EG1 Analysis - Models for Inclusive Cross Sections and Asymmetries

This writeup describes a package of Fortran code that can be used to calculate (or, rather, simulate) structure functions and asymmetries for inclusive (e,e') scattering. My goal is to develop a full set of models that can be used for **a)** radiative corrections (potentially iterative), **b)** to check our data against, and **c)** to convert asymmetries to structure functions, cross section differences etc.
The code has been deposited in the JLab CUE directory `~kuhn/Models`. The most up-to-date versions of these files will be kept under CVS by Keith Griffioen, but I will try to keep the files in `~kuhn/Models` updated as well. These files can be compiled and linked under LINUX using the g77 compiler. A sample `Makefile` is included.
The remaining files have the following names and purposes:

1. `Asym_gen.f`: Auxiliary program to generate tables of input asymmetries (see below). Not usually needed by users of the code, but can be perused to see how the algorithms are implemented.
2. `a1rr.dat`, `a2rr.dat` and `Rrr.dat` and all the variants thereof (`a1rr_dis.dat`,...) are parametrizations of the corresponding asymmetries in the resonance region (W<2). They are all needed as input data files for the main code. The files named `*_dis.dat` are fits to the DIS region, simply extrapolated into the resonance region; the files named `*_ref.dat` are my last and best attempt at modeling asymmetries **before** any information from our own data was included; the files named `*_res.dat` contain the contribution from resonances only (according to AO). A variant for the calculation of A_2 (more positive in the Delta region, see Section 7) below) is contained in the files named `*_ref_R2A2.dat`. To use this variant, copy these files over the corresponding `*_ref.dat` ones (saving a copy of the latter). The file `aparms.dat` is needed, as well; it contains the fit parameters that tell the main code how to mix the 3 different contributions to describe our data best. If you want to go back to the pre-fit version (`*_ref.dat`), simply copy `aparms0.dat` over `aparms.dat` (saving a copy of the latter) - all parameters are set to zero and the default reference model will be used.
3. `f1.in` is a sample input file for the code. The first column contains W^2, and the second Q^2. The output file would be called `f1.out` and contain Q^2, W^2, x , F_1 , F_2 , R (= the ratio of longitudinal to transverse virtual photon cross section) , A_1 , A_2 , g_1 and g_2 . To use the code, call the executable (e.g., "`strucfunc.exe`"), and answer the 2 options (Resonances or DIS only, P or D target). It will produce one output line for each input line in `f1.in`.
4. `frw_fit.f` and `frw_fit.dat` are Frank Wesselmann's parametrization of the Deep Inelastic asymmetries A_{1p}, A_{1n} and A_{1d}. I have created two updated

versions `sek_fit.f` and `sek_nce.f` (no data file needed) which should behave properly at very small Q^2. The most rescent code, `sek_nce.f` ("New Century Edition") is fitted to all the world data (including recent HERMES results at low Q^2 and our own data above W=2) and should be used as the default for the code (set variable `IA1=3`, see `strucfunc.f`. That variable is defined in `instruct.inc`.)

5. `h2model_thia.f` is a new parametrization of the world's inclusive unpolarized data on the proton in the resonance region, originally created by Thia Keppel. I have modified this code to make it behave properly at low Q^2 (down to zero).
6. `i_d2_model.f` does the same thing for the deuteron. Again, it originates from Thia Keppel and was modified for low Q^2 (below 0.5).
7. `models.f` contains most of the code for calculating structure functions, asymmetries, form factors etc.
8. `r1998.f` is the most recent parametrization of the world's data on R (see above) from SLAC, see Phys. Lett. B 452 (1999) 194-200. As an alternative in the resonance region, one can also use the code `RRicco.f` (based on G. Ricco et al, Nucl. Phys. B555, 306(1999), who use duality arguments to extrapolate the DIS results). This can be accomplished by changing the logical variable `Riccoflag` in subroutine `H2MODEL` which is contained in `models.f` (this will not affect the behavior in the DIS region).
9. `strucfunc.f` is the main calling program, used to test the code and to get structure functions and asymmetries like A_1 , A_2 . It can be used as an example how to call all the subroutines and how to set parameters.
10. `instruct.inc`, `prinplot.inc`, `radcon.inc`, `radvar.inc` and `radvarfix.inc` are include files, mostly used by `models.f` . These need to be present to properly compile everything.

NOTE: You have to link against all of the fortran routines (***.f) above to get executable code, and all of the inc.(lude) files and the parameter files must be present in the same directory.

Most of these files are based on the code development for the SLAC radiative correction code "RCSLACPOL", by Linda Stuart, with contributions from Thia Keppel, Keith Griffioen and Frank Wesselmann. I have modified some of the code specifically for the purposes of EG1 (e.g., implementing "more reasonable behavior" at low Q^2) - see below. The authors will make no warranties whatsoever, and they retain the full copyright for the code. Therefore, you may not distribute the code beyond our collaboration and may only use it for testing purposes at this point.

Status: (June 29, 2001)

- Completed low-Q^2 adaptation of code to represent known SLAC data for both **proton** and **deuteron**, including good behaviour at the real photon point. Further

improvements await the availability of new inclusive (unpolarized) cross section data from CLAS (or other places) at very small Q^2 (below 0.5).
- Completed a fit to the DIS world data for A_{1p} and A_{1n} which behaves properly at small Q^2, even in the limit of the real photon point, and is also constrained to give $A_1 = 1$ for pion threshold. This fit is used both in the region above $W^2 = 4.3$ and also as the "nonresonant background" asymmetry in the resonance region. It includes the most recent data from HERMES and EG1 (Renee).
- Implemented the most up-to-date set of form factor models (set variable `IG=20`), including Galster fit for Gen (seems to work for all new data), Bosted fit for Gmp and Gmn, and modified Bosted fit for Gep (made to agree with Hall A data from Perdrisat et al.)
- Integrated a model for A_2 in the resonance region that is based on the assumption that the DIS region is all leading twist (i.e. $A_2(W>2) = A_2^{WW}$). Also developed alternative model with more positive A_2 (see below).
- Set up a scheme to fit our own data in the resonance region using 3 different forms of $A_1(W,Q^2)$ and a set of parameters that interpolates between them.
- Still needed: Better implementation of quasi-elastic scattering (nuclear effects).

