Tracking in the SOS Spectrometer

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Contents

This document discusses the general problem of trackfinding and fitting in a field-free region and a set of solutions for the SOS spectrometer.

1 Introduction

A wire chamber measures a ψ coordinate in the plane of the chamber. The plane itself is defined by the z coordinate of the intersection of the plane with the z axis, z_0 and three rotation angles:

β: A rotation about the y axis by an angle β towards the z axis. The first SOS chamber has beta of $+\pi/4$.

γ: A rotation around the x' axis defined after the β rotation with the positive sense being y' moving towards the z' axis.

α: Within the local x-y plane defined by z_0, β, and γ, α is the rotation about the local z axis of the coordinate measuring axis, i.e. perpendicular to the wires. The measured coordinate is ψ and the orthogonal coordinate is χ. $\psi = \chi = 0$ is the intersection of the plane with the spectrometer z axis.

These coordinates are illustrated in Figure 1. In terms of the local coordinates and the four parameters of the wire plane the coordinates in the focal plane coordinate system are:

$$\begin{aligned}
z &= z_0 + \psi(\sin\alpha\sin\beta + \cos\alpha\cos\beta\sin\gamma) + \chi(-\cos\alpha\sin\beta + \sin\alpha\cos\beta\sin\gamma) \\
x &= \quad\psi(\sin\alpha\cos\beta - \cos\alpha\sin\beta\sin\gamma) - \chi(\cos\alpha\cos\beta + \sin\alpha\sin\beta\sin\gamma) \\
y &= \quad\psi\cos\alpha\cos\gamma + \chi\sin\alpha\cos\gamma
\end{aligned} \tag{1}$$

In the limit of a plane perpendicular to z, $\gamma = \beta = 0$:

$$\begin{aligned}
z &= z_0 \\
x &= \psi\sin\alpha - \chi\cos\alpha \\
y &= \psi\cos\alpha + \chi\sin\alpha
\end{aligned} \tag{2}$$

We will write the general equation in the following notation:

$$\begin{aligned}
z - z_0 &= \psi Z_\psi + \chi Z_\chi \\
x &= \psi X_\psi + \chi X_\chi \\
y &= \psi Y_\psi + \chi Y_\chi
\end{aligned} \tag{3}$$

The goal of tracking is to determine the equation of the ray which provides the best fit to the measured wire chamber coordinates. This ray is determined by the five parameters $x_t, y_t, z_t, \tan x\prime, \tan y\prime$ defining the line:

$$\begin{aligned}
x &= (z - z_t) * \tan x\prime + x_t \\
y &= (z - z_t) * \tan y\prime + y_t
\end{aligned} \tag{4}$$

