SANE: Fitting higher twists

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I. FITTING HIGHER TWISTS

1. Without BC sum rule
2. With BC sum rule
3. Without $W_{\text{threshold}}$
4. With $W_{\text{threshold}}$

A. Twist-3 distribution: D(x)

The twist-3 distribution associated with the reduced matrix element,

$$d_2 = 2\int_0^1 x^2 D(x)dx = 2d_3 \ , \tag{1}$$

is used to calculate the twist-3 contribution to the g_2 structure function as

$$g_2^{\tau 3}(x) = D(x) - \int_x^1 \frac{D(y)}{y}dy \tag{2}$$

in the massless limit. A more complicated expression exists which includes the target mass effects. We parameterize $g_2^{\tau 3}(x)$ as a function of x with p parameters and we would now like to seek constraints to limit the number of free parameters.

Next we want to solve for $D(x)$ so we take the derivative of both sides of the equation **??** and dropping the indices on g

$$\frac{d}{dx}g = \frac{D(x)}{x} + \frac{d}{dx}D(x) \tag{3}$$

and solve for $D(x)$ with the boundary condition that the function vanishes at $x = 1$. This yields the solution

$$xD(x) = -\int_x^1 y\, g'(y)\, dy \tag{4}$$

This equation provides a constraint on the parameters which can be seen as removing the constant term in a polynomial expression due to the derivative.

$$g(x) = \sum_{i=0}^{4} p_i x^i \tag{5}$$

The BC sum rule can provide a constraint too:

$$\int_0^1 dx g_2^{\tau 3}(x) = 0. \tag{6}$$

Applying the constraints gives

$$p_0 = \frac{p(2)}{3} + \frac{p(3)}{2} + \frac{3p(4)}{5} \tag{7}$$

$$p_1 = \frac{1}{30}(-40p(2) - 45p(3) - 48p(4)) \tag{8}$$

B. W threshold

If below W_{thresh} we force the twist 3 distribution $D(x_{thresh}) \to 0$ then we find that

$$xD(x) = -\int_x^1 y\, g'(y)\, dy + \int_{x_{thresh}}^1 y\, g'(y)\, dy \tag{9}$$

where

$$x_{thresh} = Q^2/(M_{thresh}^2 - M_p^2 + Q^2). \tag{10}$$

C. As a function of W

If we want to use W

$$g(x) = \sum_{i=0}^{2} p_i \left(\frac{1}{W}\right)^i \tag{11}$$

the constrained parameters are

$$p_0 = -\frac{p(2)\left(\mathrm{Mp}\sqrt{\mathrm{Mp}^2 - \mathrm{Q2}}\left(\mathrm{Mp}^2\log(\mathrm{Q2}) - \mathrm{Q2}\log\left(\frac{\mathrm{Q2}}{\mathrm{Mp}^2}\right) - 2\mathrm{Mp}^2\log(\mathrm{Mp})\right) + \left(\mathrm{Mp}^2 - \mathrm{Q2}\right)^2\sinh^{-1}\left(\sqrt{\frac{\mathrm{Mp}^2}{\mathrm{Q2}} - 1}\right)\right)}{\mathrm{Mp}\left(\mathrm{Mp}^2 - \mathrm{Q2}\right)^{3/2}\left(\mathrm{Mp}\sqrt{\mathrm{Mp}^2 - \mathrm{Q2}}\sinh^{-1}\left(\sqrt{\frac{\mathrm{Mp}^2}{\mathrm{Q2}} - 1}\right) - \mathrm{Mp}^2 + \mathrm{Q2}\right)} \tag{12}$$

$$p_1 = \frac{p(2)\left(-\mathrm{Mp}^2\log(\mathrm{Q2}) - \mathrm{Mp}^2 + 2\mathrm{Mp}^2\log(\mathrm{Mp}) + \mathrm{Q2}\right)}{\mathrm{Mp}\left(-\mathrm{Mp}\sqrt{\mathrm{Mp}^2 - \mathrm{Q2}}\sinh^{-1}\left(\sqrt{\frac{\mathrm{Mp}^2}{\mathrm{Q2}} - 1}\right) + \mathrm{Mp}^2 - \mathrm{Q2}\right)} \tag{13}$$

$$\tag{14}$$

however calculating these for higher powers becomes unwieldy. It is better to use a parameterization in x.