Extensions to low Q^2:

The original code was used for DIS experiments with beam energies of 30 - 50 GeV and beyond. Therefore, not much attention was given to the proper behaviour of all models at low energies and Q^2, especially below $Q^2 = 0.4$ (where no SLAC data exist).
My extrapolation into the low Q^2 regime is based on the following observations for the DIS region (i.e., W > 2 GeV):

1) The structure function $F_1(x,Q^2)$

scales approximately in a variable called which is basically the variable x, rescaled by $(1.0 + 0.272/Q^2)$. This even works all the way down to $Q^2 = 0$. See graph below:

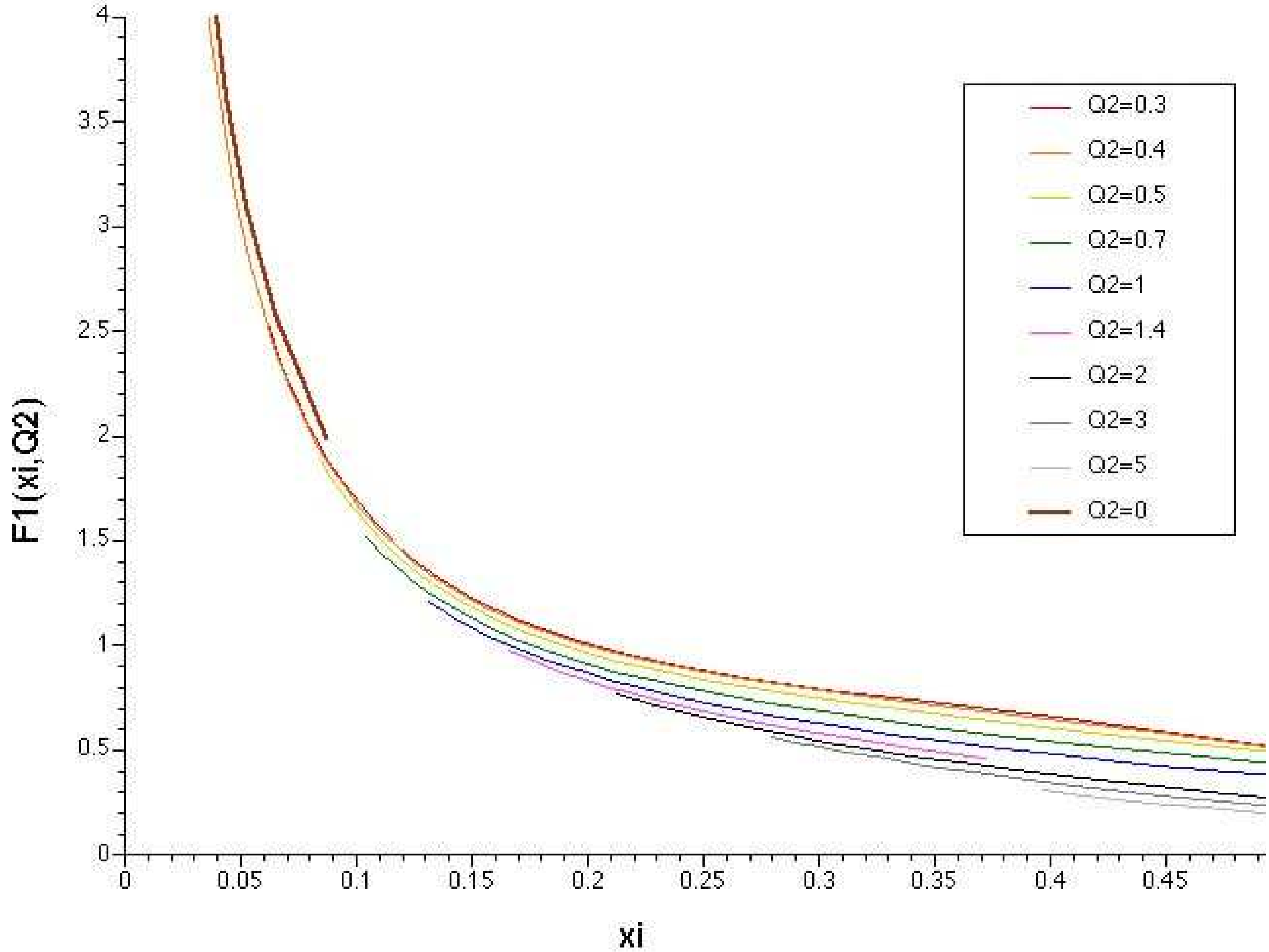


Fig. 1: Structure Function $F_1(\, ,Q^2)$ from NMC parameterization of world data, plotted against the variable which goes to =1 at pion threshold for all Q^2.

Therefore, I am calculating $F_1(x,Q^2)$ at Q^2 below 0.4 by replacing it with the value of F_1 at the same value of but at fixed $Q^2 = 0.4$, for all W>2.

The actual calculation is based on a recent fit of $F_2(x,Q^2)$ by the NMC collaboration, and the Whitlow fit to R (see above), which exist both for protons and deuterons. The neutron structure function is extracted by subtracting the proton from twice the deuteron (F_{1d} is always expressed as "per nucleon").

2) The structure function $R(x,Q^2)$

$R(x,Q^2)$ is calculated using the SLAC/Whitlow fit, as updated in 1998. This fit does not strongly dependent on Q^2 below 0.4, see the graph below:

