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RSS Moments
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DRAFT

We summarize in this short note some of the details of the RSS moments.

1 First moment of g_1^p : $\bar{\Gamma}_1^p(Q^2)$

The first moment, $\bar{\Gamma}_1^p(Q^2)$ is defined as:

$$\bar{\Gamma}_1^p(Q^2) = \int_0^{1-\epsilon} g_1(x, Q^2)dx \quad (1)$$

where we have excluded the elastic contribution at $x = 1$. The RSS resonance region data covers the range $1075 < W < 1910$, $(0.3161 < x < 0.823)$. The integration is performed using the RSS g_1 fit [2] evaluated at $Q^2 = 1.279$ GeV^2. For the resonance region, we assume a 6.82% relative systematic following the conclusion of Oscar's study [1].

1.1 DIS contribution to $\Gamma_1(Q^2)$

The remainder of the integral $(0 < x < 0.316, dx = 0.005)$ must be estimated. We discuss several alternatives in the following sections.

1.1.1 Bianchi and Thomas Model

The Regge based global fit of Bianchi and Thomas [13] can be used to provide an estimate for the unmeasured contribution to the integral. This fit is based on 238 proton data covering the range $0.3 < Q^2 < 70$ GeV^2, $4 < W^2 < 300$ GeV^2. 192 deuteron data covering the same range as the proton, and 81 neutron data covering $1 < Q^2 < 15$ GeV^2, $4 < W^2 < 70$ GeV^2. It is intended to provide a smooth extrapolation to $Q^2 = 0$. The code provides a fit to :

$$\sigma_{TT} = \frac{4\pi^2\alpha}{MK}(g_1 - \gamma^2 g_2)$$

To obtain g_1, we assume that $\gamma^2 g_2$ is negligible* in the region in question.

*This factor contributes only 2.5% relative to our DIS integral.

This fit relies on 9 parameters, each of which has an associated error. In order to propogate these errors through to the integral, we consider the DIS integral as a parameter-dependent function f:

$$f(Q^2 : p_1, p_2, \ldots, p_i, \ldots, p_N) = \int g_1(x, Q^2 : p_1, p_2, \ldots, p_i, \ldots, p_N) dx \qquad (2)$$

Neglecting any possible correlations between the fit parameters, we observe:

$$\Delta f^2 = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial p_i} \right)^2 \Delta p_i^2$$

where Δp_i is the quoted error [13] for the i^{th} parameter. In lieu of an analytic expression for the derivatives called for in eq. 3, we utilize the numerical approximation:

$$\left(\frac{\partial f}{\partial p_i} \right) \approx \frac{f(Q^2 : p_1, \ldots, p_i + \delta p_i, \ldots, p_N) - f(Q^2 : p_1, \ldots, p_i - \delta p_i, \ldots, p_N)}{2\delta p_i} \qquad (3)$$

Here, δp_i is a small differential in the parameter p_i, which we take as $0.1\Delta p_i$. The specific value 0.1 is quite arbitrary and the results are only very weakly dependent on it. With these assumptions, we find:

$$\begin{aligned} \Gamma_1^{DIS}(Q^2 = 1.279) &= \int_0^{0.316} g_1^{DIS}(x) dx \\ &= 0.0681 \pm 0.0099 \text{ (proton)} \\ &= -0.0288 \pm 0.0098 \text{ (neutron)} \end{aligned}$$

1.1.2 Oscar's Regge fit

An alternative estimate is provided by Oscar's Regge inspired fit to SLAC E143 and E155 g_1^p:

$$g_1^p = a x^b (1-x)^3 (1 + c/Q^2) \qquad (4)$$

where:

$$\begin{aligned} a &= 0.392 \pm 0.254 \\ b &= 0.00676 \pm 0.084 \\ c &= 0.0636 \pm 0.681 \end{aligned}$$

Integration and propogation of the fit errors results in:

$$\Gamma_1^{DIS}(Q^2 = 1.279) = \int_0^{0.316} g_1^{DIS}(x)dx$$
$$= 0.06833 \pm 0.00692 \text{ (proton)}$$

For full details of the fit, see Oscar's discussion in Ref. [14].

