Tensor polarization conventions

W. Cosyn
(Dated: August 20, 2024)

The goal of this note is to summarize some of the different conventions regarding spin-1 polarization in the nuclear physics literature and to propose a notation to use going forward. Parts of what is summarized here was discussed at the Tensor Friday meeting on August 2nd.

Comments welcome, if you want to see other conventions included/compared let me know, I'm not aware of everything out there, especially if it's somewhat older. This is a note in progress and will be expanded.

tl;dr summary:

1. Use notation $\mathcal{P}, \mathcal{Q}$ for degree of vector/tensor polarization.

2. Preferably use the full density matrix as this contains more information than just $\mathcal{P}, \mathcal{Q}$. Those determine the eigenvalues, while the full density matrix also tells you what the polarization direction is relative to the coordinate system (and spin quantization axis).

3. Different normalizations for the tensor polarized density matrix parameters are in use (with different symbols). No clear preference so best to be clear what the normalization is (and follow the same notation).

4. Structure functions are not partonic distribution functions. Though QCD factorization theorems give expressions for structure functions as functions of pdfs etc. Structure functions (such as $F_{[UT_{LL},L]}$ or b_1 etc.) appear in geometric decompositions of cross sections. Partonic distribution functions (incl. GPDs and TMDs, e.g. f_{1LL}) appear in the decomposition of matrix elements of certain QCD operators between polarized hadron states. Do not mix the two in word usage or symbols (i.e. don't use "the tensor polarized pdf b_1").

5. For the tensor polarized structure functions (SF), the $F_{[UT_{LL},L]}$ etc. convention used for processes with an additional identified particle in the final state gives the most intuitive dependence on polarization parameters and azimuthal angles and connection with asymmetries. Relations with inclusive b_1 to b_4 are known. Normalization of the SF follows from the tensor polarization parameters of Ref. [1]. If using different tensor polarization normalizations, use explicit numerical factors in the cross section expressions so the normalization of the structure functions does not change.

6. Use $A^V(A^{eV})$ for the vector polarized asymmetry with unpolarized (polarized) electron. Similarly $A^T(A^{eT})$ for the tensor polarized asymmetry. In inclusive electron scattering in the one photon exchange approximation only A^{eV} and A^T are non-zero. All these asymmetries also depend on the polarization direction of the target, so the notation can be augmented with for instance directional arguments (angles, L/T, $\parallel$ / $\perp$) to distinguish asymmetries with different polarization directions. The notations and corresponding directions should be clearly defined.

7. Notation for pdfs, tmds etc. is standard, see review in this volume for an overview [2]. No variations in regular use as far as I'm aware.

I. POLARIZATIONS AND DENSITY MATRIX

In quantum mechanics, ensembles of polarized particles are characterized by a density matrix $\rho(\lambda, \lambda')$, where λ, λ' refer to spin quantum numbers, quantized along a certain axis. For a spin-1 particle these take the values $\lambda = +1, 0, -1$ and the density matrix is a 3 by 3 matrix. Any density matrix is Hermitian and has unit trace

$$\sum_{\lambda} = \rho(\lambda, \lambda) = 1. \tag{1}$$

In general the spin-1 can be parametrized with 8 polarization parameters, 3 associated with vector polarization, 5 with tensor polarization (see below). As rho is a Hermitian matrix, it can always be diagonalized which corresponds to aligning the spin quantization axis with for instance the magnetic field direction used to populate the different spin states. The diagonal elements are the eigenvalues of the general density matrix and these can be identified with the probabilities n_i of populating the different spin states

$$\rho(\lambda,\lambda')[\mathrm{diag}] = \begin{pmatrix} n_+ & 0 & 0 \\ 0 & n_0 & 0 \\ 0 & 0 & n_- \end{pmatrix}. \tag{2}$$