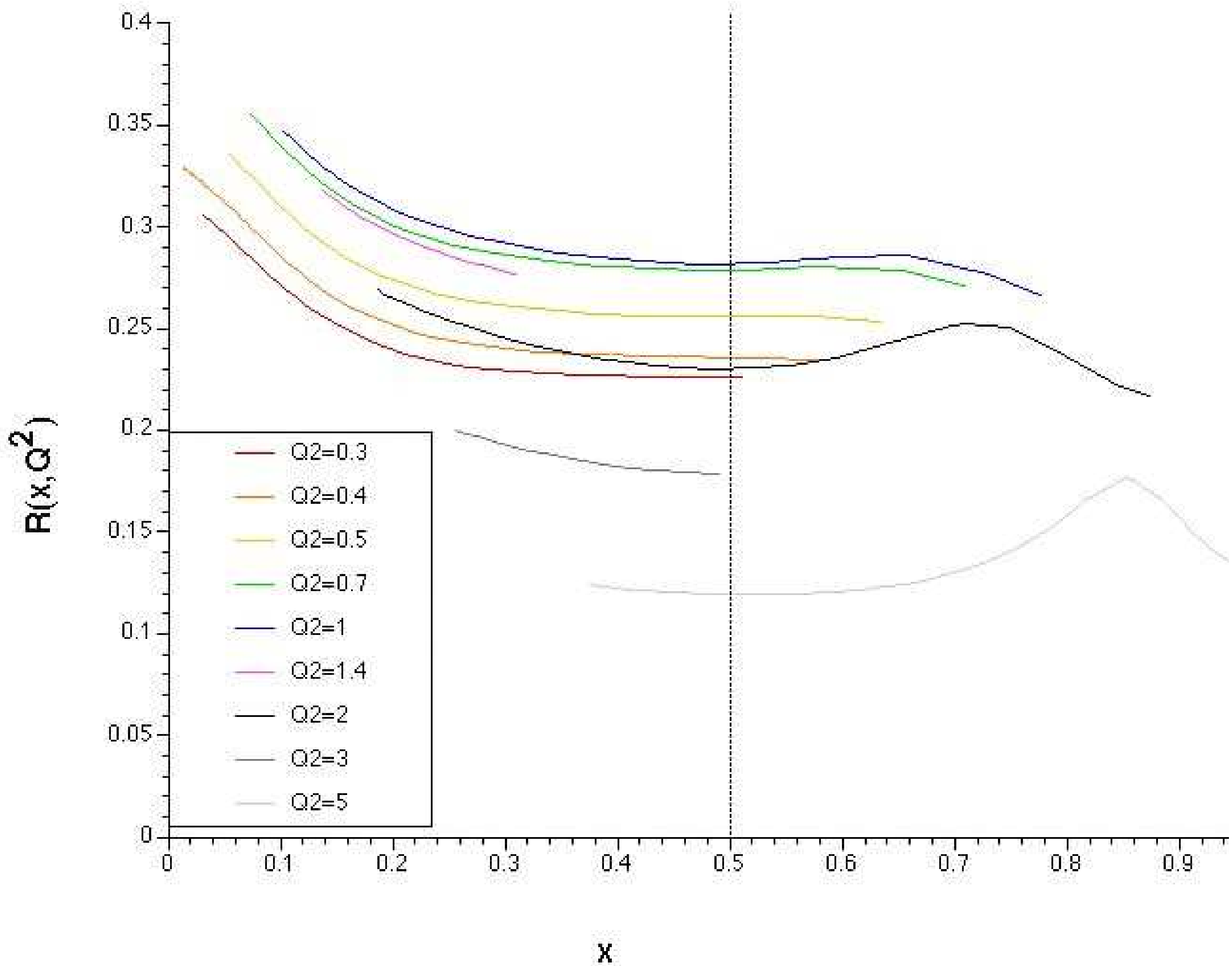


Fig. 2: Ratio $R(x,Q^2) = {}_L/{}_T$ from fit to world data (see Phys. Lett. B452(1999)194 - "R1998")

Therfore, I simply assume it's constant below $Q^2 = 0.3$. Also, the "strange bumps" at high x are probably unphysical (and all in the resonance region where R is only used for the nonresonant part of the cross section), so I assume it's actually constant beyond x = 0.5.

3) Resonance region

Fore the unpolarized structure functions, I use the most recent fit to the world data, including JLab data, performed by Thia Keppel. Her fit is also unstable at small Q^2, so I replaced it with a smooth interpolation to the data for real photons.
The Whitlow parametrization for R is also used in the resonance region, but there it multiplies only the non-resonant part of the cross section (simple assumption that $R_{Res} = 0$).
As an alternative, one can use a model by Ricco et al. (see above) to simulate R in the resonance region. This is based on a higher-twist analysis of structure function moments in the DIS region, extrapolated into the resonance region (assuming duality).
The neutron extraction from the deuteron and proton is a bit more involved, since the resonances wash out in deuterium, which would lead to negative bumps in the neutron spectrum when one subtracts the sharper proton resonances. Instead, I use a parametrization of the ratio of unsmeared/smeared cross section (based on my deuteron PWIA model) separately for the non-resonant and resonant part of F_1 and scale these two parts of F_{1d} up before subtracting the proton.

4) Combined model

The combined model of resonance/DIS unpolarized structure functions use the DIS values for $W^2 > 4.3$, the resonance values for $W^2 < 3.5$, and a smooth interpolation in between.
As an example of what the code SHOULD produce if it works properly, I have plotted below the structure function $F_1(W,Q^2)$ for several Q^2 as indicated. The data points at $Q^2=0$ are real photon data (not a parametrization).