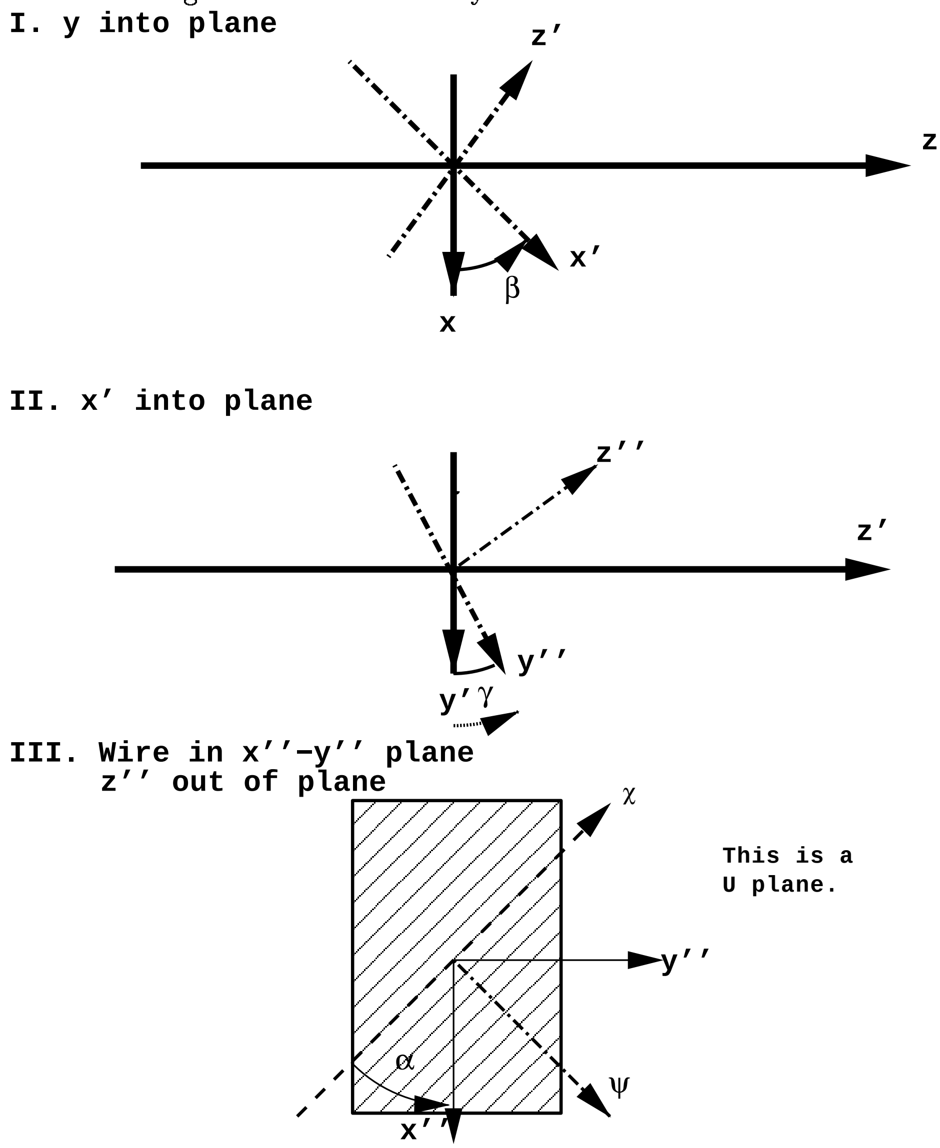
z_t can be chosen arbitrarily and will be taken to be the focal plane position.

Solving for the intersection of the wire plane with the track ray and abbreviating $\tan x\prime$ and $\tan y\prime$ as t_x and t_y then:

$$\psi_i^{fit} = \frac{t_x y_t Z_\chi - t_y x_t Z_\chi + t_x(z_0 - z_t)Y_\chi + t_y(z_t - z_0)X_\chi + Y_\chi x_t - X_\chi y_t}{t_x(Y_\psi Z_\chi - Y_\chi Z_\psi) + t_y(X_\chi Z_\psi - X_\psi Z_\chi) + (Y_\chi X_\psi - X_\chi Y_\psi)} \tag{5}$$

where the index "i" is understood on each of the plane parameters, z_0, $X_{\psi,\chi}$, $Y_{\psi,\chi}$ and $Z_{\psi,\chi}$.

Figure 1: Coordinate Systems Transformations



2 Procedure

The general steps in tracking are clear. The final goal is to fit a track to a number of hits. Starting from this, in reverse order (from last step to first) the steps are:

- General fit of track parameters to all hits on a track. Evaluate error matrix and goodness of fit
- Link space points (collections of hits in a chamber package) from each chamber to a trail track
- Resolve left-right ambiguities of hits in drift chambers associated with each point.
- Identify space points in each chamber.

In the sections below the algorithms for each step will be descibed. There are usually several possible algorithms and a choice between them will require study, and perhaps knowledge of the actual chamber performance.

3 Space Point recognition

Given the relatively small number of planes, and the fact that in the SOS chamber 1 is not parallel to chambers 2 and 3, we will need to work with space points rather than projections.