The unit trace implies

$$n_+ + n_0 + n_- = 1 \tag{3}$$

and the eigenvalues can be parametrized using two parameters, usually introduced as the vector and tensor polarization

$$\mathcal{P} = n_+ - n_-, \qquad -1 \le \mathcal{P} \le 1, \tag{4}$$
$$\mathcal{Q} = n_+ + n_- - 2n_0, \qquad -2 \le \mathcal{Q} \le 1. \tag{5}$$

and we can write

$$\rho(\lambda,\lambda')[\mathrm{diag}] = \begin{pmatrix} \frac{1}{3} + \frac{\mathcal{P}}{2} + \frac{\mathcal{Q}}{6} & 0 & 0 \\ 0 & \frac{1}{3} - \frac{\mathcal{Q}}{3} & 0 \\ 0 & 0 & \frac{1}{3} - \frac{\mathcal{P}}{2} + \frac{\mathcal{Q}}{6} \end{pmatrix}. \tag{6}$$

For details on the general spin-1 density matrix, see Refs. [1, 3, 4]. In high-energy reactions, the density matrix is written with spin quantized along the z-axis, which is constructed from the collinear axis that contains both the target momentum and that of the exchanged virtual photon. The parameters of the density matrix (vector, tensor) are then decomposed in components along (L) and perpendicular (T) that collinear axis (including azimuthal angles in the perpendicular plane). For target rest frames the collinear axis direction is opposite the 3-momentum $\boldsymbol{q}$. As that z-direction does not need to coincide with the magnetic field direction, off-diagonal elements of the density matrix are populated. The full density matrix clearly contains more information about the geometry of the polarization relative to the scattering event than just $\mathcal{P}, \mathcal{Q}$ can give.

The exact form of the density matrix is only important if one uses it in calculations. However, the parameters appear in eventual expressions of cross sections, asymmetries etc. and the normalization of the parameters is thus important for conventions. Therefore we show the expression in some of the parametrizations mentioned above and compare normalizations.

First, in Ref. [4] (where the spin-1 TMDs were introduced), the density matrix parametrization takes the form

$$\rho(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3} + \frac{1}{2}S_L + \frac{1}{3}S_{LL} & \begin{array}{c}\frac{1}{2\sqrt{2}}(S_T^x - iS_T^y) \\ + \frac{1}{2\sqrt{2}}(S_{LT}^x - iS_{LT}^y)\end{array} & \frac{1}{2}(S_{TT}^{xx} - iS_{TT}^{xy}) \\ \begin{array}{c}\frac{1}{2\sqrt{2}}(S_T^x + iS_T^y) \\ + \frac{1}{2\sqrt{2}}(S_{LT}^x + iS_{LT}^y)\end{array} & \frac{1}{3} - \frac{2}{3}S_{LL} & \begin{array}{c}\frac{1}{2\sqrt{2}}(S_T^x - iS_T^y) \\ - \frac{1}{2\sqrt{2}}(S_{LT}^x - iS_{LT}^y)\end{array} \\ \frac{1}{2}(S_{TT}^{xx} + iS_{TT}^{xy}) & \begin{array}{c}\frac{1}{2\sqrt{2}}(S_T^x + iS_T^y) \\ - \frac{1}{2\sqrt{2}}(S_{LT}^x + iS_{LT}^y)\end{array} & \frac{1}{3} - \frac{1}{2}S_L + \frac{1}{3}S_{LL} \end{bmatrix}. \tag{7}$$

The spin vector and tensor are then

$$\boldsymbol{S} = (S_T^x, S_T^y, S_L), \tag{8}$$

$$\boldsymbol{T} = \begin{pmatrix} \frac{1}{2}S_{TT}^{xx} - \frac{1}{3}S_{LL} & \frac{1}{2}S_{TT}^{xy} & \frac{1}{2}S_{LT}^x \\ \frac{1}{2}S_{TT}^{xy} & -\frac{1}{2}S_{TT}^{xx} - \frac{1}{3}S_{LL} & \frac{1}{2}S_{LT}^y \\ \frac{1}{2}S_{LT}^x & \frac{1}{2}S_{LT}^y & \frac{2}{3}S_{LL} \end{pmatrix}. \tag{9}$$