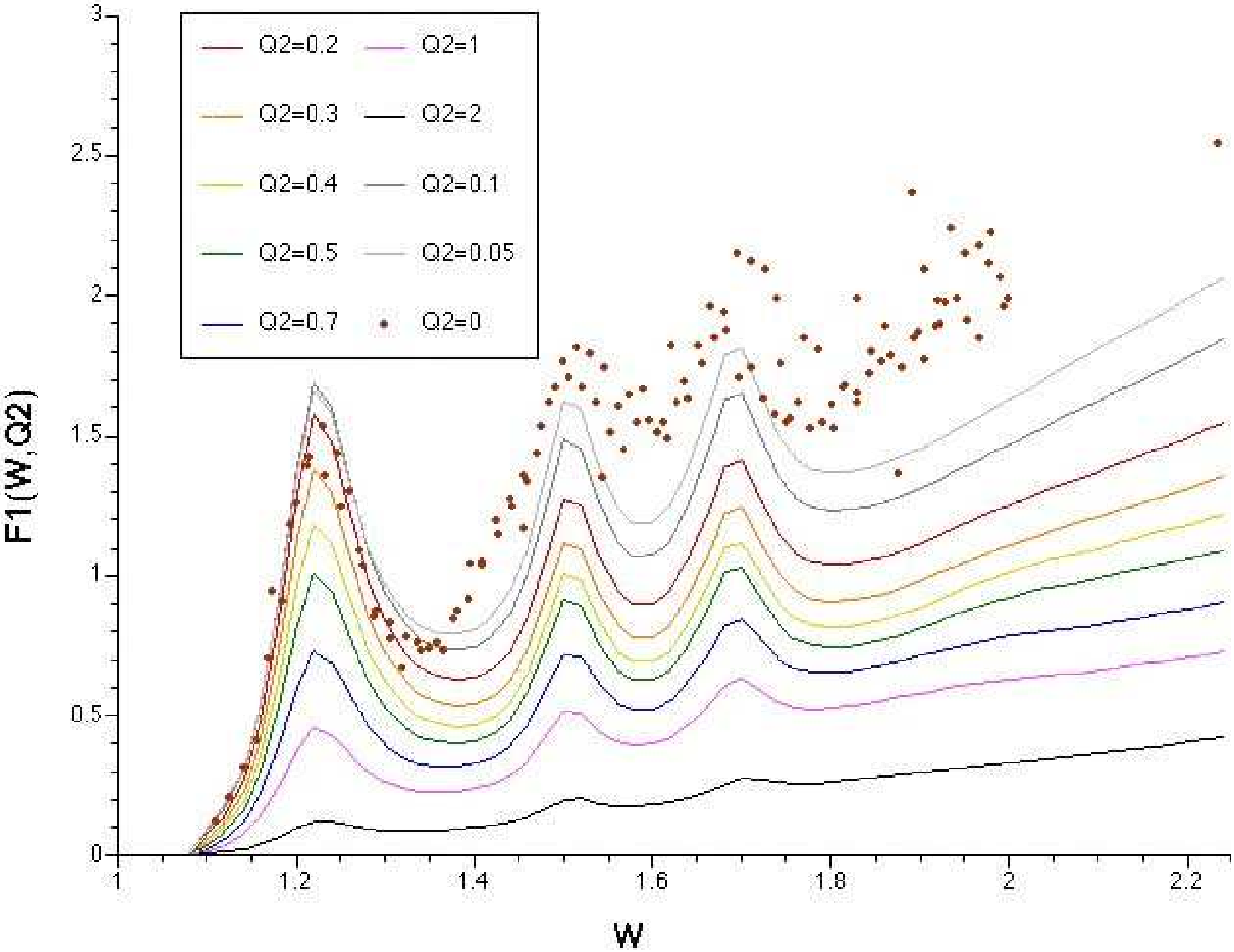


Fig. 3: Combined model for the proton structure function $F_1(W,Q^2)$ in and above the resonance region, as implemented in `models.f`

The same plot for deuterium looks like follows:

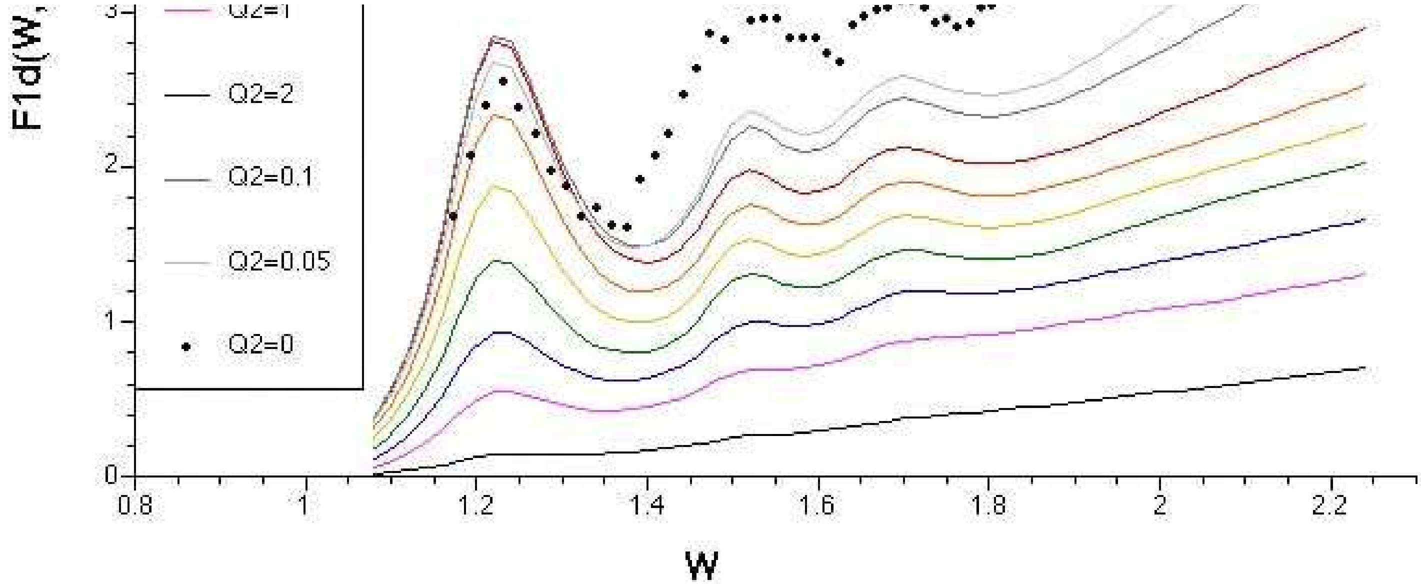
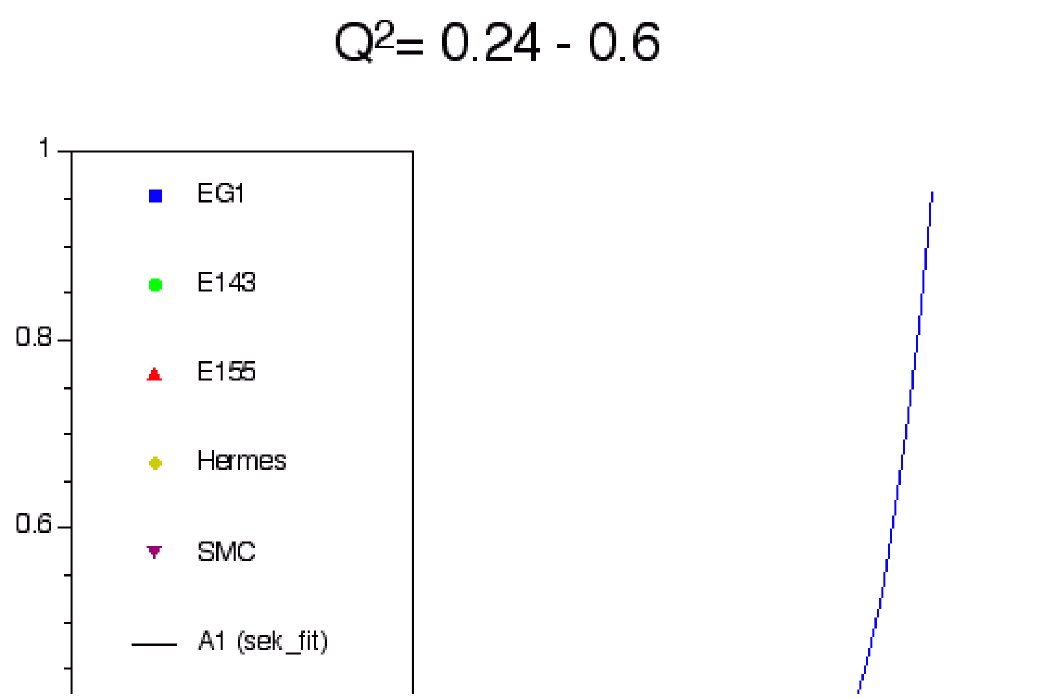


Fig. 4: Same for deuterium

5) DIS parametrization of A_1

The asymmetry A_1 in the DIS region is fit to a function of the same variable as introduced in Section 1) and Q^2. This fit is constrained to reach A_1=1 at pion threshold, and has 7 parameters (plus allows for relative adjustment of different experiments within their stated systematic normalization errors). All published data from E80, E130, E142, E143, E154, E155 (SLAC), EMC and SMC (CERN), and HERMES are used, together with our own EG1 data, for W>2. The Bianchi-prediction for the high-energy (W>2) part of the GDH sum rule is used to constrain the fit at the photon point. For the specific form of the fit, please see the subroutine `sek_nce.f`.



