The physics angles θ^* and ϕ^* in RSS with Maple V

The physics system for asymmetry calculation is defined as
$z =$ along incident electron momentum $\vec{k}$, i.e. along beam
$y =$ along cross product $\vec{k} \times \vec{k}'$ (points down in lab for $\vec{k}'$ horizontal)
$x =$ normal to y, z (points to HMS in Hall C)
First, rotate counterclockwise about z by the azimuthal scattering angle $\phi = \phi_e$:

```
Rzphi:=linalg[matrix]([[cos(phi),sin(phi),0], [-sin(phi),cos(phi),0],[0,0,1]])
```

$$Rzphi := \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(There are no subscripts in Maple V, so we use plain theta, phi for $\theta \equiv \theta_q, \phi \equiv \phi_e$).

Then, rotate about the physics y by the angle θ_q between $\vec{k}$ and $\vec{q} = \vec{k} - \vec{k}'$ from $\vec{k}$ to $\vec{q}$.

This is a CLOCKWISE rotation, since $\vec{q}$ is to the left of $\vec{k}$ and y is down, so sines are *(-1).

```
> Ryq:=linalg[matrix]([[cos(theta),0,sin(theta) ],[0,1,0],[-sin(theta),0,cos(theta)]]);
```

$$Ryq := \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Final rotation, about the rotated z^* axis

```
Rzg:=linalg[matrix]([[cos(gamma),sin(gamma),0 ],[-sin(gamma),cos(gamma),0],[0,0,1]])
```

$$Rzg := \begin{bmatrix} \cos(\gamma) & \sin(\gamma) & 0 \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> with(linalg):
> Rstar:=multiply(Rzg,Ryq,Rzphi);
```

$Rstar :=$

$[\cos(\gamma)\cos(\theta)\cos(\phi) - \sin(\gamma)\sin(\phi)\,,\ \cos(\gamma)\cos(\theta)\sin(\phi) + \sin(\gamma)\cos(\phi)\,,\ \cos(\gamma)\sin(\theta)]$

$[-\sin(\gamma)\cos(\theta)\cos(\phi) - \cos(\gamma)\sin(\phi)\,,\ -\sin(\gamma)\cos(\theta)\sin(\phi) + \cos(\gamma)\cos(\phi)\,,\ -\sin(\gamma)\sin(\theta)$
$]$

$[-\sin(\theta)\cos(\phi)\,,\ -\sin(\theta)\sin(\phi)\,,\ \cos(\theta)]$

Since there is no third rotation, i.e. γ=0,

```
Rzg1:=linalg[matrix](3,3,[1,0,0,0,1,0,0,0,1])
```

$$Rzg1 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> Rstar1:=evalm(Rzg1&*Ryq&*Rzphi);
```

$$Rstar1 := \begin{bmatrix} \cos(\theta)\cos(\phi) & \cos(\theta)\sin(\phi) & \sin(\theta) \\ -\sin(\phi) & \cos(\phi) & 0 \\ -\sin(\theta)\cos(\phi) & -\sin(\theta)\sin(\phi) & \cos(\theta) \end{bmatrix}$$

PARA:

B points along -z in all systems

```
> Bpara:=linalg[matrix](3,1,[0,0,-Bz]);
```

$$Bpara := \begin{bmatrix} 0 \\ 0 \\ -Bz \end{bmatrix}$$

```
> Bstarpara:=evalm(Rstar1 &* Bpara);
```

$$Bstarpara := \begin{bmatrix} -\sin(\theta)\, Bz \\ 0 \\ -\cos(\theta)\, Bz \end{bmatrix}$$

```
> cosstar:=Bstarpara[3,1]/Bz;
```

$$cosstar := -\cos(\theta)$$

```
> phistar:=arctan(Bstarpara[2,1]/Bstarpara[1,1] );
```

$$phistar := 0$$

$\phi^* = 0$ or 180, depending on the sign of the denominator. Here denominator is negative, so $\phi^* = 180$

PERP:

B points along -x in physics system

```
> Bperp:=linalg[matrix](3,1,[-Bx,0,0]);
```

$$Bperp := \begin{bmatrix} -Bx \\ 0 \\ 0 \end{bmatrix}$$

```
> Bstarperp:=evalm(Rstar1 &* Bperp);
```

$$Bstarperp := \begin{bmatrix} -\cos(\theta)\cos(\phi)\, Bx \\ \sin(\phi)\, Bx \\ \sin(\theta)\cos(\phi)\, Bx \end{bmatrix}$$

```
> cossstar:= Bstarperp[3,1]/Bx;
```

$$cossstar := \sin(\theta)\cos(\phi)$$

```
> phistar:=arctan(Bstarperp[2,1]/Bstarperp[1,1] );
```

$$phistar := -\arctan(\frac{\sin(\phi)}{\cos(\theta)\cos(\phi)})$$

So $\cos\theta^* = \sin\theta_q \cos\phi_e$, and $\tan\phi^*$ is in second quadrant (+ sine, − cosine), so $\phi^* = 180 + \arctan\left(\frac{\tan\phi_e}{-\cos\theta_q}\right)$