II. EVOLUTION OF HIGHER TWISTS

In [**?**] they also show that $g_2^{\tau 3}$ can be approximately evolved as a non-singlet distribution due to the very small gluon contribution (which only shows up at small x)

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2} g_2^{NS}(x, Q^2) = \frac{\alpha_s(Q^2)}{4\pi}\int_x^1 \frac{dz}{z} P^{NS}(x/z) g_2^{NS}(z, Q^2) \tag{15}$$

where the splitting function is

$$P^{NS} = \left[\frac{4C_F}{1-z}\right]_+ + \delta(1-z)\Big[C_F + \frac{1}{N_c}\Big(2 - \frac{\pi^2}{3}\Big)\Big) - 2C_F \tag{16}$$

and $C_F = (N_c^2 - 1)/(2N_c)$.

Using QCDNUM with this custom kernel implemented I can reproduce the LCWF distributions shown in FIG. 1 [**?**].

III. FIT RESULTS

Looking at the results to fitting just the SANE data shown in FIG. 2, the SANE-BETA data in the third panel with small error bars don't seem to be fit very well.

However raising the W_{min} on the fit data and including the world data helps pull the curve down for these points. This is shown in FIG. 5,

A. W_{min} = 1600 MeV

$$MinFCN = 149.632$$
$$NDf = 118$$
$$p - value = 0.97$$

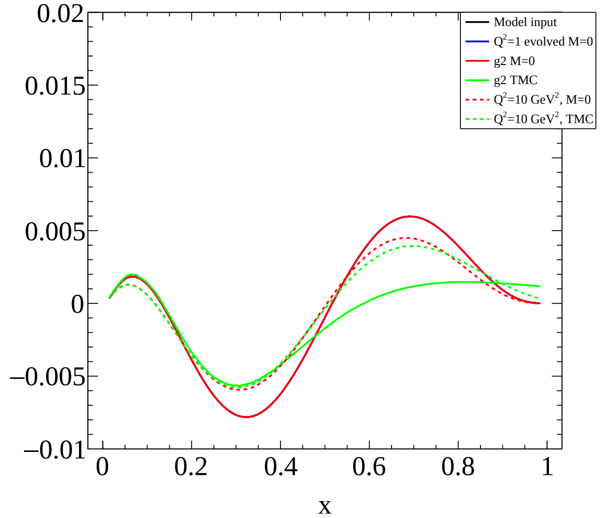

FIG. 1. Test of evolution using LCWF for comparison against the result of Braun, et.al.[**?**]

B. W_{min} = 1800 MeV

$$\begin{aligned} MinFCN &= 89.5586 \\ NDf &= 102 \\ p - value &= 0.19 \end{aligned}$$

IV. CONSTANT Q^2 DATA

See FIG. 6 and 7.

V. MOMENTS

$$\begin{aligned} A_2 &= \gamma \frac{g_T}{F_1} \\ &= \left(\frac{2Mx}{\sqrt{Q^2}} \right) \frac{g_1 + g_2}{F_1} \end{aligned} \tag{17}$$

$$\begin{aligned} I(Q^2) &= \int_0^1 dx x^2 (2g_1 + 3g_2) \\ &= \int_0^1 dx x^2 (3g_T - g_1) \\ &= \int_0^1 dx x^2 (3\frac{F_1}{\gamma} A_2 - g_1) \end{aligned} \tag{18}$$

$$M_1^n(Q^2) = \frac{\xi^{n+1}}{x^2} \left(g_1(x) \left(\frac{x}{\xi} - \frac{n^2 \xi x y^2}{(n+2)^2} \right) - \frac{(4n) x^2 y^2 g_2(x)}{n+2} \right) \tag{19}$$