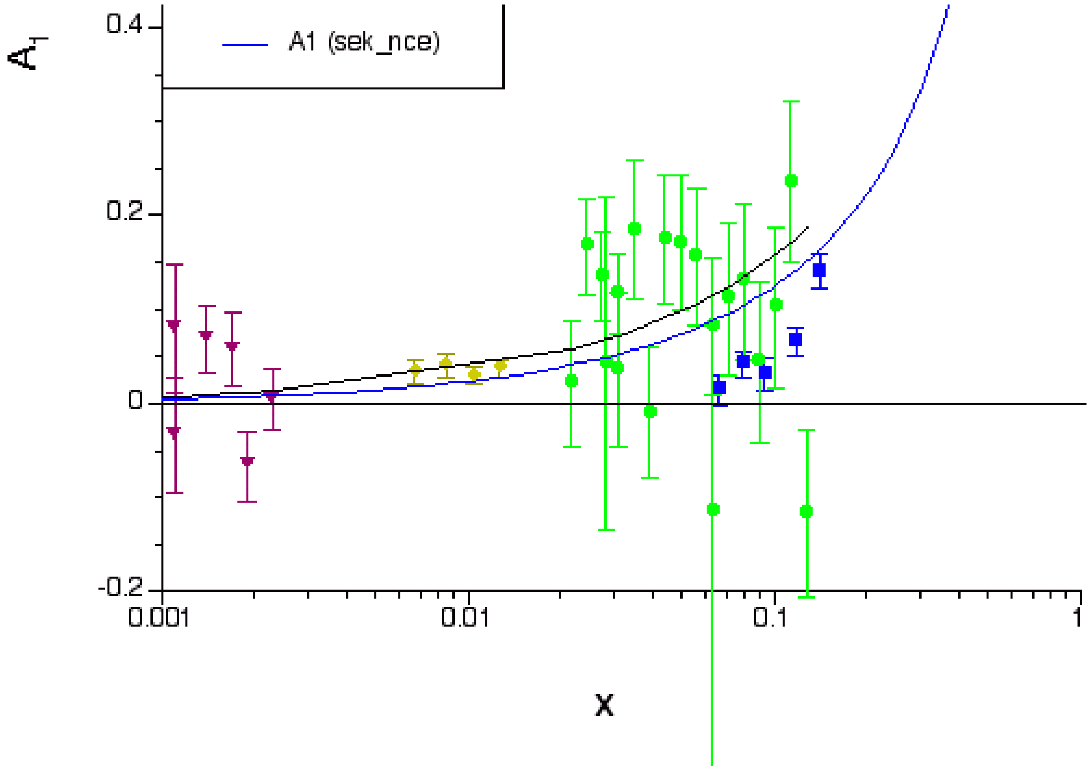


Fig. 5: x-dependence of low-Q^2 data for the proton outside the resonance region (W>2), and our fit (`sek_nce`) to the combined world data

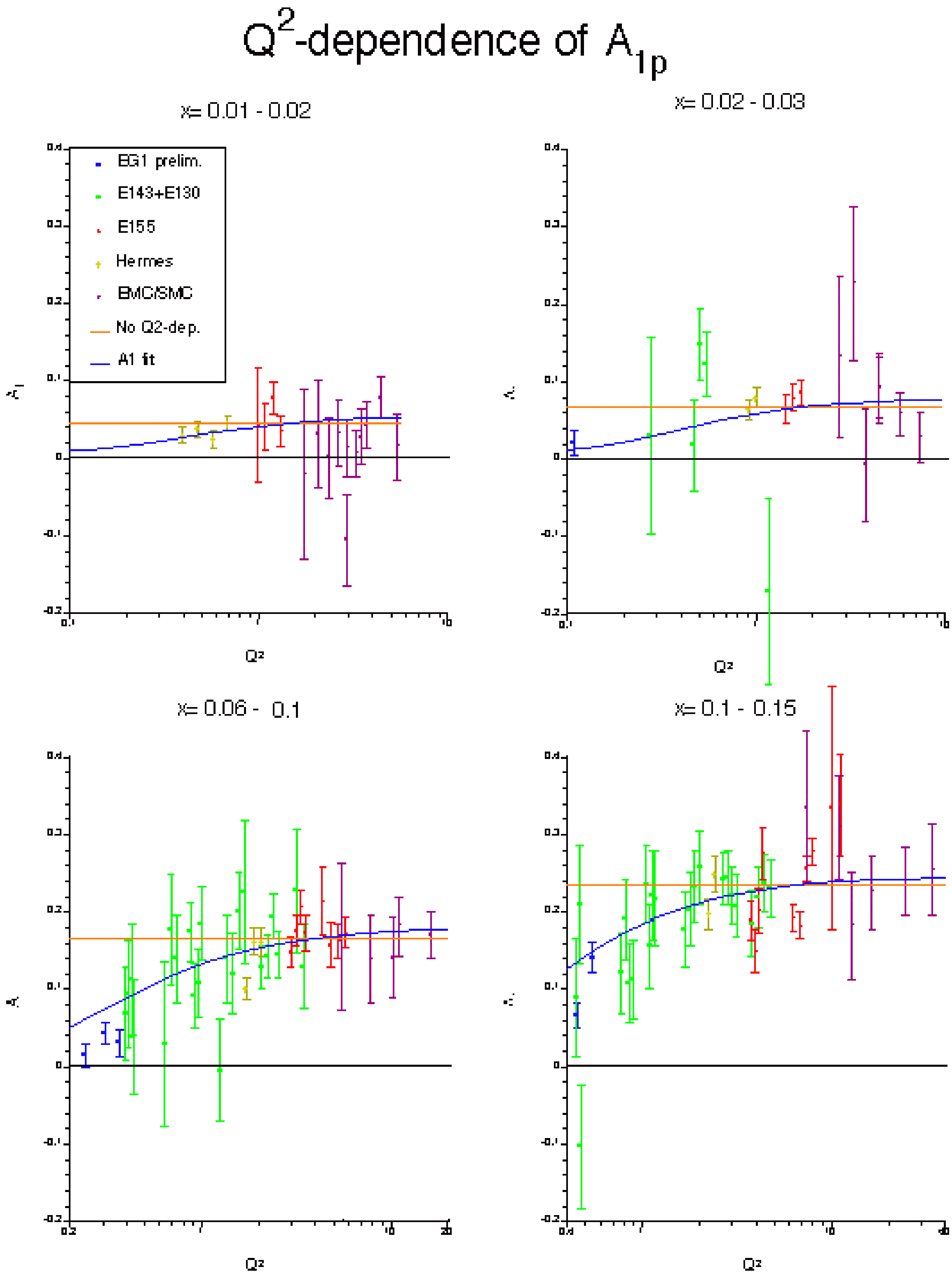


Fig. 6: Q^2-dependence of the fit together with data in selected x bins.