Next, in Ref. [1] (where tagged DIS on a polarized deutereon was concerned, the density matrix parametrization takes the form

$$\rho(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3}+\frac{1}{2}S_L+\frac{1}{2}T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi-\phi_S)} + \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi-\phi_{T_L})} & \frac{1}{2}T_{TT}e^{-i(2\phi-2\phi_{T_T})} \\ \frac{1}{2\sqrt{2}}S_T e^{i(\phi-\phi_S)} + \frac{1}{\sqrt{2}}T_{LT}e^{i(\phi-\phi_{T_L})} & \frac{1}{3}-T_{LL} & \frac{1}{2\sqrt{2}}S_T e^{-i(\phi-\phi_S)} - \frac{1}{\sqrt{2}}T_{LT}e^{-i(\phi-\phi_{T_L})} \\ \frac{1}{2}T_{TT}e^{i(2\phi-2\phi_{T_T})} & \frac{1}{2\sqrt{2}}S_T e^{i(\phi-\phi_S)} - \frac{1}{\sqrt{2}}T_{LT}e^{i(\phi-\phi_{T_L})} & \frac{1}{3}-\frac{1}{2}S_L+\frac{1}{2}T_{LL} \end{bmatrix}, \tag{10}$$

where all azimuthal angles are relative to the lepton plane following the Trento convention [5] and ϕ is the azimuthal angle of the x-axis of the reference frame that is used for the spin states. The spin vector and tensor are now

$$\boldsymbol{S} = (S_T\cos(\phi-\phi_S), S_T\sin(\phi-\phi_S), S_L)\,, \tag{11}$$

$$\boldsymbol{T} = \begin{pmatrix} \frac{1}{2}T_{TT}\cos(2\phi-2\phi_{T_T})-\frac{1}{2}T_{LL} & \frac{1}{2}T_{TT}\sin(2\phi-2\phi_{T_T}) & T_{LT}\cos(\phi-\phi_{T_L}) \\ \frac{1}{2}T_{TT}\sin(2\phi-2\phi_{T_T}) & -\frac{1}{2}T_{TT}\cos(2\phi-2\phi_{T_T})-\frac{1}{2}T_{LL} & T_{LT}\sin(\phi-\phi_{T_L}) \\ T_{LT}\cos(\phi-\phi_{T_L}) & T_{LT}\sin(\phi-\phi_{T_L}) & T_{LL} \end{pmatrix}. \tag{12}$$

Comparing expressions one observes that for the tensor polarization parameters

$$T_{LL} = \frac{2}{3}S_{LL} \qquad -\frac{2}{3} \le T_{LL} \le \frac{1}{3}, \quad -1 \le S_{LL} \le \frac{1}{2}, \tag{13}$$

$$T_{LT} = \frac{1}{2}S_{LT} = \frac{1}{2}\sqrt{(S_{LT}^x)^2+(S_{LT}^y)^2} \qquad 0 \le T_{LT} \le \frac{1}{2}, \quad 0 \le S_{LT} \le 1, \tag{14}$$

$$T_{TT} = S_{TT} = \sqrt{(S_{TT}^{xx})^2+(S_{TT}^{yy})^2} \qquad 0 \le T_{TT}, S_{TT} \le 1. \tag{15}$$

The overall degree of polarization is defined as

$$0 \le d = \left(\tfrac{3}{4}\boldsymbol{S}^2 + \tfrac{3}{2}\boldsymbol{T}^2\right) \le 1, \tag{16}$$

$$0 \le \boldsymbol{S}^2 \le 1, \tag{17}$$

$$0 \le (\boldsymbol{T}^2 = \sum_{ij} T_{ij}^2) \le \tfrac{2}{3}, \tag{18}$$

$$\boldsymbol{T}^2 = \frac{1}{6}\mathcal{Q}^2 = \frac{3}{2}T_{LL}^2 + 2T_{LT}^2 + \frac{1}{2}T_{TT}^2 = \frac{2}{3}S_{LL}^2 + \frac{1}{2}\left(S_{LT}^2+S_{TT}^2\right), \tag{19}$$

where in the last equation the invariance of the Frobenius norm of the matrix was used. The limits are valid for any possible mixed state. From Eq. (19) one sees that, for a mixed state with general polarization direction, the degree of tensor polarization is distributed across the different tensor polarization intensities. Pure states have $d = 1$. These equations clearly show that any pure state has tensor polarization, and a pure state can never be formed with only T_{LT} or T_{TT}.