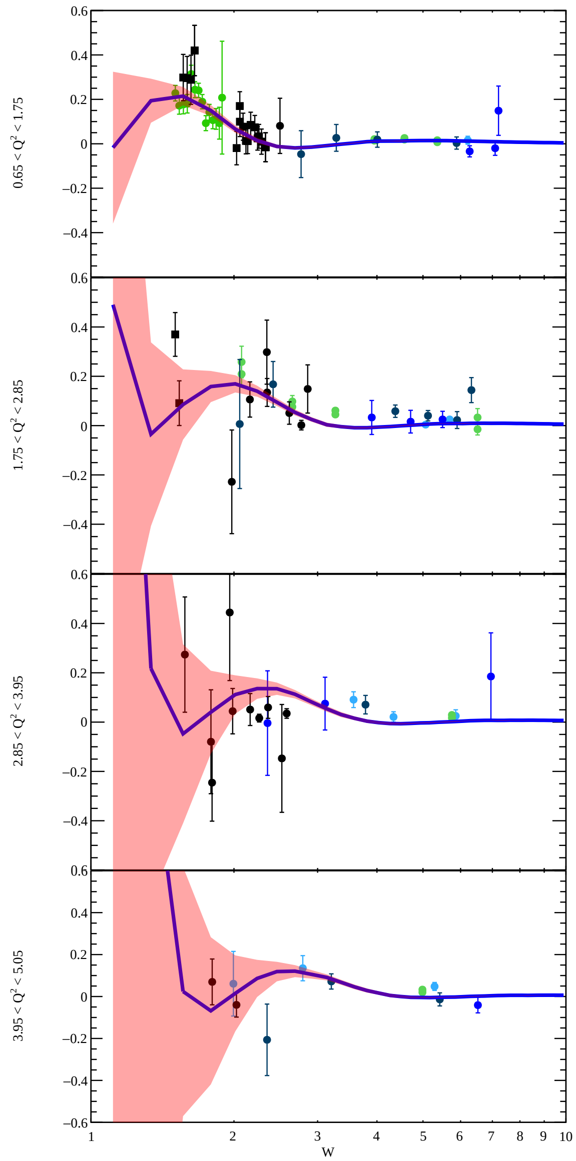

FIG. 2. Result using only SANE data and a low $W_{min} = 1500$ MeV. The black circles are SANE-BETA and the black squares are SANE-HMS.

where $y^2 = M^2/Q^2$.

$$M_2^n(Q^2) = \frac{\xi^{n+1}}{x^2}\left(\frac{xg_1(x)}{\xi} + g_2(x)\left(\frac{nx^2}{(n-1)\xi^2} - \frac{nx^2y^2}{n+1}\right)\right) \tag{20}$$

The moments above are just definitions. The structure functions inserted into each depend on the order in twist one wishes to examine. If we restrict the analysis to twist-3 we define the input structure functions to be

$$g_{1,2} \equiv g_{1,2}^{\tau 2} + g_{1,2}^{\tau 3}\,. \tag{21}$$

The Nachtmann moments were derived in the twist-3 OPE analysis and their usefulness becomes apparent when these