The first attempt will be to use wire center positions without drift time information. The angle resolution is then about $1/20 \approx 50$ mr comparable to the angular divergence so we will assume that the track is parallel to the z axis, i.e. $\tan x\prime = 0$ and $\tan y\prime = 0$. Then eq. (5) simplifies to:

$$\psi_i^{fit} = \frac{Y_\chi x_t - X_\chi y_t}{Y_\chi X_\psi - X_\chi Y_\psi} \tag{6}$$

even for planes with a β and γ rotation.

Let me define the new combination of constants:

$$X_i^{SP} = \frac{Y_\chi}{Y_\chi X_\psi - X_\chi Y_\psi} \tag{7}$$

$$Y_i^{SP} = \frac{-X_\chi}{Y_\chi X_\psi - X_\chi Y_\psi} \tag{8}$$

$$\psi_i^{fit} = X_i^{SP} * x_t - Y_i^{SP} * y_t \tag{9}$$

For a group of hits in a chamber package, we will first find the intersection of each intersecting pair (omitting pairs of parallel wires). Then all combinations of pairs are tested to see if the squared distance between the intersections are less than a constant, the space point criterion (variable name: `space_point_criterion`) which may be different in each chamber because of the β rotations.

Finally a list is made of the separate space points, the x_t and y_t positions of each point, the number of hit-pair-combinations which are linked to the space point, and a list of the hits linked to each space point. The minimum requirement is that the point is found in at least one hit-pair-combination. Externally, this will be made more restrictive so that there are enough degrees of freedom to fit a stub to the chamber space point.

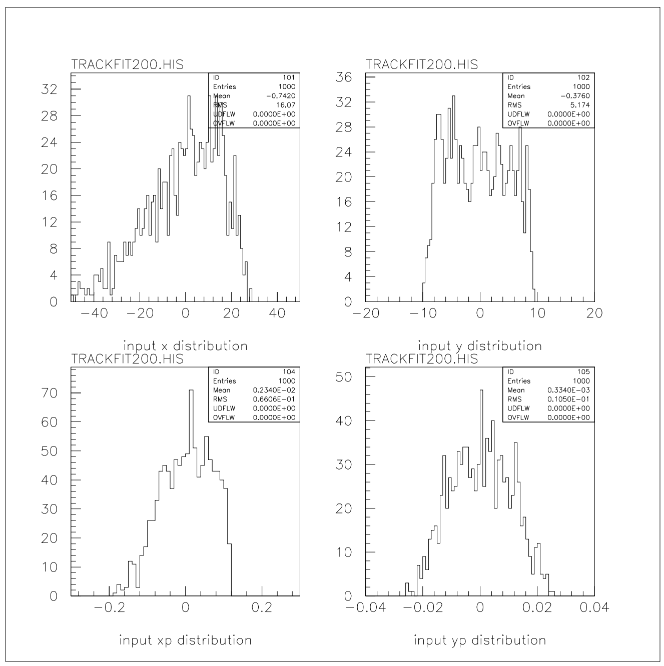
This procedures is carried out by the subroutine `S_PATTERN_RECOGNITION` which calls the subroutine `find_space_points` to identify potential space points and the subroutine `select_space_points` to require the minimum number of hits (`min_hit(i)`) and combinations (`min_combos(i)`) where i is the chamber number.

