DRAFT
BigCal resolution for SANE
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The resolution of BigCal required to measure the spin asymmetries in SANE should be as close as possible to the proposal's 5% $\sqrt{E}$[GeV]. However, the actual available resolution will depend on the condition of the glass at the end of the *GEp*-III experiment, and the effectiveness of the procedure used to repair the radiation damage.

In order to estimate the minimum acceptable resolution, some physics criteria need to be considered:

- the measurement of d_2 integrates over the inelastic scattering region up to $x(W{=}\pi_{\mathrm{threshold}})$, so the inelastic and elastic contributions to the asymmetries can be clearly separated. This implies that the highest measured x bin should have no elastic contamination. This can be achieved if the upper edge of the bin is defined to be at least one sigma away from $x = 1$, in terms of W. If the resolution dW is poor, the upper limit of the d_2 integral may need to be lower than $x(W{=}\pi_{\mathrm{threshold}})$. But it should be kept in mind that, although the integral is weighted by x^2, the potential loss of x range is partly compensated by the decrease of g_2 as it approaches $x = 1$.
- the proposed test of local spin duality depends on the ability to resolve resonance regions. At the proposed resolution, the lowest W that could be included was about 1.35 GeV, so only the higher resonances could be tested. But the *RSS* results show that local spin duality does not obtain, and it is specially absent for the range $M <= W < \sim$ 1.4 GeV. For spin duality it would not be important to resolve better than about 400 MeV in W at $W = M$, so this requirement is less demanding than that of the d_2 integral.
- The extrapolation of A_1 to $x = 1$ improves with high x data, but it is less affected by the resolution in x since the highest bin could include the elastic point.

Therefore, the main question seems to be what is the highest value of x that can be measured with minimal elastic asymmetry contamination, to determine a purely inelastic result for d_2, given an effective scattered electron energy resolution.

The proposal's Table 2 lists the resolutions that could be achieved assuming the glass can be calibrated at the 5% $\sqrt{E}$[GeV]. A more detailed version of that table is given below as Table 1, which shows the values of the resolutions in final energy E', x, Q^2 and W and the upper edge of the highest inelastic x bin, calculated for the expected CEBAF energies at the time of the run and for representative values of Q^2, corresponding to angles in the range covered by BETA. This table is the starting reference for comparisons with worsened resolution examples.

	E	E'	θ	Q²	W	x	dE'=σE'	dx	dQ²	dW
	Rad 'D'amage?									
	u	σ√(E') = 5%								
Bin center	5.910	2.994	32.3	5.473	0.9383	1.000	0.087	0.059	0.167	0.171
Bin edge	5.910	2.735	32.3	**4.999**	**1.356**	**0.839**	0.083	0.048	0.159	0.113
Bin center	5.910	2.509	32.3	4.586	1.636	0.718	0.079	0.040	0.152	0.090
	5.910	2.442	39.2	6.508	0.9383	1.000	0.078	0.055	0.215	0.189
Bold = x max	5.910	2.251	39.2	**6.000**	**1.322**	**0.874**	0.075	0.048	0.206	0.129
	5.910	2.082	39.2	5.550	1.586	0.773	0.072	0.042	0.198	0.103
Energies in	5.910	2.171	43.4	7.017	0.9383	1.000	0.074	0.054	0.244	0.201
GeV	5.910	2.011	43.4	**6.500**	**1.303**	**0.888**	0.071	0.047	0.234	0.139
	5.910	1.868	43.4	6.039	1.557	0.796	0.068	0.043	0.225	0.112
	4.720	2.688	31.8	3.813	0.9383	1.000	0.082	0.072	0.123	0.144
	4.720	2.468	31.8	**3.500**	**1.267**	**0.828**	0.079	0.056	0.117	0.102
	4.720	2.274	31.8	3.225	1.498	0.703	0.075	0.046	0.112	0.083
	4.720	2.133	40.6	4.855	0.9383	1.000	0.073	0.063	0.170	0.162
	4.720	1.977	40.6	**4.500**	**1.236**	**0.874**	0.070	0.054	0.164	0.118
	4.720	1.838	40.6	4.184	1.451	0.774	0.068	0.047	0.158	0.097
	4.720	1.860	46.0	5.366	0.9383	1.000	0.068	0.061	0.200	0.173
	4.720	1.734	46.0	**5.000**	**1.218**	**0.892**	0.066	0.054	0.193	0.129
	4.720	1.619	46.0	4.671	1.424	0.803	0.064	0.048	0.186	0.106