Since the two Regge methods provide essentially the same result, we utilize Oscar's results to take advantage of the reduced systematic error.

1.1.3 PDFs

In addition to the above mentioned DIS estimates, we can also form the DIS integral from the available target mass corrected PDF's. I have not attempted to propogate the PDF errors to g_1, but for reference, here are the results:

Γ_1^{DIS}	PDF
0.0796	AAC00 NLO-1 with TMC
0.0861	AAC00 NLO-2 with TMC
0.0827	GRSV NLO-1 with TMC
0.0830	GRSV NLO-2 with TMC

2 Notes on the Plot quantities

The various theory curves and world data in the Γ_1 plots, (Figs. 1,2), are explained here in detail. In all cases[†], the inner error bars represent statistical uncertainty, while the outer represents the total uncertainty.

1. SMC : Here, the value comes from Table XXVII in the E143 long paper [3]. The original SMC proton publications are Refs. [4]. SMC measured at an average Q^2 of 10 GeV^2. $\Gamma_1(10) = 0.136$. The 5 GeV^2 number is a global analysis of the (then) available world data evolved to lower Q^2. $\Gamma_1(5) = 0.140$.

2. E155 : Single point at $Q^2 = 5$. From Ref. [6].

[†]Umm..., except for SMC where only total error is shown

3. E143 : data from Table XXXVIII ($Q^2 = 0.5$ and 1.2) and Table XXVI ($Q^2 = 2, 3, 5$) of Ref. [3].

4. Hermes : From latest long paper, Ref. [7].

5. EG1A : For the resonance region, I plot Table C.10 of R. Fatemi's thesis [9], which has an upper W limit on the integration of 2 GeV. This differs slightly from RSS's upper limit of 1.910 GeV. Also note that this "EG1A resonance" is quite different than what is in the official R. Fatemi PRL [8]. In the PRL, the resonance integral includes all measured data, so the upper limit of integration ranges from 2.0 to 2.6 GeV, as detailed in Table 5.2 of R. Fatemi's thesis [9].

6. EG1A Resonance+DIS : This is not shown on the plot since it is consistent with the more precise and recent EG1B result. However, note that the EG1A DIS estimate comes from the Hall-B "Models" integrated to $x = 10^{-5}$. The systematic error of the DIS contribution arises from looking at the variation of three possible models: the Simula model, the Hall B fit prior to EG1A, and the Hall B fit including EG1A data.

7. EG1B : The EG1B data is split between two files with differing incident energy. There are four Q^2 bins of overlap in the range $0.6 < Q^2 < 1.0$ GeV^2. Here I took the statistics weighted average to combine. Only the total integral is shown.

8. AO : Ref. [10].

9. PQCD : Ref. [11].

10. MAID : MAID 2003 model, Ref. [12]. For refererence, we show the MAID model integrated over the resonance region($W_\pi < W < 2$ GeV) and RSS's slightly more restricted range ($W_\pi < W < 1.910$ GeV).