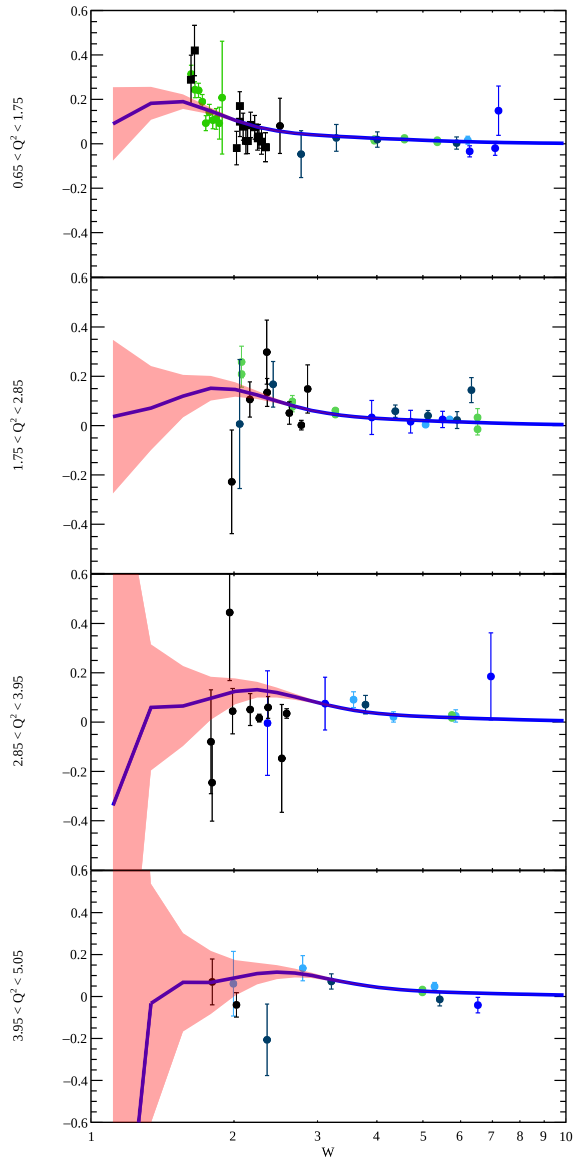

FIG. 3. Result using world data with $W_{min} = 1600$ MeV. The black circles are SANE-BETA and the black squares are SANE-HMS.

structure functions used

$$\begin{aligned} M_2^3(Q^2) &= d_3 \\ &\simeq \int dx \Big[\frac{1}{2} x^2 (2g_1(x) + 3g_2(x)) \\ &\qquad - \frac{3}{4} y^2 \left(x^4 (4g_1(x) + 5g_2(x)) \right) \\ &\qquad + \frac{3}{2} x^6 y^4 (6g_1(x) + 7g_2(x)) + \mathcal{O}\left(y^6\right) \Big] \\ &= d_2/2 + \mathcal{O}\left(d_8\, y^6\right) \end{aligned} \tag{22}$$

where the higher order terms in the last equation appear only due finite series expansion.

Note that a twist-4 OPE analysis would spoil the Nachtmann moments like the twist-3 OPE does the WW relation. Reduced twist-4 matrix elements would appear at all higher orders in y^2.

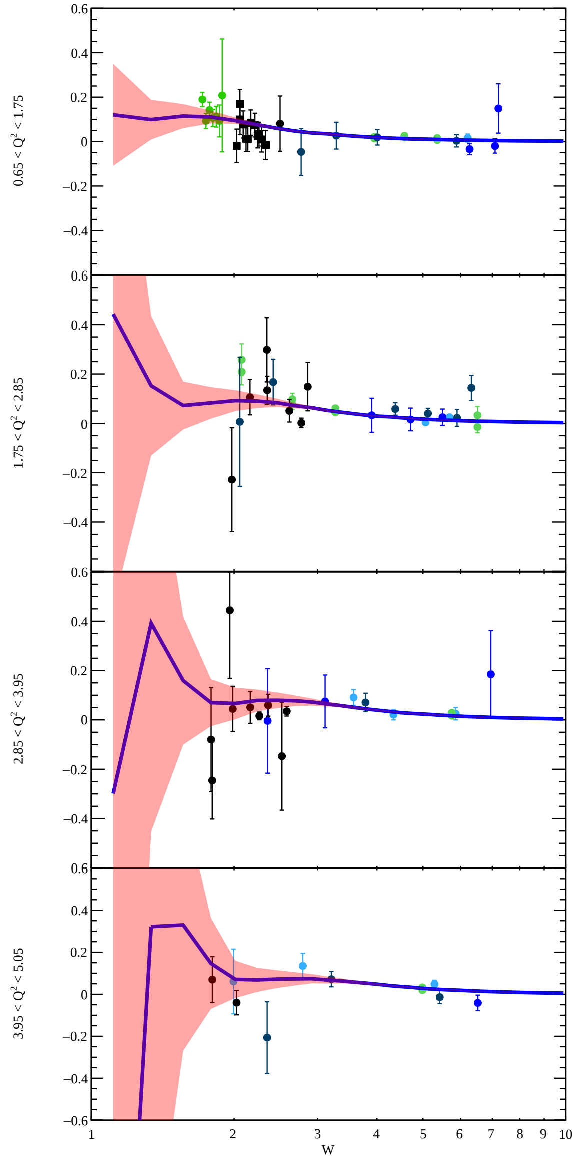

FIG. 4. Result using world data with a higher $W_{min} = 1700$ MeV. The black circles are SANE-BETA and the black squares are SANE-HMS.