Table 1. Proposal resolutions

A study of the response to radiation damage of the TF1 glass used in BigCal was reported in [1]. An expression representing the energy resolution, including the reduction of glass transparency $R = A_d/A_0$, where A_d and A_0 are the PMT responses to light produced in damaged and undamaged glass in terms of R and the detected particle's energy E is given there as Eq. (8)

$$\frac{\sigma_E}{E} = .01\sqrt{1+\frac{10}{\sqrt{E}}+10\ln^2(R)+\frac{33}{RE}}$$

There is a typo in the publication, where the last term is given as $30R/E$. The R in the numerator typo is easy to spot because it would imply improved resolution with reduced transparency ($0 < R \leq 1$). Also, 30 should actually be 10/.3 and the coefficient of $\ln^2 R$ is really 100/9. Figure 4 of the reference can only be reproduced with the expression as given above, as is shown on Figure 1 below, which displays the resolution for energies from 5 to 200 GeV, which are shown in the reference, plus two extra curves for 1 and 2 GeV, that are relevant to SANE. The terms in the expression are numerical values for parameters that have relative uncertainties of 10% to 20%.

Figure 1 also shows that even for perfectly transparent glass, the resolution is about 6.6% at 1 GeV, indicating that the proposal's 5% value may be on the favorable end of the range allowed by the uncertainties in the parameterization of the resolution. Table 2 shows the same information shown on Table 1, based on the above expression with $R = 1$. The last column shows the resolution calculated with the above parameterization, which is about 30% worse than the proposal's.

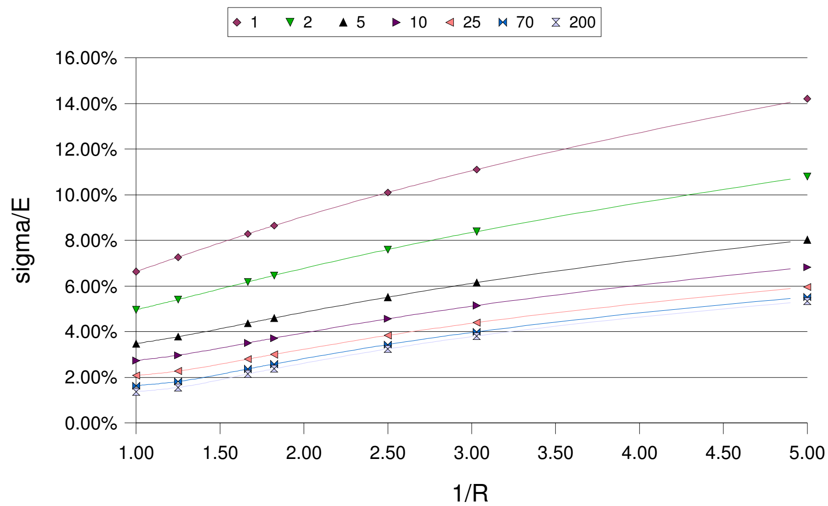

Figure 1. Energy resolution as a function of glass transparency for energies from 1 to 200 GeV.