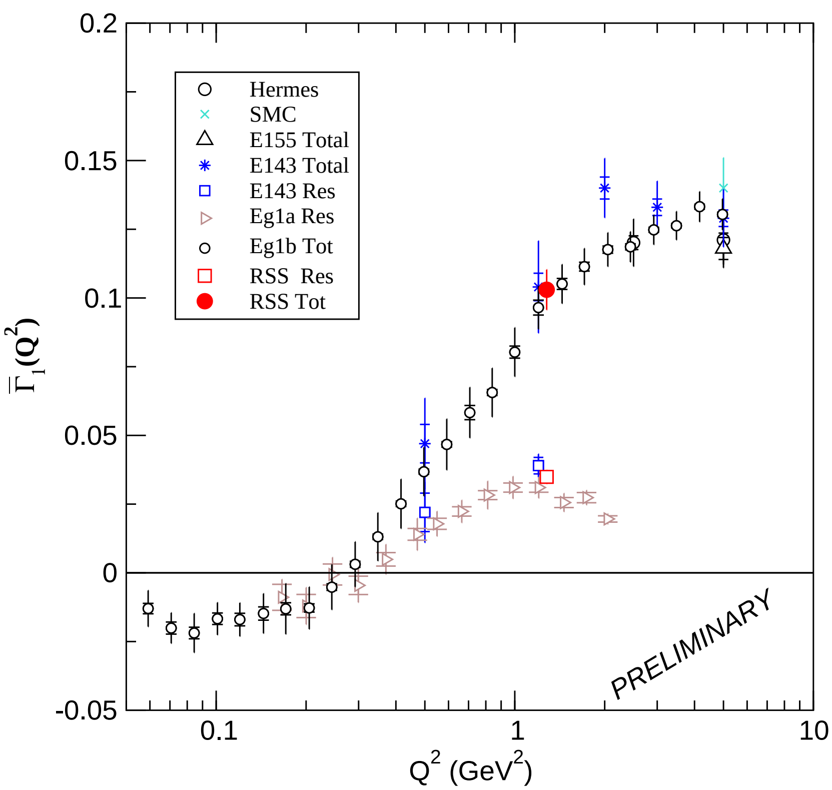

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Figure 1: $\Gamma_1^p(Q^2 = 1.279)$.

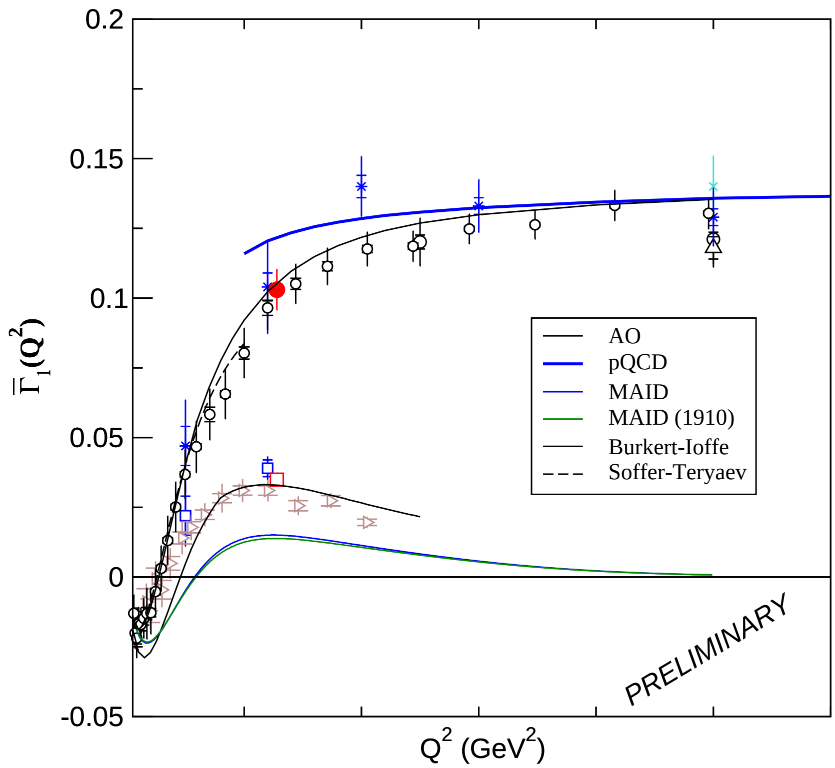

Figure 2: $\Gamma_1^p(Q^2 = 1.279)$.

3 First moment of g_2^p : $\Gamma_2^p(Q^2)$

The Burkhardt-Cottingham sum rule [16] states that:

$$\Gamma_2^p(Q^2) = \int_0^1 g_2(x, Q^2)dx = 0 \tag{5}$$

3.1 DIS contribution to $\Gamma_2(Q^2)$

In practice, we measure to some lowest $x = x_0$, and the remaining part of the integral

$$\Gamma_2^{p,DIS}(Q^2) = \int_0^{x_0} g_2(x, Q^2)dx \tag{6}$$

must be estimated.