Note that there are still other normalizations in use in the literature. Ref. [3] for instance has

$$\boldsymbol{T}^{\text{Ref. [3]}} = \sqrt{\frac{3}{2}}\,\boldsymbol{T}^{\text{Ref. [1]}}. \tag{20}$$

To avoid confusion, be explicit about the normalization when introducing tensor polarization parameters.

II. STRUCTURE FUNCTIONS

Structure functions are invariant variables (depending on kinematic invariants) that appear in the geometric decomposition of the cross section. In the standard approach, the dependence on polarization parameters and azimuthal angles (angle between lepton and hadron plane, angles from polarization parameters) is entirely kinematic and can be separated from the dynamics which is encoded in the SF. This decomposition does not need any specific assumptions about the nature of the dynamics. Separating all the polarization and azimuthal angle dependence makes the connection between SF and experimentally measured asymmetries and/or azimuthal modulations straightforward.

For tensor polarization, historically tensor polarized structure functions were first introduced for **inclusive** electron scattering in Ref. [6] as b_1 to b_4. These definitions make connections with partonic probability densities straightforward in the parton model (similar to F_1, F_2 and g_1, g_2). This definition, however, does not have the straightforward connection with tensor polarized observables.

For the SF for polarized spin-1 processes a similar construction as the spin-1/2 case can be considered. For the unpolarized and vector polarized sector, this leads to identical formulas as the spin-1/2 case where 18 SF appear, see Refs. [7, 8]. The tensor polarized sector has 23 additional SF. Partial expressions (ϕ-independent) appear in Ref. [1], the derivation of the full expression will appear elsewhere [] and the final result is included here

$$
\begin{aligned}
d\sigma = {} & \frac{2\pi y^2 \alpha_{\text{em}}^2}{Q^6}\, dx dQ^2 \frac{d\psi_{l'}}{2\pi} \\
& \times \left(\mathcal{F}_U + \mathcal{F}_S + \mathcal{F}_T\right)\, d\Gamma_{P_h},
\end{aligned}
\tag{21}
$$

where $\mathcal{F}_U, \mathcal{F}_S$ are written in Refs. [7, 8] and

$$
\begin{aligned}
& \mathcal{F}_T \\
& = T_{LL} \Big[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L} \\
& \qquad + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_h F_{UT_{LL}}^{\cos\phi_h} \\
& \qquad + \epsilon \cos 2\phi_h F_{UT_{LL}}^{\cos 2\phi_h} \Big] \\
& + T_{LL}(2\lambda_e) \sqrt{2\epsilon(1-\epsilon)} \sin\phi_h F_{LT_{LL}}^{\sin\phi_h} \\
& + T_{LT} \Big[\cos(\phi_h - \phi_{T_L}) \\
& \qquad \times \left(F_{UT_{LT},T}^{\cos(\phi_h-\phi_{T_L})} + \epsilon F_{UT_{LT},L}^{\cos(\phi_h-\phi_{T_L})} \right) \\
& \qquad + \epsilon \cos(\phi_h + \phi_{T_L}) F_{UT_{LT}}^{\cos(\phi_h+\phi_{T_L})} \\
& \qquad + \epsilon \cos(3\phi_h - \phi_{T_L}) F_{UT_{LT}}^{\cos(3\phi_h-\phi_{T_L})} \\
& \qquad + \sqrt{2\epsilon(1+\epsilon)} \cos\phi_{T_L} F_{UT_{LT}}^{\cos\phi_{T_L}} \\
& \qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(2\phi_h - \phi_{T_L}) F_{UT_{LT}}^{\cos(2\phi_h-\phi_{T_L})} \Big] \\
& + T_{LT}(2\lambda_e) \Big[\sqrt{1-\epsilon^2} \sin(\phi_h - \phi_{T_L}) F_{LT_{LT}}^{\sin(\phi_h-\phi_{T_L})} \\
& \qquad + \sqrt{2\epsilon(1-\epsilon)} \sin\phi_{T_L} F_{LT_{LT}}^{\sin\phi_{T_L}} \\
& \qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(2\phi_h - \phi_{T_L}) F_{LT_{LT}}^{\sin(2\phi_h-\phi_{T_L})} \Big] \\
& + T_{TT} \Big[\cos(2\phi_h - 2\phi_{T_T}) \\
& \qquad \times \left(F_{UT_{TT},T}^{\cos(2\phi_h-2\phi_{T_T})} + \epsilon F_{UT_{TT},L}^{\cos(2\phi_h-2\phi_{T_T})} \right) \\
& \qquad + \epsilon \cos 2\phi_{T_T} F_{UT_{TT}}^{\cos 2\phi_{T_T}} \\
& \qquad + \epsilon \cos(4\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(4\phi_h-2\phi_{T_T})} \\
& \qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(\phi_h-2\phi_{T_T})} \\
& \qquad + \sqrt{2\epsilon(1+\epsilon)} \cos(3\phi_h - 2\phi_{T_T}) F_{UT_{TT}}^{\cos(3\phi_h-2\phi_{T_T})} \Big] \\
& + T_{TT}(2\lambda_e) \Big[\sqrt{1-\epsilon^2} \sin(2\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(2\phi_h-2\phi_{T_T})} \\
& \qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(\phi_h-2\phi_{T_T})} \\
& \qquad + \sqrt{2\epsilon(1-\epsilon)} \sin(3\phi_h - 2\phi_{T_T}) F_{LT_{TT}}^{\sin(3\phi_h-2\phi_{T_T})} \Big] .
\end{aligned}
\tag{22}
$$

