Appendices

Appendix A

Asymmetry Extraction

The difference of cross sections of opposite spin directions is

$$\begin{aligned}\Delta\sigma &= \sum_{s'}\left[\frac{d^2\sigma}{d\Omega dE'}(k,s,P,S;k',s') - \frac{d^2\sigma}{d\Omega dE'}(k,s,P,-S;k',s')\right] \\ &= \frac{8m\alpha^2 E'}{q^4 E}\{[(q\cdot S)(q\cdot s) + Q^2(s\cdot S)]MG_1 \\ &+ Q^2[(s\cdot S)(P\cdot q) - (q\cdot S)(P\cdot s)]\frac{G_2}{M}\},\end{aligned} \tag{A.1}$$

where k^μ is the 4-momentum of the incoming electron, k'^μ is of the scattered, P^μ is the initial 4-momentum of the proton, S^μ is the initial covariant spin 4-vector of the proton, s^μ is of the incoming electron, and s'^μ is of the outgoing electron. Other definition is the same as ep scattering process in the Introduction. In this context G_1 and G_2 are the spin structure functions, where $G_1 = \frac{g_1}{M^2\nu}$ and $G_2 = \frac{g_2}{M\nu}$.

Fig. A.1 shows the target coordinate in the lab frame. The target is positioned at the origin, and the beam direction is defined to follow z-axis. The x-axis points at the BETA side, i.e. the beam left, therefore HMS is in the opposite side. This detector direction can be controlled by φ. Actually, HMS φ

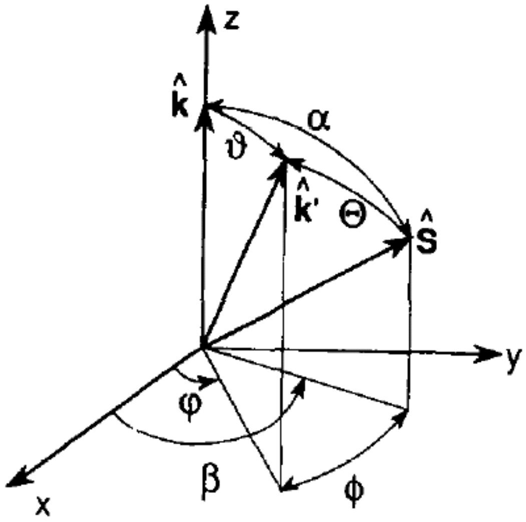

Figure A.1: Coordinate system of the target [1]

is defined already in this manner, i.e. $\varphi \approx 180°$. After some algebra, following Ref. [1],

$$\Delta\sigma = \frac{-4\alpha^2 E'}{Q^2 E}\left[(E\cos\alpha + E'\cos\Theta)MG_1 + 2EE'(\cos\Theta - \cos\alpha)G_2\right]. \quad (A.2)$$

As $\cos\Theta$ is obtained by other angles like

$$\cos\Theta = \sin\vartheta\sin\alpha\cos(\beta - \varphi) + \cos\vartheta\cos\alpha, \quad (A.3)$$

the parallel setting, where target spin is directing 180° from the beam direction, has

$$\alpha = 180°, \beta = 0, \cos\Theta = -\cos\vartheta, \cos\alpha = -1. \quad (A.4)$$

So, $\Delta\sigma_{180}$, the parallel setting cross section difference of $\Delta\sigma$, is

$$\Delta\sigma_{180} = \frac{-4\alpha^2 E'_{180}}{Q^2_{180}E_{180}}[-(E_{180} + E'_{180}\cos\vartheta_{180})MG_1 + Q^2_{180}G_2], \quad (A.5)$$

where subscript 180 means the kinematic variable from the parallel setting. Likewise the near-pependicular setting, where target spin is directing 80° from the beam direction, has

$$\alpha = 80^\circ, \beta = 0, \cos\Theta = \sin\vartheta \sin 80^\circ \cos\varphi + \cos\vartheta \cos 80^\circ, \cos\alpha = \cos 80^\circ. \quad (A.6)$$

$\Delta\sigma_{80}$, the near-perpendicular setting cross section difference of $\Delta\sigma$, is

$$\begin{aligned}\Delta\sigma_{80} = \frac{-4\alpha^2 E'_{180}}{Q^2_{180} E_{180}} [(E_{80} \cos 80^\circ \\ + E'_{80}(\sin\vartheta_{80} \cos\varphi_{80} \sin 80^\circ + \cos\vartheta_{80} \cos 80^\circ)) M G_1 \\ + 2E_{80} E'_{80}(\sin\vartheta_{80} \cos\varphi_{80} \sin 80^\circ \\ + \cos\vartheta_{80} \cos 80^\circ - \cos 80^\circ) G_2],\end{aligned} \quad (A.7)$$

where subscript 180 means the kinematic variable from the near-perpendicular setting. These cross section difference is divided by two times of the unpolarized cross section, which is

$$\sigma^{unpol.} \equiv \frac{d^2\sigma^{unpol.}}{d\Omega dE'} = \frac{2\alpha^2 E'}{Q^2 E} \frac{F_1}{M D'}, \quad (A.8)$$

where $D' = \frac{1-\epsilon}{1+\epsilon R}$ as ϵ defined in Eq. (**??**), while F_1 and R are unpolarized structure functions, to get the asymmetries.

The kinematics of parallel and near-perpendicular asymmetries are different for SANE HMS resonance region, though their W, Q^2, x_{Bj}, and ν are almost same, with maximum offset of each W bin is 3%. So relation between (A_{180}, A_{80}) and (g_1, g_2) and also (A_1, A_2) should be carefully examined. First, (A_{180}, A_{80}) and (g_1, g_2) have the following relation:

$$A_{180} = \frac{-D'_{180}}{F_{1,180}} [-\frac{E_{180} + E'_{180} \cos\vartheta_{180}}{E_{180} - E'_{180}} g_1 + \frac{Q^2_{180}}{(E_{180} - E'_{180})^2} g_2], \quad (A.9)$$

$$\begin{aligned}A_{80} = \frac{-D'_{80}}{F_{1,80}} [\frac{E_{80} \cos 80^\circ + E'_{80}(\sin\vartheta_{80} \cos\varphi_{80} \sin 80^\circ + \cos\vartheta_{80} \cos 80^\circ)}{E_{80} - E'_{80}} g_1 \\ + \frac{2E_{80} E'_{80}(\sin\vartheta_{80} \cos\varphi_{80} \sin 80^\circ + \cos\vartheta_{80} \cos 80^\circ - \cos 80^\circ)}{(E_{80} - E'_{80})^2} g_2].\end{aligned} \quad (A.10)$$

Second, (g_1, g_2) and (A_1, A_2), with average F_1 and γ, have the following:

$$g_1 = \frac{F_1}{1+\gamma^2}(A_1 + \gamma A_2), \tag{A.11}$$

$$g_2 = \frac{F_1}{1+\gamma^2}(-A_1 + \frac{A_2}{\gamma}) \tag{A.12}$$

Above equations can be inverted as 2x2 matrices to get reverse relation. But the usual relation between (A_{180}, A_{80}) and (A_1, A_2) becomes ambiguous. So, it is not good to use the following:

$$A_{\parallel} = D(A_1 + \eta A_2), \tag{A.13}$$

$$A_{\perp} = d(A_2 - \zeta A_1), \tag{A.14}$$

though they are usual formulae when the kinematics are completely same. Instead we can use the new relations keeping track of variable of each setting. All the calculation followed it, and the errors were propagated using it.

Bibliography

[1] M. Anselmino, M. Boglione, and F. Murgia, Phys. Lett. B **362**, 164 (1995).