$$I(Q^2) \simeq d_2 + 3y^2 d_4 + 6y^4 d_6 + \mathcal{O}\left(d_8\, y^6\right) \tag{23}$$

$$\lim_{M\to 0} M_2^3(Q^2) = \frac{I(Q^2)}{2} = \frac{d_2}{2} = d_3 \tag{24}$$

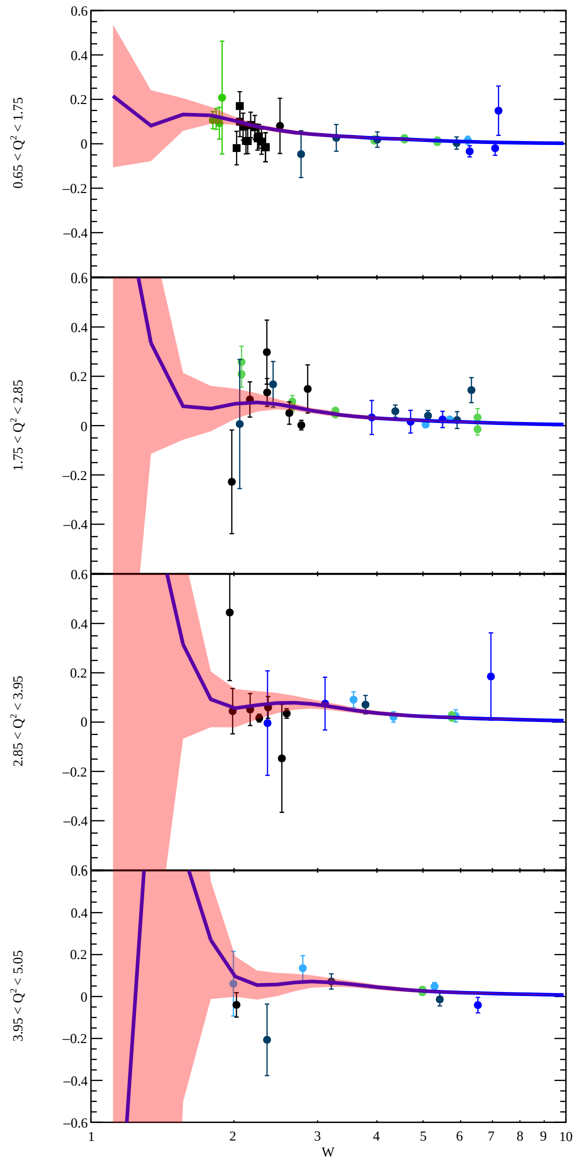

FIG. 5. Result using world data with W_{min} = 1800 MeV. The black circles are SANE-BETA and the black squares are SANE-HMS.