The parametrizations for A_{1n} is similar.

6) Resonance Region parametrization of A_1

I have used 2 ingredients to cook this one up: The DIS fit above, simply extrapolated into the resonance region (as function of x and Q^2), and the output from AO for a grid in W and Q^2 with non-resonant (Born) terms turned off. From this input, I calculate a "reference model" (using the program `Asym_gen.f,` with output `a1rr_ref.dat`) that weights the DIS fit with the non-resonant part of the cross section (see 4) and the AO output with the resonant part. In case of the neutron, because the inherent instability of extracting these two parts of the cross section from deuteron and proton parametrizations, I simply assume that the ratio $_{nres}/_{res}$ is the same as for the proton (for the purpose of calculating A_1). Again, this combination is used in the resonance region W<2 (read in from the file and interpolated) and the DIS parametrization is used above W^2=4.3, with a smooth interpolation in between. The deuteron asymmetry is calculated as the weighted (by F_{1p} and F_{1n}) average of proton and neutron, and multiplied with 0.925 to account for the D-state effect in deuterium.

In the final step, all three tables of A_{1p} - DIS, reference and Resonance - are used to fit our own EG1 data with a linear combination of them (depending on W and Q^2), with a total of 9 parameters each for proton and neutron. The result of such a fit, and the individual contributions, are shown below:

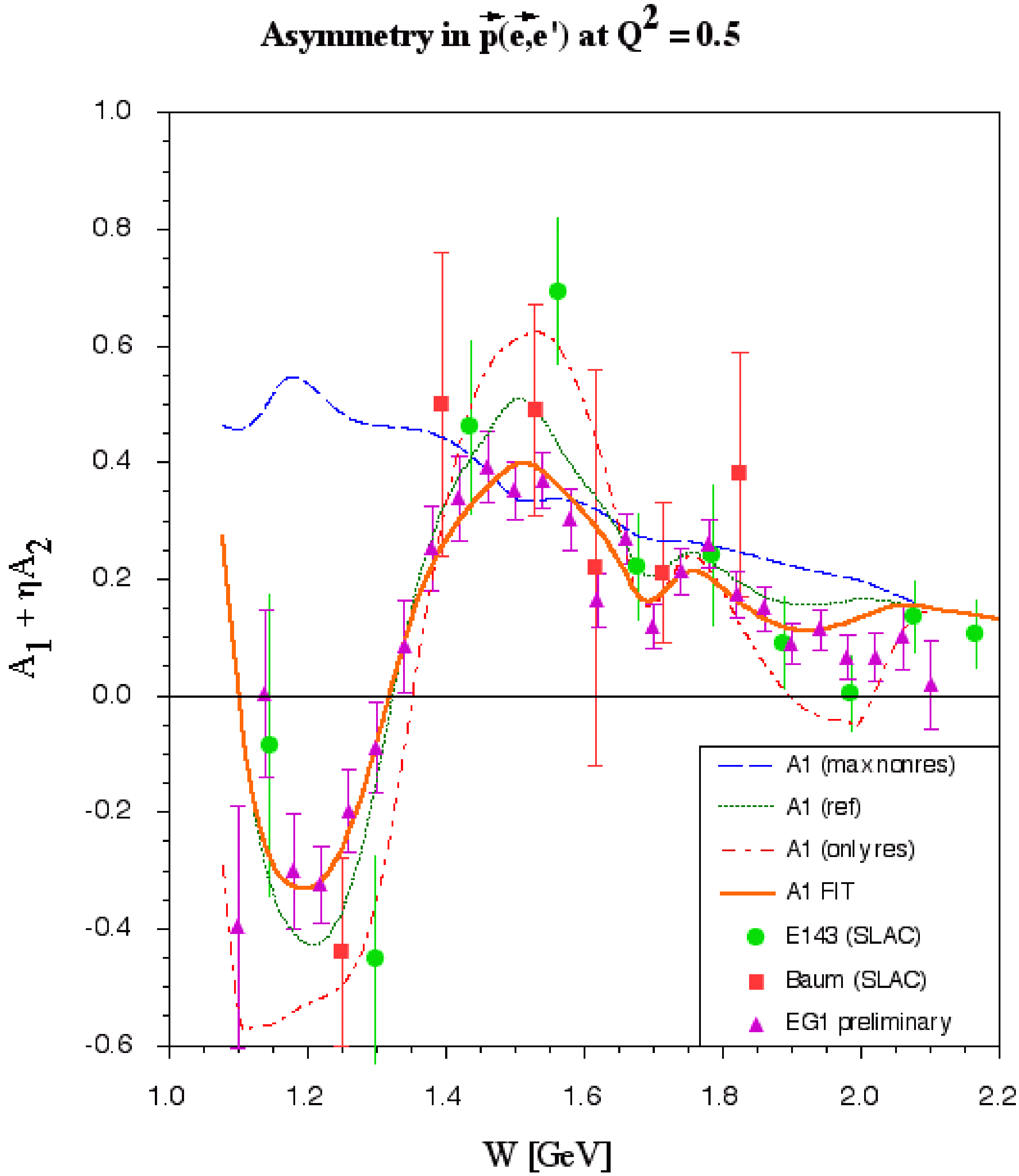
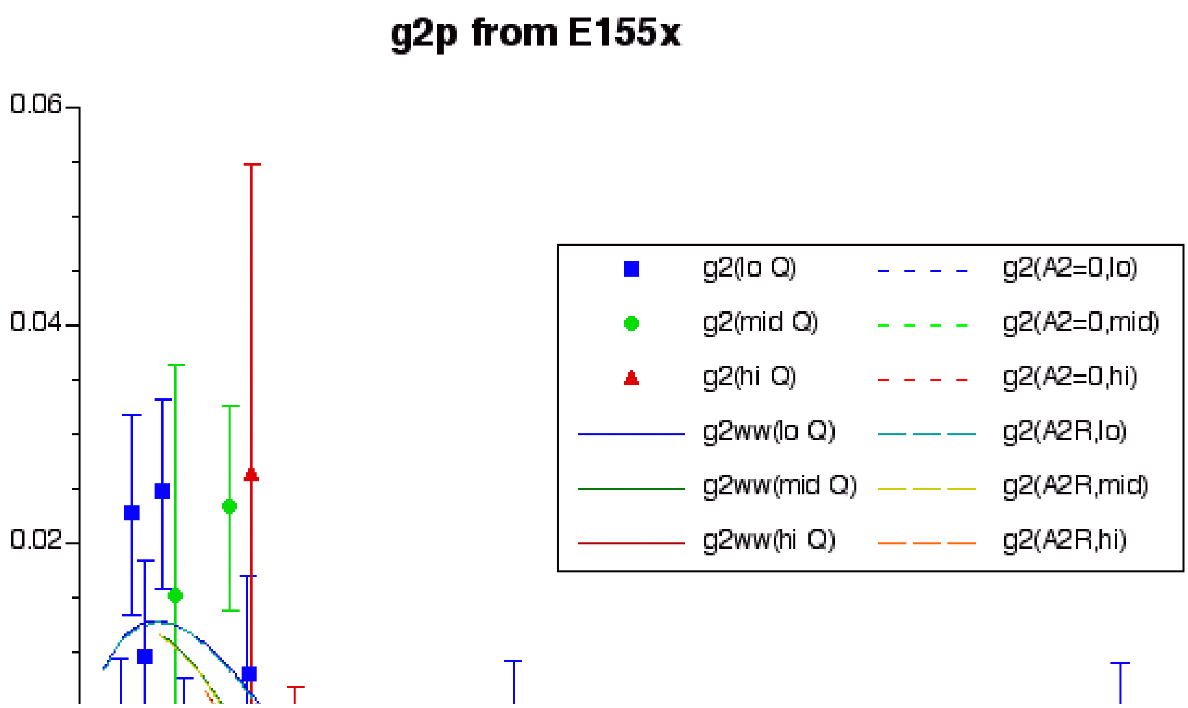