4 Resolve Left-Right Ambiguities

If all staggered planes fire, one could resolve the left-right ambiguity of the two drift planes by requiring the hits to be between the wire centers. However this is not robust if one of the two planes is missing. With 5 hits we can fit a stub to each chamber and pick the fit with the best χ^2 to determine the left-right ambiguity. With four hits in a chamber there may still be an ambiguity. In that case we may have to assume values for the slopes to resolve the left-right ambiguity. To make the fit linear, we work in a coordinate system perpendicular to the planes. In terms of the fit results in the primed coordinate system, the track parameters in the focal plane coordinate system are:

$$\begin{aligned}
t_x &= \frac{t_{x'} - \tan\beta}{1 + t_{x'}\tan\beta} \\
x_t &= x_{t'}\cos\beta - x_{t'}t_x\sin\beta \\
t_y &= \frac{t_{y'}}{t_{x'}\sin\beta + \cos\beta} \\
y_t &= y_{t'} - x_{t'}t_y\sin\beta
\end{aligned} \tag{10}$$

Figure 2: Input Track Distributions in SOS



The coordinates of the intersection of the ϕ axis which is normal to the wire plane and contains the $x = y = z = 0$ focal plane origin (parallel to z'' axis of Figure 1.III) are defined to be ψ_0, χ_0, ϕ_0:

$$
\begin{aligned}
\psi_0 &= \frac{-z_0 Z_\psi (Z_\chi^2 + X_\chi^2 + Y_\chi^2) + z_0 Z_\chi (Z_\chi Z_\psi + X_\chi X_\psi + Y_\chi Y_\psi)}{(Z_\chi^2 + X_\chi^2 + Y_\chi^2)(Z_\psi^2 + X_\psi^2 + Y_\psi^2) + (Z_\chi Z_\psi + X_\chi X_\psi + Y_\chi Y_\psi)^2} \\
\chi_0 &= \frac{-z_0 Z_\chi (Z_\psi^2 + X_\psi^2 + Y_\psi^2) + z_0 Z_\psi (Z_\chi Z_\psi + X_\chi X_\psi + Y_\chi Y_\psi)}{(Z_\chi^2 + X_\chi^2 + Y_\chi^2)(Z_\psi^2 + X_\psi^2 + Y_\psi^2) + (Z_\chi Z_\psi + X_\chi X_\psi + Y_\chi Y_\psi)^2} \\
|\phi_0| &= \sqrt{(z_0 + Z_\psi \psi_0 + Z_\chi \chi_0)^2 + (X_\psi \psi_0 + X_\chi \chi_0)^2 + (Y_\psi \psi_0 + Y_\chi \chi_0)^2}
\end{aligned} \tag{11}
$$

Minimizing χ^2 leads to the following matrix equation.

$$
\sum_i a_{i,j} (\psi^i - \psi_0^i)^2 / \sigma_i^2 = \sum_{i,k} a_{i,j} a_{i,k} t_k / \sigma_i^2 \tag{12}
$$

where $t_1 = x_{t'}$, $t_2 = y_{t'}$, $t_3 = t_{x'}$ and $t_4 = t_{y'}$. The coefficents, $a_{i,k}$ where i labels the plane number (and is an understood index on all the geometrical parameters: X, Y, and Z) and k the track parameter are:

$$
\begin{aligned}
a_{i,1} &= \frac{Y_\chi^s}{X_\psi^s Y_\chi^s - X_\chi^s Y_\psi^s} \\
a_{i,2} &= \frac{-X_\chi^s}{X_\psi^s Y_\chi^s - X_\chi^s Y_\psi^s} \\
a_{i,3} &= \phi_0^i a_{i,1} \\
a_{i,4} &= \phi_0^i a_{i,3}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
X_\chi^s &= -\cos\alpha & \qquad X_\psi^s &= \sin\alpha \\
Y_\chi^s &= \sin\alpha & \qquad Y_\psi^s &= \cos\alpha
\end{aligned} \tag{14}
$$

The success of this approach is quite dependent on the resolution σ of the wire chambers. For the SOS gemeotry, with 200 μm σ, in roughly 30% of the stubs a left-right ambiguity was miss-assigned. The input distributions of tracks transmitted through the SOS is shown in figure 2 and the resolutions of the stub fit is indicated by the residual distributions in figure 3. Since the resolution in t_y is poorer than the range of input t_y, we will do the stub

