SANE Higher Twists

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d_2 Extraction

$$d_2 = 3\int x^2 \bar{g}_2(x)dx$$

$$\bar{g}_2(x) = g_2^{exp} - g_2^{WW}$$

$$g_1^{exp}(x,Q^2) = g_1^{\tau=2}(x,Q^2) + g_1^{\tau=3}(x,Q^2) + g_1^{\tau=4}(x,Q^2) + ...$$

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The Wandzura-Wilczek relation gives the twist-2 contribution to g_2

$$g_2^{WW}(x) = g_2^{\tau=2}(x) = -g_1^{\tau=2}(x) + \int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_1^{\tau=2}(y)$$

but we need only the leading twist $g_1^{\tau=2}$, not g_1^{exp}.

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but we need only the leading twist $g_1^{\tau=2}$, not g_1^{exp}.
However, target mass and finite Q^2 corrections need to be made. The important twist-3 contribution to g_1 shows up as a target mass correction[?] in the Blumlein-Tkabladze relation

$$g_1^{\tau=3}(x) = \frac{4M^2x^2}{Q^2}\left[g_2^{\tau=3}(x) - 2\int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_2^{\tau=3}(y)\right] \quad (1)$$

Need to disentangle different twist contributions

Twist-2 TMCs

$$g_1^{exp}(x,Q^2) = g_1^{\tau 2+TMC}(x,Q^2) + g_1^{\tau 3+TMC}(x,Q^2) + g_1^{\tau 4}(x,Q^2) + \ldots$$
$$g_2^{exp}(x,Q^2) = g_2^{\tau 2+TMC}(x,Q^2) + g_2^{\tau 3+TMC}(x,Q^2) + g_2^{\tau 4+TMC}(x,Q^2) + \ldots$$

Target Mass Corrections

- Power corrections of order M^2/Q^2
- This power does not necessarily correspond to the twist (depending on how you define twist).

Following the latest JAM paper's notation:

Twist-2 TMC corrections to g_1

$$g_1^{(\tau 2+TMC)}(x,Q^2) = \frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \left(\frac{\rho^2-1}{\rho^4}\right)\int_\xi^1 \frac{dz}{z}\left[\frac{(x+\xi)}{\xi} - \frac{(3-\rho^2)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$
$$= g_1^{(\tau 2)}(x,Q^2; M\to 0)$$

where $\rho^2 = \left(1 + \frac{4M^2x^2}{Q^2}\right)$

Twist-2 TMCs

Twist-2 TMC corrections to g_2

$$g_2^{(\tau 2+TMC)}(x,Q^2) = -\frac{x}{\xi\rho^3} g_1^{(\tau 2)}(\xi,Q^2) + \frac{1}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{x}{\xi} - (\rho^2-1) + \frac{3(\rho^2-1)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$
$$= g_2^{WW}(x,Q^2; M\to 0)$$

where $\rho^2 = \left(1+\frac{4M^2x^2}{Q^2}\right)$

$$g_2^{WW}\left[g_1^{(\tau 2)}\right] = -g_1^{\tau 2}(x) + \int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_1^{\tau 2}(y)$$

$$g_2^{WW}\left[g_1^{(\tau 2+\mathbf{TMC})}\right] = -g_1^{\tau 2+\mathbf{TMC}}(x) + \int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_1^{\tau 2+\mathbf{TMC}}(y)$$

$$g_2^{WW}\left[g_1^{(\tau 2+\tau\mathbf{3})}\right] \neq -g_1^{\tau 2+\tau\mathbf{3}}(x) + \int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_1^{\tau 2+\tau\mathbf{3}}(y)$$

Twist-3 TMCs

Twist-3 TMC corrections to g_1

$$g_1^{\tau=3}(x) = \frac{4M^2x^2}{Q^2}\left[g_2^{\tau=3}(x) - 2\int_x^1 \frac{\mathrm{dy}}{\mathrm{y}} g_2^{\tau=3}(y)\right] \tag{2}$$

B & T emphasized the need to take full account of this type of term if twist-3 terms are kept in the cross sections. I am not sure if this means using this equation, or the more accurate equation which has terms of high order in M^2/Q^2

Backup Slides

Simultaneously disentangling

Q^2 evolution of twist-3 distributions

From the work of Braun, et.al.[?], we have the (large N$_c$) evolution equations

$$Q^2 \frac{d}{dQ^2}\Delta q_T^{+,S} = \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y}\left(P_{qq}^T(x/y)\Delta g_T^{+,S}(y) + P_{qg}^T(x/y)\Delta g_T(y)\right)$$

$$Q^2 \frac{d}{dQ^2}\Delta g_T = \frac{\alpha_s}{4\pi}\int_x^1 \frac{dy}{y}\left(P_{gg}^T(x/y)\Delta g_T(y)\right)$$

$$g_2^{\tau=3,LL}(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \int_x^1 \frac{\mathrm{dy}}{\mathrm{y}}\left[\Delta q_T^{+}(y)\right]$$

In [?] they also show that g_2 can be approximately evolved as a non-singlet distribution due to the very small gluon contribution (which only shows up at small x)

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} g_2^{NS}(x,Q^2) = \frac{\alpha_s(Q^2)}{4\pi}\left(\int_x^1 \frac{dz}{z} P^{NS}(x/z) g_2^{NS}(z,Q^2)\right)$$

Ignoring contributions beyond twist-3 effects, we see that $g_1(x)$ and $g_2(x)$ are completely defined when Δq along with $g_2^{\tau=3}$ (or equivalently Δq_T and Δq_T) are given at some input scale Q_0^2.