Fig. 7: A_1 fit (with corresponding values for A_2 added) in the resonance region, compared with EG1 and SLAC data.

7) Parametrization of A_2

To extract A_1 or g_1 from our data, we must account for the contribution from the unknown second asymmetry, A_2. While there are no data (published) in the resonance region, we have some general rules to go by, and we know that in the DIS region, A_2 is well described by its twist-2 part, A_2^{WW}. This (Wandzura-Wilczek) part is completely

determined by an integral over $g_1(y)/y$, integrated from the kinematic point x in question to the elastic limit (including the elastic peak). Since in turn g_1 can be expressed with A_1 and A_2, there is a straightforward "bootstrap" method to extract all needed information from a parametrization of A_1 alone (see relevant pieces of code in Asym_gen.f and the subroutine g1g2 in models.f).
However, A_2^{WW} cannot be "the whole story" for all x, at least if one believes the Burkardt-Cottingham sum rule, which states (slightly adapted) that the integral over $g_T = A_2{}^*F_1{}^*$ /Q must equal the integral over g_1, again including the elastic peak. The Wandzura-Wilczek prescription guarantees that this is true EXCEPT for the contribution from the elastic peak, where both g_1 and A_2 are fixed by nucleon elastic form factors and the WW prescription breaks down. My model of A_2 assumes a simple quadratic (in W) form of the NON-WW ("higher twist") part of A_2 which has just the right size to compensate for the mismatch in the elastic peak, plus goes smoothly to zero at W=2. Beyond the resonance region, the WW relationship holds. In principle, the model would also accommodate specific values for A_2 for certain resonance put in "by hand" - these are set to zero for the "standard" (reference) model. However, as a variant I have also included the option to set these "resonant additions to g_T" equal to the limiting value given by the Soffer-inequality $|A_2| < \mathrm{sqrt}(R[A_1+1]/2)$. The resulting alternative parametrization for the "reference" value of A_2 is contained in the file `a2rr_ref_R2A2.dat`. The figure below shows how well this prescription describes the existing DIS data - admittedly not a very strong piece of evidence since we're mostly interested in the resonance region.
More information on my A_2 fit can be found in a technical note.



