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Rates and Error calculations for measurement of A_{zz}

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(to be continued by Ellie)

Abstract

The tensor asymmetry A_{zz} can be extracted from:

$$\sigma = \sigma_u \Big[1 - P_z P_B A_{\parallel} + P_{zz} A_{zz} \Big] \tag{1}$$

For unpolarized beam,

$$\sigma = \sigma_u \Big[1 + P_{zz} A_{zz} \Big] \tag{2}$$

1 Rates

1.1 General expressions

The total rates for ND3 are:

$$\begin{aligned} R_T &= \mathcal{A}\Big[L_{He}\sigma_{He} + L_N \sigma_N + L_D \sigma_D \Big] && (3) \\ &= \mathcal{A}\Big[L_{He}\sigma^u_{He} + L_N \sigma^u_N + L_D \sigma^u_D \Big(1 + \frac{1}{2} N_D P_{zz} A_{zz} \Big) \Big] && (4) \end{aligned}$$

with $\mathcal{A}$ is defined as the acceptance ($\Delta\Omega\Delta E'$). The quantity N_D is the D-state contribution to the deuterium ground state wave function (only the D-state can contribute to b1). The luminosity L_A is defined as follows:

$$L_A = N_e * N_A \tag{5}$$

with $N_A = \mathcal{N} \frac{\rho_A}{M_A} z_A$ and $N_e = I_{beam}/e$.
Also $\mathcal{N}$ is the Avogadro's number. The quantities ρ_A, M_A and z_A are the density, the atomic or molecular mass and the thickness of the nuclear species A. Therefore we have:

$$\begin{aligned} N_{\rm He} &= \mathcal{N} \frac{\rho_{\rm He}}{M_{\rm He}} z\ (1 - p_f) = \mathcal{N}\ \mathcal{D}_{\rm He}\ z\ (1 - p_f) && (6) \\ N_{\rm ND_3} &= \mathcal{N} \frac{\rho_{\rm ND_3}}{M_{\rm ND_3}} z\ p_f = \mathcal{N}\ \mathcal{D}_{\rm ND_3}\ z\ p_f && (7) \\ N_{\rm N} &= \mathcal{N} \frac{\rho_{\rm ND_3}}{M_{\rm ND_3}} z\ p_f = \mathcal{N}\ \mathcal{D}_{\rm ND_3}\ z\ p_f && (8) \\ N_{\rm D} &= 3\ \mathcal{N} \frac{\rho_{\rm ND_3}}{M_{\rm ND_3}} z\ p_f = 3\ \mathcal{N}\ \mathcal{D}_{\rm ND_3}\ z\ p_f && (9) \\ & && (10) \end{aligned}$$

where $\mathcal{D}_{\mathcal{A}} = \rho_{\rm A}/M_{\rm A}$. The factor 3 in the expression of $l_{\rm D}$ take into account that there are three deuterium atoms in the ammonia molecule. The total rate can be finally expressed as follows:

$$R_T = \mathcal{A}\ N_e\ \mathcal{N}\ z\ \left[\mathcal{D}_{He}(1-p_f)\sigma^u_{He} + \mathcal{D}_{ND3}\ p_f\left(\sigma^u_N + 3\sigma^u_D(1+\frac{1}{2}N_D P_{zz} A_{zz})\right)\right] \quad (11)$$

with $R_T = R_U + R_D$. The rate coming from other nuclear species than deuterium is written as:

$$R_U = \mathcal{A}\ N_e\ \mathcal{N}\ z\ (\mathcal{D}_{He}(1-p_f)\sigma^u_{He} + \mathcal{D}_{ND3}\ p_f\ \sigma^u_N). \quad (12)$$

and the deuterium rate can then be extracted:

$$R_D = \mathcal{A}\ N_e\ \mathcal{N}\ z\ \mathcal{D}_{ND3}\ p_f\ 3\sigma^u_D\left(1+\frac{1}{2}N_D P_{zz} A_{zz}\right) \quad (13)$$

1.2 Expression of the measured asymmetry

From Refs. [1] and [2], the enhancement of the tensor polarization with solid polarized targets can be done via the "hole burning“ method by pushing down either one of the $|m_z| = 1$ states moving its population to $m_z = 0$. But this necessarily enhances the absolute vector polarization, $|m_+ - m_-|$ because one of the m_1's stays fixed. So, as it is said in Ref. [1], the improvement in P_{zz} comes only from better P_z. The asymmetry would come from counting events with m_+, m_- and m_0 for opposite P_z's*:

$$P_z^+ = m_+ - m_- \quad \text{with } m_+ > m_- \quad (14)$$
$$-P_z^- = m_+ - m_- \quad \text{with } m_+ < m_- \quad (15)$$

and for the P_{zz}'s:

$$P_{zz}^+ = P_{zz}(P_z^+) = m_+ + m_- - 2m_0^+ = 2m_+ - P_z^+ - 2m_0^+ \quad (16)$$
$$P_{zz}^- = P_{zz}(P_z^-) = m_+ + m_- - 2m_0^- = 2m_- - P_z^- - 2m_0^- \quad (17)$$