One way to estimate the unmeasured contribution to Γ_2 as $x \to 0$ is to assume that the Wandzura-Wilczek [17] relation holds:

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy \tag{7}$$

and that

$$\begin{aligned} \int_0^{x_0} g_2(x, Q^2)dx &\simeq \int_0^{x_0} g_2^{WW}(x, Q^2)dx && (8) \\ &= x_0 \int_{x_0}^1 \frac{g_1(x, Q^2)}{x} dx && (9) \\ &\equiv x_0 \left[g_2^{WW}(x_0, Q^2) + g_1(x_0, Q^2) \right] && (10) \end{aligned}$$

where Eq. 9 follows from integration by parts[‡], and Eq. 10 follows directly from the definition of g_2^{WW}. Both E155x [18], and E94010 [19] used Eq. 9 for an estimate of the DIS contribution to Γ_2, with g_1 evaluated from their own data. E155x made no estimate of the uncertainty of this approximation, while E94010 assumed a (somewhat arbitrary) 20% error on this contribution.

In practice, Eqs. 9 and 10 give numerically identical results, independent of which g_1 model is used. While Eq. 8 is analytically equivalent to the other

[‡]See for example Ref. [20]

formulations, it can give numerically different results because of the rapidly changing behaviour of g_2^{WW} as $x \to 0$[§].

Eqs. 8-10 all depend on the choice of g_1 used when forming g_2^{WW}. We observe that there are four possible scenarios:

I: Evaluate Eq. 9 with g_1 given by the fit to our own measured data. This is the approach chosen by E155x and E94010, and has the advantage of avoiding dependence on any particular DIS model. However, it is not clear that the g_2^{WW} formed from resonance data is suitable for a DIS estimate. Conversely, we should note that at our low Q^2, g_2^{WW} evaluated from our g_1 does not reproduce our g_2 (significant higher-twist).

II: Evaluate Eq. 8 with a g_2^{WW} that has been formed with a DIS g_1 fit for $0 \leq x < x_0$ and from the RSS resonance fit in the region $x_0 \leq x \leq 1$. This gives results quite similar to Scenario I, with a maximum deviation of about 11%, depending on which DIS g_1 fit is chosen (see Table 1). However, the g_2^{WW} formed is a resonance-DIS hybrid and hard to describe or interpret.

III: Evaluate Eq. 9 with g_1 given by a DIS fit extrapolated into the resonance region. This is the simplest to interpret and describe, however, it is not clear that the DIS extrapolations of g_1 into the resonance region are valid. Perhaps this approach can be justified when the DIS fit in question exhibits duality with our data? For reference, we list the global duality ratios for each of the DIS fits considered in Table 2. The AAC target mass corrected pdf comes closest to matching our data globally in the resonance region.

IV: Evaluate Eq. 8 with a g_2^{WW} that has been formed entirely from a DIS g_1 fit over the entire range $0 \leq x \leq 1$. This is analytically equivalent to Scenario III, although it can give slightly different numerical results for the reasons discussed previously.

[§]This behaviour enhances the integral sensitivity the numerical step size chosen. In addition, the PDFs are not defined at $x = 0$ and instead have higher limits of applicability: $x = 10^{-4}$ for AAC and $x = 10^{-9}$ for GRSV respectively. However, when the integration step size is sufficiently small, and the x lower limit sufficiently near zero, Eq. 8 approaches Eq.10 to better than a percent.

DIS model	Scenario I	Scenario II	Scenario III	Scenario IV
BT	0.0264	0.0254	0.0187	0.0178
AAC-1	0.0264	0.0239	0.0275	0.0251
AAC-2	0.0264	0.0257	0.0275	0.0269
GRSV-1	0.0264	0.0251	0.0297	0.0284
GRSV-2	0.0264	0.0251	0.0297	0.0284
OR Regge	0.0264	0.0253	0.0156	0.0147

Table 1: Estimates of $\int_0^{x_0}$ in the four different scenarios, and using various different models of g_1. Note, BT neglects $\gamma^2 g_2$

I believe we should reject Scenario II due to it's hybrid nature which is difficult to interpret meaningfully. In any case, it gives similar results to scenario I. Also, as we note above, scenario IV is analytically equivalent to scenario III. This leaves scenario I and III as the only independent possibilities.