Figure 3: 4 Parameter Stub Fit residuals with 200 μm resolution

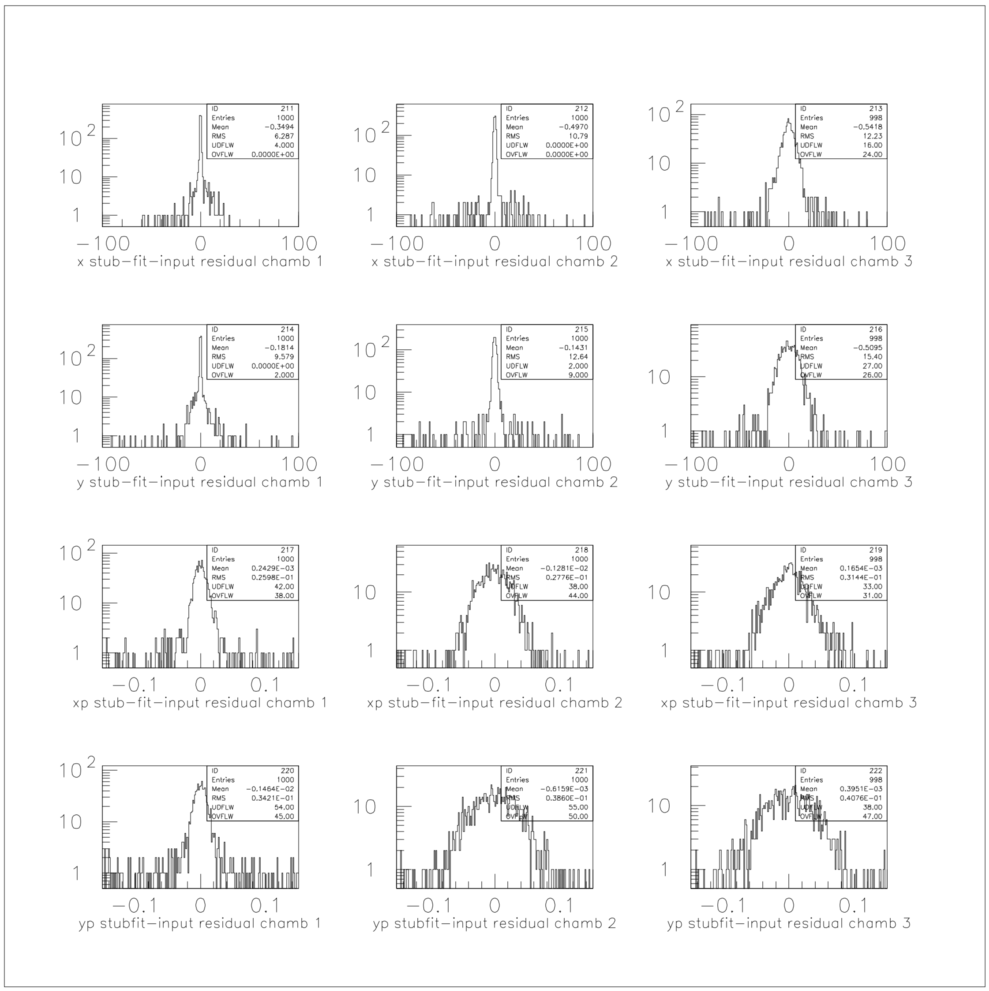
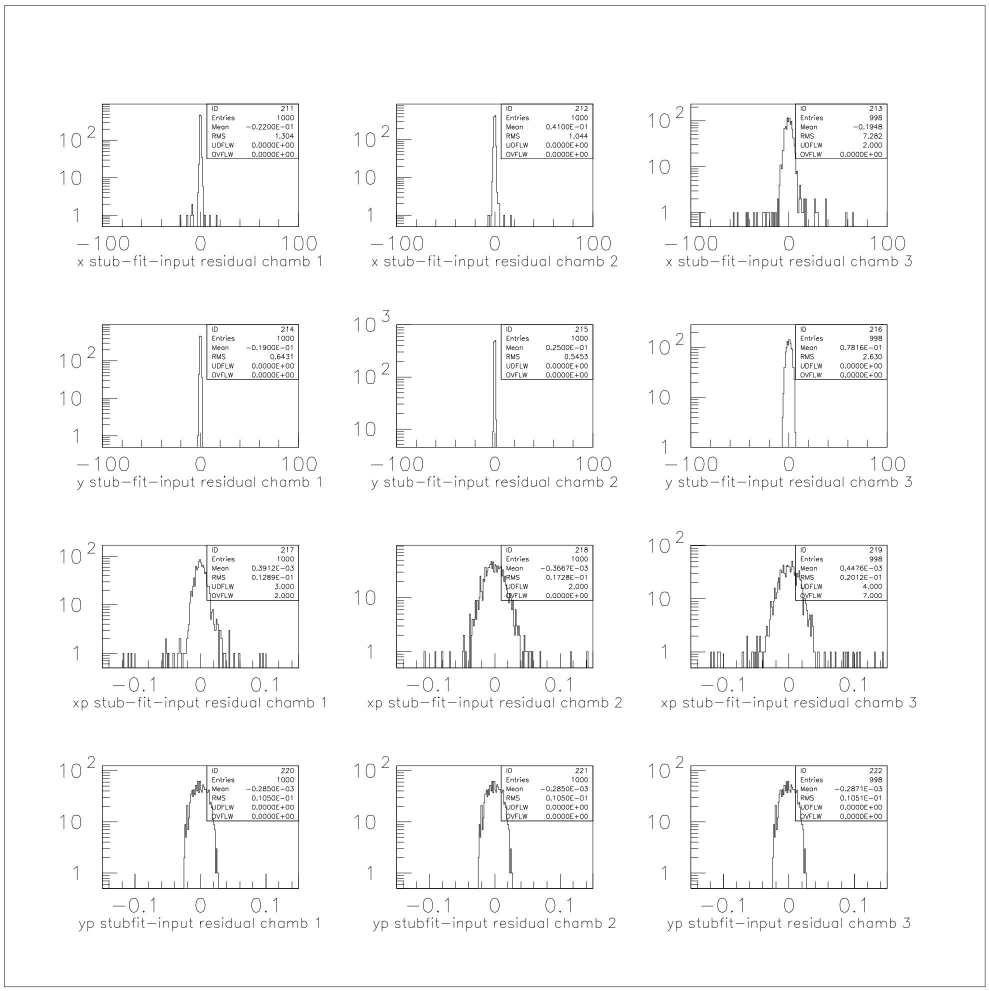