Note that m_0 populations won't necessarily be the same.

$$R_T^+ - R_T^- = (R_U^+ + R_D^+) - (R_U^- + R_D^-) \quad (18)$$
$$= \mathcal{A}\ N_e\ \mathcal{N}\ z\ \mathcal{D}_{ND3}\ p_f\ 3\sigma^u_D \frac{1}{2} N_D A_{zz}(P_{zz}^+ - (-P_{zz}^-)) \quad (19)$$

with

$$P_{zz}^+ + P_{zz}^- = (2m_+ - P_z^+ - 2m_0^+) + (2m_- - P_z^- - 2m_0^-) \quad (20)$$
$$= 2(m_+ + m_-) - 2(m_0^+ + m_0^-) - (P_z^- + P_z^+) \quad (21)$$

*m_+, m_- and m_0 represent the normalized populations.

In order to access A_{zz}, we will have to take data with $P_{zz} < 0$ and $P_{zz} > 0$. Simplifications could be done assuming we are using the same target cup and the same integrated luminosity is seen for each polarization stage. Also if $-P_z^- \sim P_z^+$ and $m_0^+ \sim m_0^- = m_0$, we get:

$$P_{zz}^+ + P_{zz}^- = 2(m_+ + m_- - 2m_0) = 2P_{zz} \tag{22}$$

and

$$R_D^+ - R_D^- = \mathcal{A}\ N_e\ \mathcal{N}\ z\ \mathcal{D}_{ND3}\ p_f\ 3\sigma_D^u \frac{1}{2} N_D A_{zz} P_{zz} \tag{23}$$

$$\begin{aligned} R_D^+ - R_D^- &= \mathcal{A}\ N_e\ \mathcal{N}\ z\ \mathcal{D}_{ND3}\ p_f\ 3\ \sigma_D^u \frac{1}{2} N_D A_{zz} P_{zz} && (24) \\ R_D^+ + R_D^- &= 2\mathcal{A}\ N_e\ \mathcal{N}\ z\ \mathcal{D}_{ND3}\ p_f\ 3\ \sigma_D^u && (25) \end{aligned}$$

$$\begin{aligned} A_{meas} &= f \frac{R_D^+ - R_D^-}{R_D^+ + R_D^-} && (26) \\ &= \frac{1}{4}\ f\ N_D\ P_{zz}\ A_{zz} && (27) \end{aligned}$$

Table 1: Values used in the rate estimates

ρ_{ND_3}	1.007 g.cm^{-3}
M_{ND_3}	20 g.mol^{-1}
$p_f(ND_3)$	0.80
$f(ND_3)$	6/20
z	3 cm
P_{zz}	0.25
N_D	0.05

2 statistical error

$$A_{zz} = \frac{4}{f\ N_D\ P_{zz}} A_{meas} \tag{28}$$

$$\delta A_{zz} = \frac{4}{f\ N_D\ P_{zz}} \delta A_{meas} \tag{29}$$

With $N_{+(-)} = R_D^{+(-)} * T_{+(-)}$, T being the time in second,

$$\delta A_{zz} = \frac{4}{f\ N_D\ P_{zz}} \frac{2}{(N_+ + N_-)^2} \sqrt{N_+\ N_-\ (N_+ + N_-)} \tag{30}$$

Because A_{zz} is very small, we can assume $N_+ \simeq N_- \simeq N/2$ and therefore the statistical error on A_{zz} becomes:

$$\delta A_{zz} = \frac{4}{f\ N_D\ P_{zz}} \frac{1}{\sqrt{N}} \tag{31}$$

Time needed to make the measurement:

$$T \quad = \quad \left(\frac{4}{f\ N_D\ P_{zz}\ \delta A_{zz}} \right)^2 \frac{1}{R_D} \tag{32}$$

$$\tag{33}$$

I believe that N_D shouldn't appear (Patricia).

3 kinematics choice

Table 2: default

x	Q^2	W	E_P	θ_0	θ_q
0.15	2.011	3.504	3.856	12.50	6.580
0.25	2.020	2.634	6.695	9.50	14.105
0.35	3.381	2.676	5.852	13.16	14.107
0.45	2.754	2.061	7.738	10.32	22.261
0.55	3.811	2.000	7.308	12.50	22.253

4 systematics

References

[1] T.W. Meyer and E.P. Schilling, Tensor polarized deuteron targets for intermediate energy physics experiments, BONN-HE-85-06 (1985)

[2] S. Bueltmann, D. Crabb, Y. Prok. UVa Target Studies, UVa Polarized Target Lab technical note, 1999.