Nitrogen Correction

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Nitrogen correction, C_N is the factor to eliminate the nitrogen polarization. Unlike RSS using ^{15}N, SANE has ^{14}N for ammonia. According to RSS technote 2005-01, the nitrogen correction is

$$C_N = 1 + \frac{A_{15}}{A_1}\frac{f_{15}}{f_1}\frac{P_{15}}{P_1}, \tag{1}$$

where A's are asymmetries, f's are dilution factor, and P's are polarization with subscript denotes the mass number of each nucleus. In this formula, the ratio of asymmetries is approximated by shell model.

For SANE, nitrogen asymmetry should use different model. And nitrogen polarization should be re-calculated by equal spin temperature (EST) model. SMC and CLAS eg1-DVCS has their own approximation for these, and actually CLAS followed the SMC analysis:

$$C_N = 1 + \frac{\eta_N}{\eta_p}\frac{A_N\sigma_N}{A_p\sigma_p}\frac{P_N}{P_p}, \tag{2}$$

where η is the number of nuclei of a given species, and σ the cross section per nucleus with subscript is the name of nucleus. So $\frac{\eta_N}{\eta_p}$ is $1/3$. Considering spin-1 nitrogen-14 nucleus as a spinless carbon surrounded by extra proton and neutron,

$$\frac{\sigma_N}{\sigma_p} = -\frac{1}{3}(\sigma_p A_p + \sigma_n A_n) \approx -\frac{1}{3}\sigma_d A_d, \tag{3}$$

where subscript n means neutron, d deuteron. So, the ratio of (Asymmetry)X(Cross section) of deuteron and proton becomes important.

$$\sigma^{unpol.} = \frac{d^2\sigma^{unpol.}}{d\Omega dE'} = \frac{2\alpha^2 E'}{Q^2 E}\frac{F_1}{MD'}, \tag{4}$$

where $D' = \frac{1-\epsilon}{1+\epsilon R}$ as ϵ is the virtual photon polarization, while F_1 and R are unpolarized structure functions, to get the asymmetries. So, the ratio of cross sections per nucleon of deuteron and proton is

$$\frac{\sigma^d}{\sigma^p} = \frac{F_1^d D'^p}{F_1^p D'^d}, \tag{5}$$

where superscript p is for proton and d for deuteron.

The kinematics of parallel and near-perpendicular asymmetries are different for SANE HMS resonance region, though their W, Q^2, x_{Bj}, and ν are almost same, with maximum offset of each W bin is 3%. So relation between (A_{180}, A_{80}) and (g_1, g_2) and also (A_1, A_2) should be carefully examined. First, (A_{180}, A_{80}) and (g_1, g_2) have the following relation:

$$A_{180} = \frac{-D'_{180}}{F_{1,180}}[-\frac{E_{180} + E'_{180}\cos\vartheta_{180}}{E_{180} - E'_{180}} g_1 + \frac{Q^2_{180}}{(E_{180} - E'_{180})^2} g_2], \tag{6}$$

$$\begin{aligned} A_{80} = \frac{-D'_{80}}{F_{1,80}}[\frac{E_{80}\cos 80^\circ + E'_{80}(\sin\vartheta_{80}\cos\varphi_{80}\sin 80^\circ + \cos\vartheta_{80}\cos 80^\circ)}{E_{80} - E'_{80}} g_1 \\ + \frac{2E_{80}E'_{80}(\sin\vartheta_{80}\cos\varphi_{80}\sin 80^\circ + \cos\theta_{80}\cos 80^\circ - \cos 80^\circ)}{(E_{80} - E'_{80})^2} g_2]. \end{aligned} \tag{7}$$

So, when we calculate $\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p}$, $\frac{\sigma^d}{\sigma^p}$ is cancelled out by the factor of $\frac{A_d}{A_p}$, i.e. $\frac{D'^d F_1^p}{F_1^d D'^p}$.

If we approximate $A_{180} \propto g_1$ and $A_{80} \propto g_2$, according to following discussion,

$$\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p} \approx \frac{g_1^d}{g_1^p}, \tag{8}$$

for A_{180}, and

$$\frac{\sigma^d}{\sigma^p}\frac{A_d}{A_p} \approx \frac{g_2^d}{g_2^p}, \tag{9}$$

for A_{80}.

And for the ratio of polarization, $\frac{P_N}{P_p}$, it is known that EST usually matches well with actual polarization measurements.

$$P_p = \tanh\left(\frac{\hbar\omega_p}{2kT_s}\right), P_N = \frac{r^2-1}{r^2+r+1}, r = \exp\left(\frac{\hbar\omega_N}{kT_s}\right), \tag{10}$$

where ω is the Larmor angular frequency, T_s is equal spin temperature. $\omega_p = 267.513 \times 10^6 B$ and $\omega_N = 19.331 \times 10^6 B$, where B is the magnetic field in Tesla.

As SANE has average proton polarization of 68 % , $P_p = 0.68$ gives that T_s is 0.00616, which is 6.16 mK. So, $P_N = 0.0797$. If we assume 10 % error of EST method, $\frac{P_N}{P_p} = 0.12 \pm 0.01$.