Figure 4: 3 Parameter Stub Fit residuals with 200 μm resolution



fit fixing $t_y = 0$. This 3 parameter fit is much more successful with less then 3% errors in the left-right determination. The results for the stub residuals are given in Figure 4.

The determination of the left-right ambiguity is done in the subroutine S_LEFT_RIGHT which loops over all left-right combinations and then calls the subroutine find_best_stub. find_best_stub contructs the $a_{i,j}$ coefficents and uses the subroutine solve_three_by_three to analytically invert eq. 12. find_best_stub then rotates the stub track parameters back to the focal plane coordinate system.

5 Linking Space Points to Tracks

The subroutine S_LINK_STUBS loops over all space points and links those into a track when the difference of each of the stub fit track parameters are less than the test criteria sxt_track_criterion,syt_track_criterion, and sxpt_track_criterion. (There is no requirement on the t_y since it was forced to be zero). These requirements are kept quite loose and we will later place more restrictive requirements on th χ^2 of the track fit to eliminate close combinations.

6 Track Fitting

We will minmize χ^2 to fit the track parameters, although another statistic could have been chosen. If we ignore multiple scattering (an unacceptable assumption) then χ^2 is given by:

$$\chi^2 = \sum_{i=1}^{nplanes} (\psi_i - \psi_i^{fit})^2/\sigma_i^2 \tag{15}$$

Solving for the intersection of the wire plane with the track ray and abbreviating $\tan x\prime$ and $\tan y\prime$ as t_x and t_y then:

$$\psi_i^{fit} = \frac{t_x y_t Z_\chi - t_y x_t Z_\chi + t_x(z_0 - z_t)Y_\chi + t_y(z_t - z_0)X_\chi + Y_\chi x_t - X_\chi y_t}{t_x(Y_\psi Z_\chi - Y_\chi Z_\psi) + t_y(X_\chi Z_\psi - X_\psi Z_\chi) + (Y_\chi X_\psi - X_\chi Y_\psi)} \tag{16}$$

where the index "i" is understood on each of the plane parameters, z_0, $X_{\psi,\chi}$, $Y_{\psi,\chi}$ and $Z_{\psi,\chi}$.

In general χ^2 is not simply a quadratic function of the track variables since it contains terms like $(t_x y_t)^2$ and so minimizing χ^2 is not a linear problem. However if $\beta = \gamma = 0$, with perpendicular planes, then:

$$Z_\chi = Z_\psi = 0$$

$$\psi_i^{fit} = t_x(z_0 - z_t)\sin\alpha + t_y(z_0 - z_t)\cos\alpha + x_t \sin\alpha + y_t \cos\alpha$$

and one can do a simple linear least squares fit.

The problem is that in SOS, the first 6 planes in chamber 1 must have a β rotation. So the more general non-linear expression must be used.

At the present time, the CERN Library routine MINUIT is used to to the track fitting. This is probably too slow for final code. We will postpone studies of this until multiple scattering is properly included.

7 Noise Hits

The effect of noise hits on the algorithms was investigated by introducing random noise in each plane with a hit probability `sos_noise`. Multiple noise hits in each plane are allowed. The parameters were set to the following values:

parameter	value
`space_point_criterion`	3.0,2.2,2.2
`min_hit`	5,5,5
`min_combos`	3,3,3
`sxt_track_criterion`	30. cm
`syt_track_criterion`	10. cm
`sxpt_track_criterion`	0.1 rad
wire chamber σ	.02 cm

A track was considered found if χ^2 per degree of freedom of the track fit was less than 10. 1000 tracks were presented as input.

Noise Probability	Number of tracks
0.0	996
0.1	970
0.2	944
0.3	923
0.4	846
0.5	770

The primary problem is that pattern recognition included extra hits in the space points which lead to very large χ^2. One could iterate the fit, either at the stub or the track level and try to eliminate these hits. This is an example of the trade off involved in pattern recognition.

Tighting space_point_criterion would reduce the number of extra hits contained in each point at the expense of missing some true hits.

The following is a study to tune space_point_criterion. For this study, min_combos was set to 6 and the noise probability was 0.1.

space_point_criterion chamber 1,2,3	Number of tracks	duplicates	No tracks
3.0,2.2,2.2	970		
2.2,2.2,2.2	973		
2.0,2.0,2.0	974		
1.8,1.8,1.8	973	3	5
1.6,1.6,1.6	977		
1.4,1.4,1.4	977	2	4
1.2,1.2,1.2	980	3	3
1.0,1.0,1.0	1036	60	3
0.9,1.2,1.2	1012	35	3
1.0,1.2,1.2	1032	60	3

Once space_point_criterion becomes equal to the wire spacing, we find a lot of duplicate tracks because the same space point is found twice in chamber 1. It appears that values of 1.2 to 1.4 are reasonable choices for space_point_criterion. Because I test on the distance between pairs in the x-y direction, it is hard to put the wire chamber tilt in properly.

With space_point_criterion set to 1.2 cm, and a noise probability of 0.5, the number of found tracks increase from 770 (see table above) to 847 with 15 duplicates and 25 missed tracks.