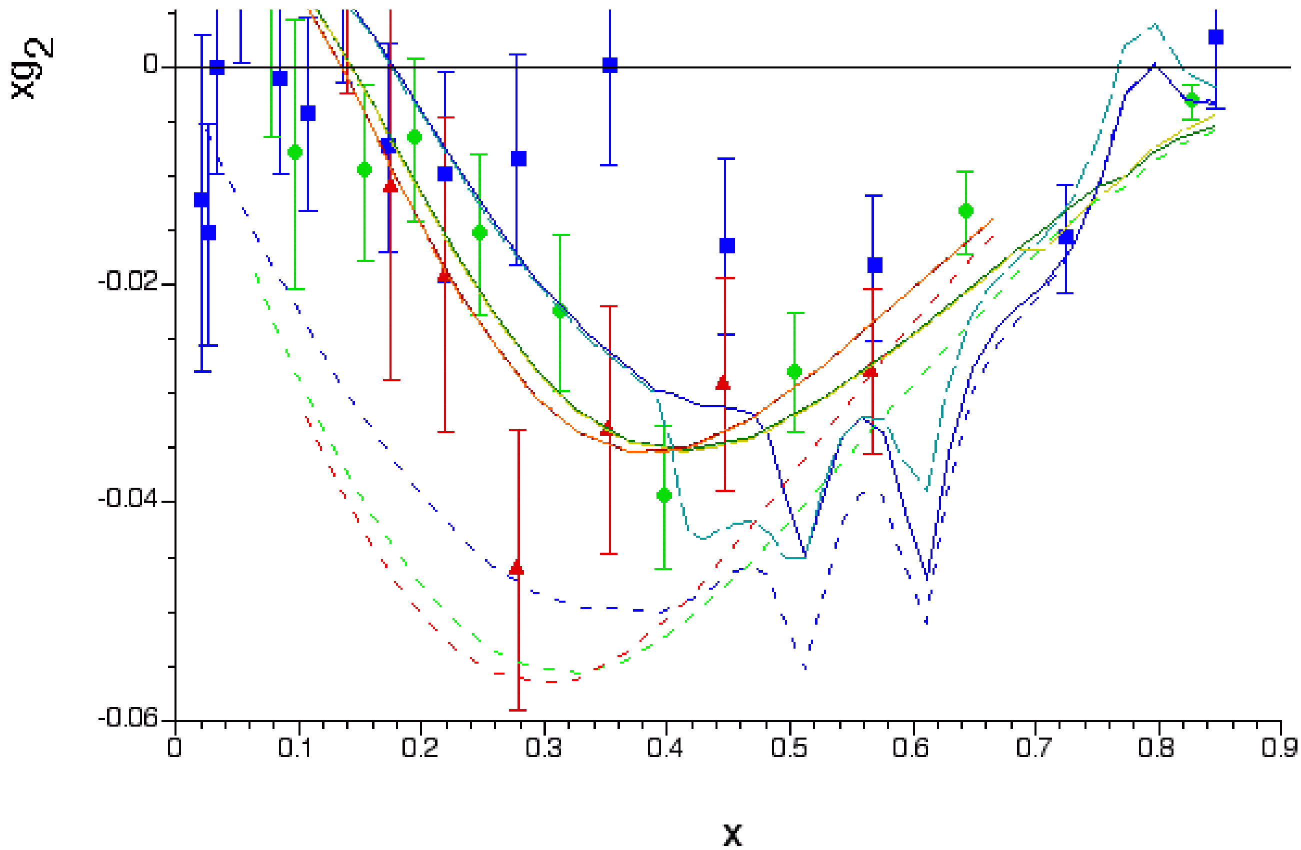


Fig. 8: Comparison of several models for $A_2(x,Q^2)$ with existing data on $g_2(x,Q^2)$ from SLAC E155x. The data were taken with beam energies of 29 GeV and 32 GeV in 3 spectrometers, at angles of 2.75, 5.5 and 10 degrees (corresponding to "lo" Q^2 = 1 -2 , "mid" Q^2 = 2.5 - 7, and "hi" Q^2 = 5 - 18 GeV^2). The low-Q^2 data reach into the resonance region (above x = 0.45), but are not very reliable there (because the resolution of the spectrometer approaches x/x = 100%). However, the sign and general trend are well reproducted by the model, even in the resonance region. The "standard" (reference) model for A_2 yields the solid lines, while the simple assumption $A_2 = 0$ gives the short-dashed lines. The variant including sqrt(R) for the higher-twist part of g_T (see text above) is shown with long dashes (nearly indistinguishable from the solid lines except for the lowest Q^2 range, where it yields more positive values for A_2 and g_2 up to the second resonance region, which is then compensated by more negative values in the higher W part of the resonance region). A final, extreme choice would be $g_2 = 0$ throughout (equivalent with A_2 = sqrt(Q^2)/ A_1).

Both MAID and AO can also calculate A_2 in the resonance region. Below I show A_{2p}

(with the same E155x "low Q^2" data) with my model and MAID as well as the Soffer positivity bound. The result from AO is similar to MAID but usually lies between MAID and my model. From this figure, I would conclude that the average of my model and MAID may be a good estimate of A_2, with the error given by the maximum deviation from either my model or the line $A_2 = 0$. (Note that the behavior of MAID at large W and Q^2 is likely wrong - it has to converge to the well-established WW behavior. This is not surprising because MAID usually doesn't do a good job at high W, anyway).

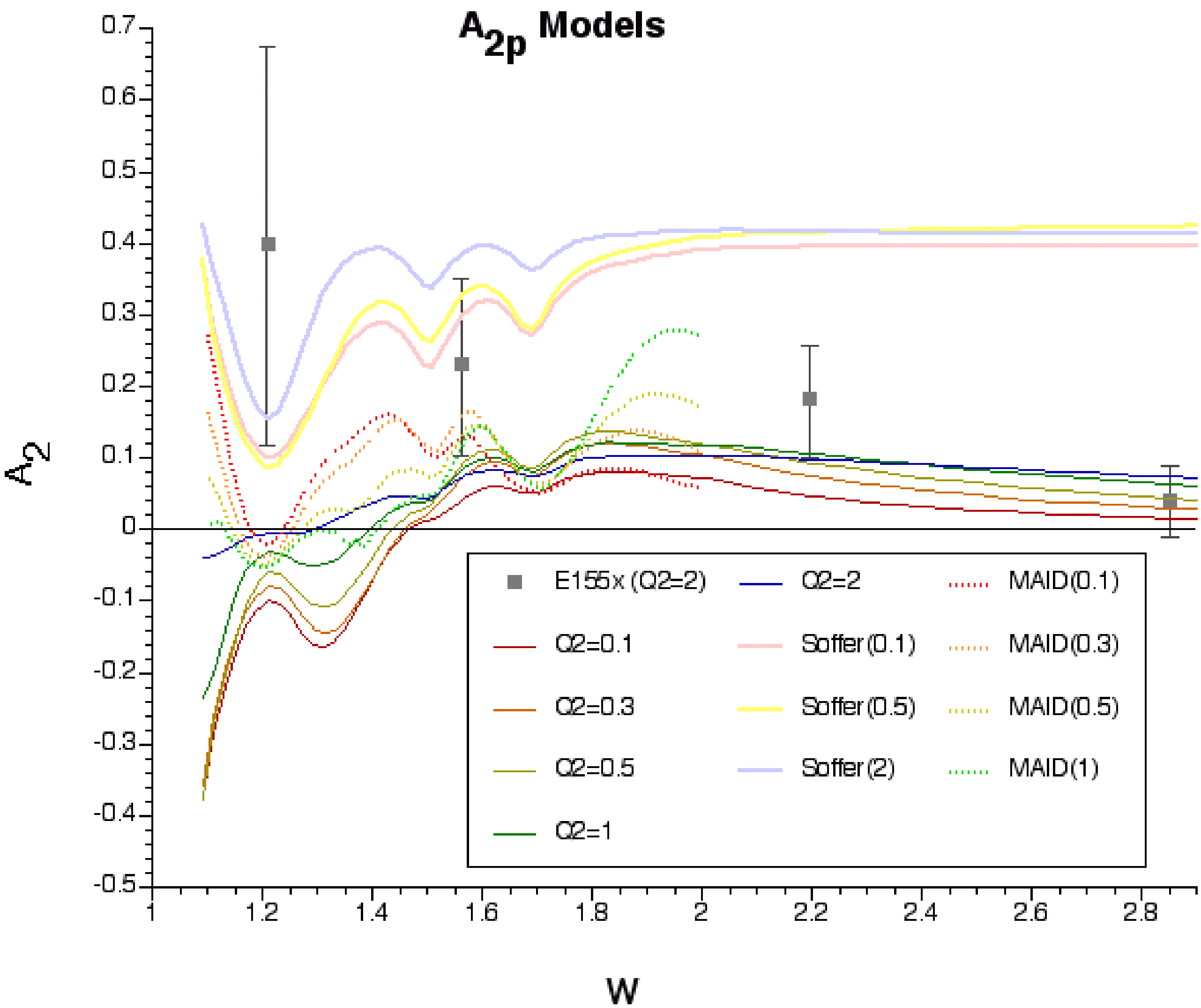


Fig. 9: Comparison of A_2 models. The lines named "Soffer" indicate the positivity bound.

More background information about inclusive asymmetries and spin structure functions can be found in [my HUGS write-up].

Please contact Sebastian Kuhn with any comments, additions, ideas...

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