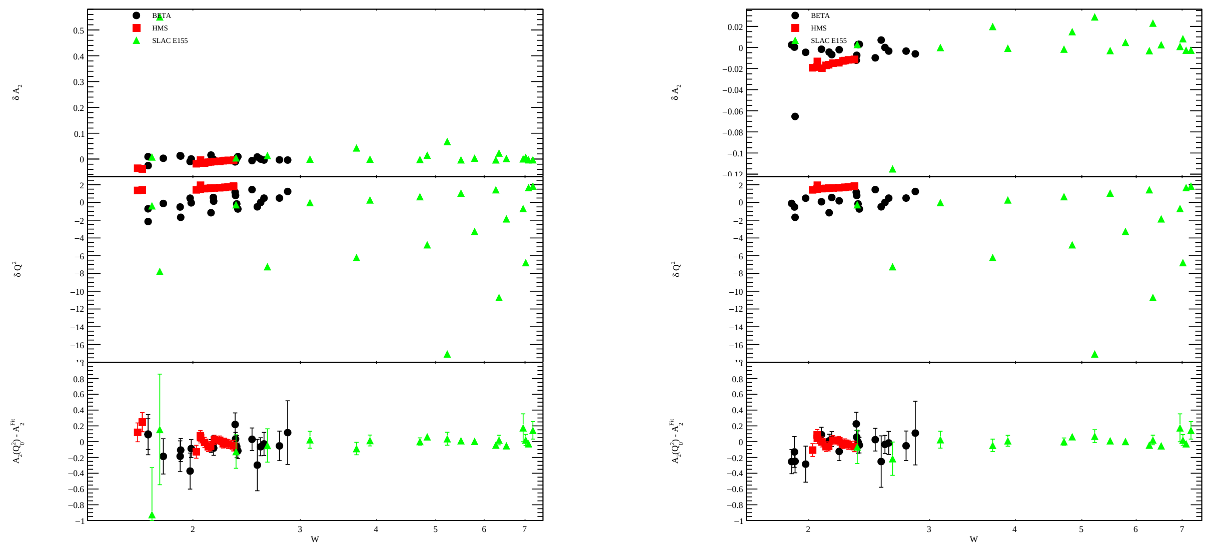

FIG. 6. Corrections for constant $Q_0^2 = 3$ (GeV/c)^2 with the fit using $W_{min} = 1600$ MeV (left) and $W_{min} = 1800$ MeV (right). The upper panel shows the difference between A_2 calculated at each data point's Q_i^2 and the constant Q_0^2, the middle pannel shows the difference $Q^2 - Q_i^2$, and the lower panel shows the difference between the measured A_2 and the calculated A_2 at Q_i^2.

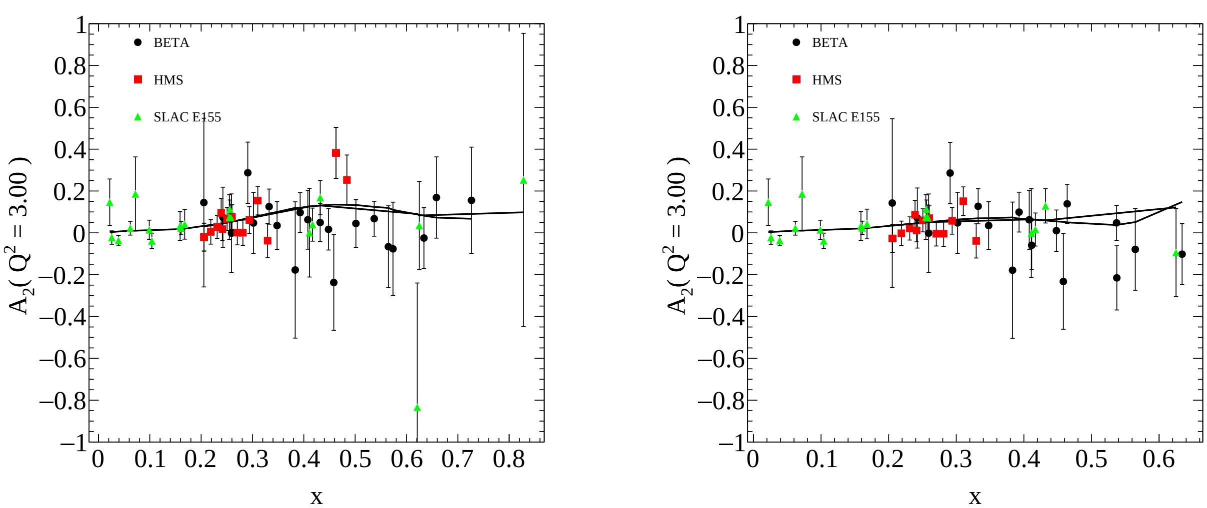

FIG. 7. Data with a constant $Q^2 = 3$ (GeV/c)^2 correction applied from the fit using $W_{min} = 1600$ MeV (left) and $W_{min} = 1800$ MeV (right).