For the inclusive case the relation between the inclusive SF defined in such a way and the $b_1 - b_4$ is given by

$$\begin{aligned}
&\int d\Gamma_{P_h} F_{UT_{LL},L} \\
&= \frac{1}{x}\left[2(1+\gamma^2)xb_1 - (1+\gamma^2)^2\left(\frac{1}{3}b_2 + b_3 + b_4\right)\right. \\
&\left. -(1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) - \left(\frac{1}{3}b_2 - b_3\right)\right], \\
&\int d\Gamma_{P_h} F_{UT_{LL},T} \\
&= -\left[2(1+\gamma^2)b_1 - \frac{\gamma^2}{x}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right)\right], \\
&\int d\Gamma_{P_h} F_{UT_{LT}}^{\cos\phi_{T_L}} \\
&= -\frac{\gamma}{2x}\left[(1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) + \left(\frac{2}{3}b_2 - 2b_3\right)\right], \\
&\int d\Gamma_{P_h} F_{UT_{TT}}^{\cos 2\phi_{T_T}} = -\frac{\gamma^2}{x}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right).
\end{aligned} \tag{23}$$

where x is the Bjorken variable that has bounds $0 \le x \le 1$ and $\gamma = 2x\frac{M}{Q}$ with M the target rest mass. Note that these relations are quite a bit more involved than the equivalent ones in the unpolarized (F_1, F_2 vs $F_{UU,L}, F_{UU,T}$) and vector polarized (g_1, g_2 vs $F_{LS_L}, F_{LS_T}^{\cos\phi_S}$) sectors. This also complicates the extraction of b_1 from tensor polarized asymmetries.

Regarding normalization of the SF, the agreement is to keep the normalization so that the polarization parameters multiplying the SF in the cross section are those of Eq. (10) [1]. If a different normalization is used, include explicit numerical factors in the cross section expressions such that the normalization of the SF does not change. As an example, if one uses the polarization parameters of Ref. [4], the first terms in Eq. (22) change to

$$\mathcal{F}_T = \frac{2}{3}S_{LL}\left[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L} + \ldots\right. \tag{24}$$

III. ASYMMETRIES

To isolate the vector or tensor polarized structures in the cross section of Eq. (21), incoherent superpositions of different deuteron ensembles must be taken to isolate that specific part of the density matrix (and cross section). Using pure deuteron states we can take the following asymmetry which will isolate the structure sensitive to deuteron vector polarization (and unpolarized electron)

$$A^V = \frac{d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = +1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = +1) - d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = -1) - d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = -1)}{d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = +1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = +1) + d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = -1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = -1)} \tag{25}$$

and similarly for those sensitive to vector and electron polarization

$$A^{eV} = \frac{d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = +1) - d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = +1) - d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = -1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = -1)}{d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = +1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = +1) + d\sigma(\lambda_e = +\frac{1}{2}, \Lambda_d = -1) + d\sigma(\lambda_e = -\frac{1}{2}, \Lambda_d = -1)} \tag{26}$$

