2
0.0256	Eq. 7 with g1 from OR
0.0263	Eq. 8 (what I use)

Maximum spread is 9.7%.

3.2 Plot Quantities

1. Elastic : $\Gamma_2^{el} = G_M^P(G_E^P - G_M^P)\tau/(2(1+\tau))$ with the form factors from Ref. [15]. I assume 5% relative uncertainty on Γ_2^{el}. At $Q^2 = 1.3$ $\Gamma_2^{el} \approx \Gamma_2^{res}$.

2. MAID : MAID 2003 model, Ref. [12].

3. E155x : From Ref. [17]. E155x covered the range $0.02 < x < 0.8$ at $Q^2 = 5$ GeV^2. $\Gamma_2^{E155x}(5) = 0.044 \pm 0.008 \pm 0.003$. The uncertainty is dominated by statistics. The average of E155x,E143 and E155 gives $\Gamma_2^{E155x}(5) = 0.042 \pm 0.008$, so we use the E155x number instead. For the unmeasured contribution $0 < x < 0.02$, they assume $\int_0^x g_2^{WW}(y)dy = x(g_2^{WW} + g_1(x)) = 0.020$, and include no error for this contribution. This brings the E155x number to $0.044 \pm 0.008 \pm 0.003$.

4. For the resonance region, we assume a 6.82% relative systematic following the conclusion of Oscar's study [1]. Note this study was only for Γ_1. Stat. error on the resonance fit is scaled data stat error.

References

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[3] E143 : P.L. Anthony *et al.*, Phys. Rev. **D58** (1998) 112003.