Simplified Expressions to get A_1 and A_2 from A_{80} and A_{180} in SANE

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The formulas for $A_{1,2}(A_{80}, A_{180})$ in Jonathan Mulholland's tech. note[1] can be simplified to depend on the measured kinematic quantities E, E', θ and ϕ as much as possible, instead of using calculated variables like Q^2 and ν. There are many cancellations that reduce the expressions to these simple forms:

$$A_1 = \frac{1}{D'}\left(\frac{E - E'\cos\theta}{E + E'} A_{180} + \frac{E'\sin\theta\cos 80°}{(E+E')\cos\phi\sin 80°} A_{180} + \frac{E'\sin\theta}{(E+E')\cos\phi\sin 80°} A_{80} \right)$$

$$A_2 = \frac{1}{D'}\frac{1}{2E}\left(\sqrt{Q^2}\, A_{180} - \sqrt{Q^2}\frac{(E - E'\cos\theta)\cos 80°}{E'\sin\theta\cos\phi\sin 80°} A_{180} - \sqrt{Q^2}\frac{E - E'\cos\theta}{E'\sin\theta\cos\phi\sin 80°} A_{80} \right)$$

These expressions match the formulas in eq. (12) of *RSS* tech. note[2] on the spin asymmetries A_1 and A_2 and measured asymmetries A_{80}, A_{180} when 80° is replaced by -90°, according to the SANE vs *RSS* angle conventions.

It is easy to see that the ϕ-dependent factor in front of A_{180} is the same as the factor in front of A_{80}, with just an extra constant coefficient cos 80°, so it can be calculated directly from the 80° data. In this way, the factors for each asymmetry are associated with their originating data set, without unwanted admixtures.

Rearranging the factors by common kinematic terms we have

$$A_1 = \frac{1}{D'}\left(\frac{E - E'\cos\theta}{E + E'} A_{180} + \frac{E'\sin\theta}{(E+E')\cos\phi}\frac{A_{180}\cos 80° + A_{80}}{\sin 80°} \right)$$

$$A_2 = \frac{1}{D'}\frac{1}{2E}\left(\sqrt{Q^2}\, A_{180} - \sqrt{Q^2}\frac{E - E'\cos\theta}{E'\sin\theta\cos\phi}\frac{A_{180}\cos 80° + A_{80}}{\sin 80°} \right)$$

where the term $(A_{180}\cos 80° + A_{80})/\sin 80°$ is similar to what Hovhannes Baghdasaryan[3] has been calling $A_{90}(0)$, but here it applies at all values of ϕ, not just over a restricted range. Because we took data at 80° we have a linear combination of A_{90} and A_{180}: $A_{80} = A_{90}\sin 80° - A_{180}\cos 80°$. The kinematic factor in front of this "A_{90}" must come from the 80° data.

1 http://twist.phys.virginia.edu/~jrm2zq/sf_deriv.pdf
2 http://twist.phys.virginia.edu/~or/e01006/analysis/aparaper.pdf
3 https://hallcweb.jlab.org/experiments/sane/wiki/index.php/Asymmetry_extraction