		Rad 'D'amage?										
R=Ar/A0	1	**D**	σ√(E'=1) = 6.63%									
With Theta Constant:		E	E'	θ	Q²	W	x	dE'=σE'	dx	dQì	dW	dE'(Rad. D)
Elastic	Bin center	5.910	2.871	33.7	5.702	0.9383	1.000	0.085	0.084	0.251	0.253	0.123
	Bin edge	5.910	2.518	33.7	**5.000**	**1.499**	**0.785**	0.079	0.062	0.165	0.147	0.114
	Bin center	5.910	2.231	33.7	4.431	1.831	0.642	0.075	0.049	0.154	0.112	0.106
		5.910	2.324	41.0	6.729	0.9383	1.000	0.076	0.077	0.227	0.276	0.108
	Bold = x max	5.910	2.072	41.0	**6.000**	**1.443**	**0.833**	0.072	0.063	0.214	0.167	0.101
		5.910	1.862	41.0	5.392	1.756	0.710	0.068	0.053	0.202	0.129	0.095
		5.910	2.058	45.3	7.229	0.9383	1.000	0.072	0.075	0.257	0.290	0.101
		5.910	1.850	45.3	**6.500**	**1.414**	**0.853**	0.068	0.064	0.243	0.180	0.095
		5.910	1.675	45.3	5.884	1.715	0.740	0.065	0.055	0.231	0.140	0.089
		4.720	2.610	32.9	3.960	0.9383	1.000	0.081	0.100	0.129	0.210	0.116
		4.720	2.306	32.9	**3.500**	**1.382**	**0.773**	0.076	0.071	0.120	0.133	0.108
		4.720	2.057	32.9	3.122	1.660	0.625	0.072	0.055	0.113	0.103	0.101
		4.720	2.053	42.1	5.004	0.9383	1.000	0.072	0.087	0.179	0.231	0.101
		4.720	1.847	42.1	**4.500**	**1.331**	**0.835**	0.068	0.070	0.169	0.153	0.095
		4.720	1.672	42.1	4.074	1.589	0.712	0.065	0.059	0.161	0.121	0.089
		4.720	1.784	47.7	5.511	0.9383	1.000	0.067	0.084	0.209	0.245	0.093
		4.720	1.618	47.7	**5.000**	**1.304**	**0.859**	0.064	0.071	0.199	0.167	0.088

Table 2. Resolutions based on Eq. (8) of [1]for perfect light transmission.

The statistical errors on d_2 for the proposal's resolution and a ~7% effective resolution comparable to that obtained using the R dependent formula, are shown on Table 3. The errors have been estimated SANE's GEANT simulation, based on the nominal beam energies 6 GeV and 4.8 GeV. The loss of high x range does not seem to have a significant impact on the measurement.

Ref. [1] also states that the radiation damage to the glass transparency has an exponential dependence on the number of incident particles. Specifically, for TF1 glass, R is reduced by $1/e$ after a dose of $(5\pm1)\times10^{10}$ 30 GeV pions. Although particles of this energy are not present in our case (the damage decreases with decreasing particle energy,) it can be used to get a crude estimate of the dose received by BigCal in the period starting on Feb. 6, 2008 at 17:10 (EPICS time) and ending on Feb. 10, at 1:42. During this period, the EPICS logger shows that the Hall received about 7 h of 35μA beam and 72 of 40 μA beam, or a maximum charge of 10.5 C. A report by Wei Luo [2] indicates that the glass response decreased by 20% during that period, so the number of equivalent pions absorbed by BigCal is

$$N\pi = -5\times10^{10}\ln 0.8 = 1.1\times10^{10}$$

or 1×10^{9}/C. This allows us to estimate the maximum radiation damage we can expect for BigCal at the end of *GEp*-III, corresponding to a charge of about 180 C, or 18×10^{10} pions

$$R = e^{\left(\frac{-18\times10^{10}}{5\times10^{10}}\right)} = e^{-3.6} = 0.027$$

As expected, BigCal's glass will be essentially opaque. However, [2] also reports that the UV light treatment of BigCal's glass in March resulted in an improvement that can be fitted to project a better than 65% transparency after 30 days of irradiation, assuming a 7% initial transmission of LED light. Since the fit is exponential, the difference between a ~ 3% and a 7% starting values is a matter of a few additional hours.