We note that all the scenario III pdf results agree within 13% with scenario I. The Regge fits (BT and OR) disagree with Scenario I by 28% and 41% respectively. If we exclude AAC-2 and GRSV-2, since they are not independent of AAC-1 and GRSV-2¶, we have five separate estimates of the DIS contribution to Γ_2. The average is 0.0236, with a standard deviation of 0.0054, or a relative spread of 22.9%. From this I conclude that our previous estimate of 20% error on the DIS contribution to Γ_2 is reasonable, and I propose we continue to use Scenario I for the DIS estimate.

3.2 Plot Quantities

Fig. 3 displays $\Gamma_2 = \int g_2(x, Q^2)dx$.

1: Elastic : $\Gamma_2^{el} = G_M^P(G_E^P - G_M^P)\tau/(2(1+\tau))$ with the form factors from Ref. [15]. I assume 5% relative uncertainty on Γ_2^{el}. At $Q^2 = 1.3$ $\Gamma_2^{el} \approx \Gamma_2^{res}$. (Note, need to update plot with Mark's elastic instead of Mergell).

2: MAID : MAID 2003 model, Ref. [12].

¶If we consider the AAC-2 and GRSV-2 as independent we find average is 0.025 ± 0.0052 or 21% relative.

DIS model	Global duality
BT	0.73
AAC-1	1.14
AAC-2	1.14
GRSV-1	1.22
GRSV-2	1.22
OR Regge	0.60

Table 2: Global duality ratio: $(\Gamma_1^{DIS}/\Gamma_1^{Res})$ for each of the DIS g_1 fits considered.

3: E155x : From Ref. [18]. E155x covered the range $0.02 < x < 0.8$ at $Q^2 = 5$ GeV^2. $\Gamma_2^{E155x}(5) = -0.044 \pm 0.008 \pm 0.003$. The uncertainty is dominated by statistics. The average of E155x, E143 and E155 gives $\Gamma_2^{E155x}(5) = -0.042 \pm 0.008$, which is quite similar to the result from E155x alone, so we use the E155x number instead. For the unmeasured contribution $0 < x < 0.02$, they assume $\int_0^x g_2^{WW}(y)dy = x(g_2^{WW} + g_1(x))$ $= 0.020$, and include no uncertainty on this contribution. This brings the E155x number to $-0.024 \pm 0.008 \pm 0.003$.

4: For the resonance region, we assume a 6.82% relative systematic following the conclusion of Oscar's study [1]. Note this study was only for Γ_1. The statistical error on the resonance fit is scaled from the data statistical error, keeping the relative value constant.

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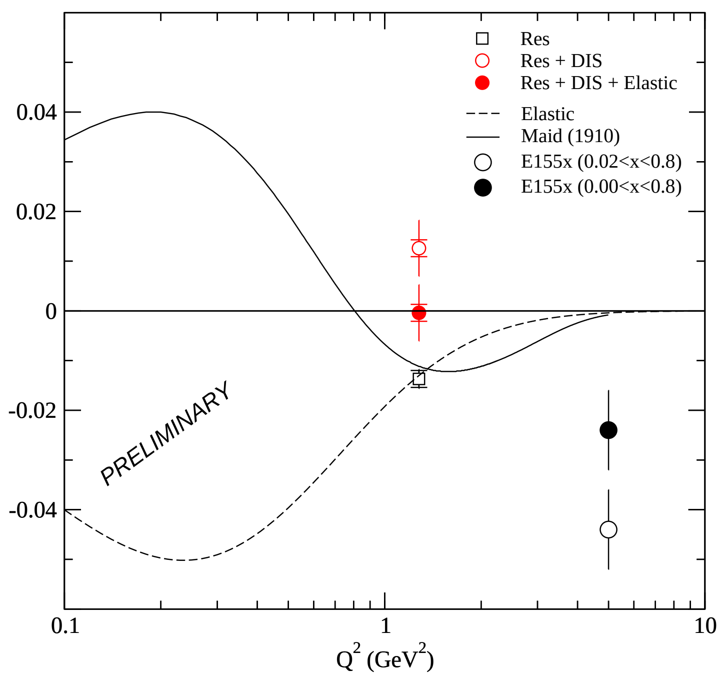

Figure 3: $\Gamma_2^p(Q^2 = 1.279)$.

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