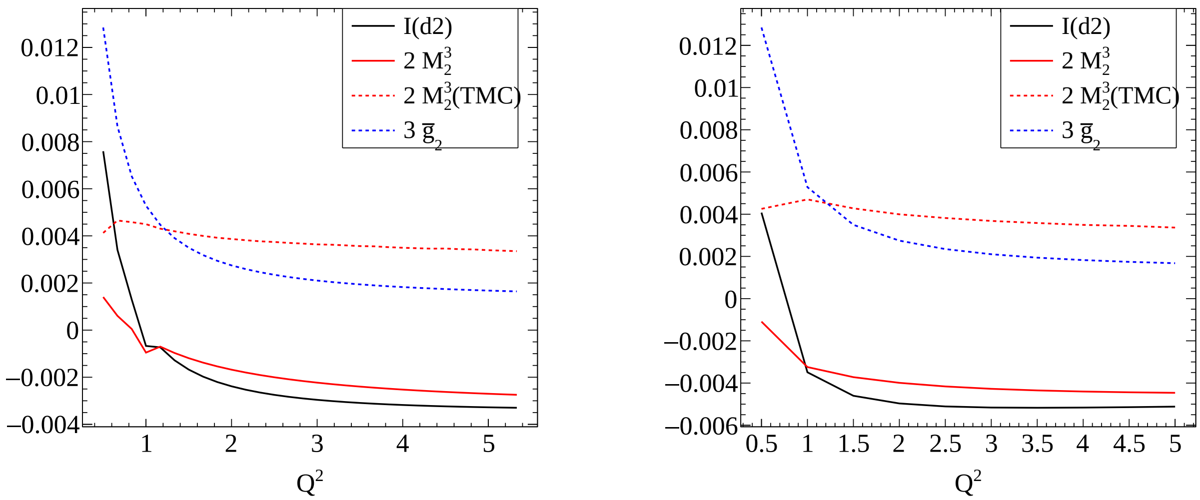

FIG. 8. Moments from A_2 fit with W_{min} = 1600 MeV and using statistical polarized PDFs (left) and the JAM polarized PDFs.

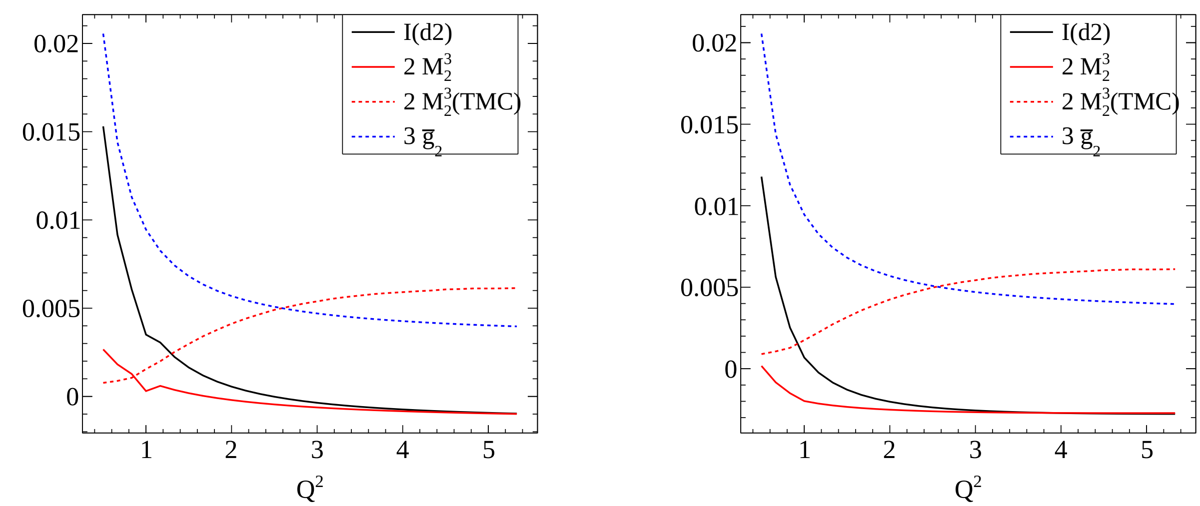

FIG. 9. Moments from A_2 fit with W_{min} = 1800 MeV and using statistical polarized PDFs (left) and the JAM polarized PDFs.

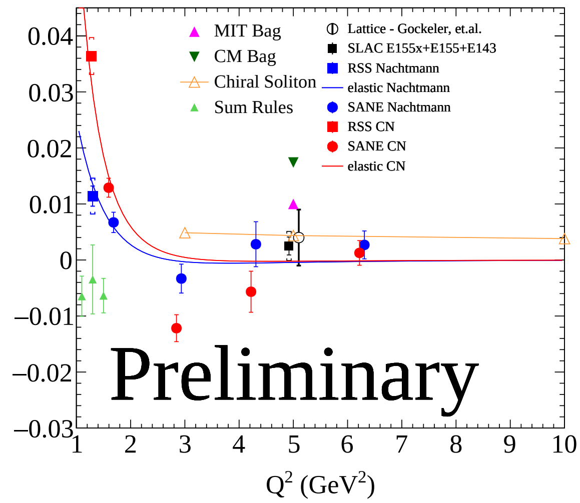

FIG. 10. Moments extracted directly from the data (need to update this result).