Similarly to isolate tensor polarization one takes the asymmetries

$$A^T = \frac{d\sigma(\Lambda_d = +1) + d\sigma(\Lambda_d = -1) - 2d\sigma(\Lambda_d = 0)}{d\sigma(\Lambda_d = +1) + d\sigma(\Lambda_d = -1) + d\sigma(\Lambda_d = 0)}, \tag{27}$$

where no electron polarization is considered. A similar expression for the double asymmetry A^{eT} can be written. These asymmetries are bounded by

$$-1 \le A^V, A^{eV} \le 1, \tag{28}$$

$$-2 \le A^T, A^{eT} \le 1. \tag{29}$$

In inclusive scattering in the one photon exchange approximation, only A^{eV} and A^{T} are non-zero.

Note that different considered polarization directions will yield different asymmetries. A change in polarization direction changes the values of the polarization parameters for the pure states. These different values result in different linear combinations of the SF that enter the asymmetry and thus different asymmetries. It is therefore desirable to be explicit about which polarization direction is considered and the augment the asymmetry notation with additional variables that distinguish different directions. This could be the angles of the direction in a reference frame of choice or for certain standard polarization directions (along, perpendicular to the electron beam) the asymmetry can be indexed with $\|, \perp$ and the values of the polarization parameters can be calculated for these cases. These are the so-called *effective polarizations*, see Ref. [1] for examples.

In certain theory publications showing discussing tensor asymmetries, the asymmetry is not defined as above but directly as a certain ratio of structure functions [9, 10]. For tensor asymmetries this can mean the normalization of the asymmetry differs from that of Eq. (28). *[Have to check factor, but looks like 1/2].*

For the tensor asymmetry, A_{zz} has also been used (the HERMES result [11], JLab proposal etc.). The use of A^{T} over A_{zz} is preferred as A_{zz} implies polarization along a specific (z) axis but between theory and experiment different choices of z-axis are common (virtual photon direction, electron direction). In general, tensor asymmetries can be discussed or measured for any polarization direction.

IV. PARTONIC DISTRIBUTION FUNCTIONS

The notation for PDFs, TMDs is standard for the tensor polarized sector, see Ref. [2] for a review in this volume.

[1] W. Cosyn and C. Weiss, Polarized electron-deuteron deep-inelastic scattering with spectator nucleon tagging, Phys. Rev. C **102**, 065204 (2020), arXiv:2006.03033 [hep-ph].

[2] S. Kumano, Parton distribution functions and fragmentation functions of spin-1 hadrons, (2024), arXiv:2406.01180 [hep-ph].

[3] E. Leader, *Spin in Particle Physics (Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology)* (Cambridge University Press, 2005).

[4] A. Bacchetta and P. J. Mulders, Deep inelastic leptoproduction of spin-one hadrons, Phys. Rev. D **62**, 114004 (2000), arXiv:hep-ph/0007120.

[5] A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Single-spin asymmetries: The Trento conventions, Phys. Rev. D **70**, 117504 (2004), arXiv:hep-ph/0410050.

[6] P. Hoodbhoy, R. Jaffe, and A. Manohar, Novel Effects in Deep Inelastic Scattering from Spin 1 Hadrons, Nucl.Phys. **B312**, 571 (1989).

[7] M. Diehl and S. Sapeta, On the analysis of lepton scattering on longitudinally or transversely polarized protons, Eur. Phys. J. C **41**, 515 (2005), arXiv:hep-ph/0503023.

[8] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, Semi-inclusive deep inelastic scattering at small transverse momentum, JHEP **02**, 093.

[9] W. Leidemann, E. L. Tomusiak, and H. Arenhovel, Inclusive deuteron electrodisintegration with polarized electrons and a polarized target, Phys. Rev. C **43**, 1022 (1991).

[10] S. Jeschonnek and J. W. Van Orden, Target Polarization for H-2-polarization (e, e-prime p)n at GeV energies, Phys. Rev. C **80**, 054001 (2009), arXiv:0907.3712 [nucl-th].

[11] A. Airapetian *et al.* (HERMES), First measurement of the tensor structure function b(1) of the deuteron, Phys. Rev. Lett. **95**, 242001 (2005), arXiv:hep-ex/0506018.