Q^2 range GeV^2	**<Q^2>** GeV^2	**Lowest W** GeV	**Resolution** $\sigma\sqrt{E'}$)	**High x**	**d2 error** (stat)
2.5 - 3.5	3.107	1.100	5.0%	0.713	3.6%
	3.107	1.350	6.6%	0.713	3.6%
	3.107	1.480	8.0%	0.713	3.6%
3.5 - 4.5	4.069	1.100	5.0%	0.929	2.4%
	3.998	1.350	6.6%	0.825	2.5%
	3.951	1.480	8.0%	0.776	2.8%
4.5 - 5.5	4.890	1.100	5.0%	0.940	3.4%
	5.014	1.350	6.6%	0.842	3.6%
	5.000	1.480	8.0%	0.796	3.8%
5.5 - 6.5	5.912	1.100	5.0%	0.909	6.7%
	5.922	1.350	6.6%	0.879	7.6%
	5.928	1.480	8.0%	0.837	7.8%

Table 3. Upper edges of highest x bin and expected statistical errors for the indicated ranges of Q^2,

Based on these estimates and the apparent equality of LED and cherenkov light transmission observed in the BigCal UV treatment tests, Table 4 shows the same information as Table 2, for R = 0.65. The corresponding impact on the statistical precision is shown on Table 3.

		Rad 'D'amage?										
R=Ar/A0	0.65	**D**	σ√(E'=1) = 7.98%									
With Theta Constant:		E	E'	θ	Q²	W	x	dE'=σE'	dx	dQì	dW	dE'(Rad. D)
Elastic	Bin center	5.910	2.793	34.6	5.849	0.9383	1.000	0.084	0.099	0.309	0.307	0.145
	Bin edge	5.910	2.388	34.6	**5.000**	**1.578**	**0.756**	0.077	0.071	0.168	0.166	0.132
	Bin center	5.910	2.072	34.6	4.340	1.935	0.603	0.072	0.055	0.156	0.125	0.121
		5.910	2.246	42.2	6.875	0.9383	1.000	0.075	0.092	0.235	0.335	0.127
	Bold = x max	5.910	1.960	42.2	**6.000**	**1.514**	**0.809**	0.070	0.073	0.219	0.191	0.117
		5.910	1.730	42.2	5.295	1.852	0.675	0.066	0.060	0.205	0.146	0.109
		5.910	1.982	46.7	7.371	0.9383	1.000	0.070	0.090	0.266	0.353	0.118
		5.910	1.748	46.7	**6.500**	**1.480**	**0.832**	0.066	0.074	0.250	0.208	0.110
		5.910	1.556	46.7	5.786	1.807	0.708	0.062	0.064	0.235	0.159	0.103
		4.720	2.558	33.7	4.058	0.9383	1.000	0.080	0.118	0.133	0.254	0.138
		4.720	2.206	33.7	**3.499**	**1.449**	**0.742**	0.074	0.080	0.122	0.151	0.126
		4.720	1.928	33.7	3.059	1.750	0.584	0.069	0.060	0.114	0.115	0.116
		4.720	1.999	43.2	5.107	0.9383	1.000	0.071	0.103	0.184	0.281	0.119
		4.720	1.761	43.2	**4.500**	**1.390**	**0.810**	0.066	0.081	0.173	0.176	0.110
		4.720	1.567	43.2	4.004	1.671	0.677	0.063	0.067	0.163	0.137	0.103
		4.720	1.729	49.0	5.612	0.9383	1.000	0.066	0.100	0.216	0.298	0.109
		4.720	1.541	49.0	**5.000**	**1.359**	**0.838**	0.062	0.082	0.204	0.192	0.102
		4.720	1.383	49.0	4.490	1.628	0.717	0.059	0.070	0.193	0.151	0.096

Table 4. Resolutions for reduced light transmission.

1 A. Inyakin et al. NIM 215 (1983) 103
2 Wei Luo, private communication