THE MAGNETIC FIELD DIRECTION MEASUREMENTS FOR THE A_1^n AND d_2^n EXPERIMENTS IN HALL C

Murchhana Roy, Wolfgang Korsch

July, 2020

Table of Contents

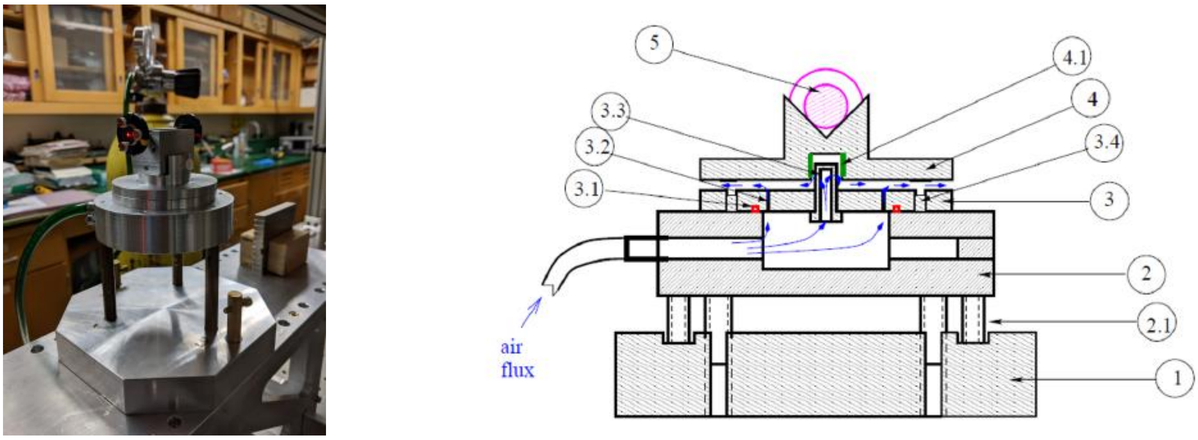
List of Figures

List of Tables

1 The Horizontal Compass

The experiments A_1^n and d_2^n required the target holding field direction to be measured precisely to about ± 0.1 ° in absolute Hall C coordinate system. In Hall C, the longitudinal polarization direction is the direction of the target spins pointing parallel or antiparallel to the beam direction in the horizontal plane and the transverse polarization direction is the direction of the target spins pointing perpendicular to the beam direction in the horizontal plane. If the target polarization direction is a little different from 90 ° or 270 ° (the horizontal transverse polarization directions) w.r.t beam direction, the longitudinal asymmetry contributes to the total asymmetry in same order as the transverse asymmetry. To account for the contribution of the longitudinal asymmetry, the holding field direction needed to be measured very precisely.

In order to measure the horizontal magnetic field directions, a novel air-compass was conceptualized and developed at the University of Kentucky and susequently modified at Jefferson Lab as the commercially available compasses cannot achieve the desired level of precision.



(a) The compass

(b) Sectional view through center

Figure 1: Design of the horizontal compass

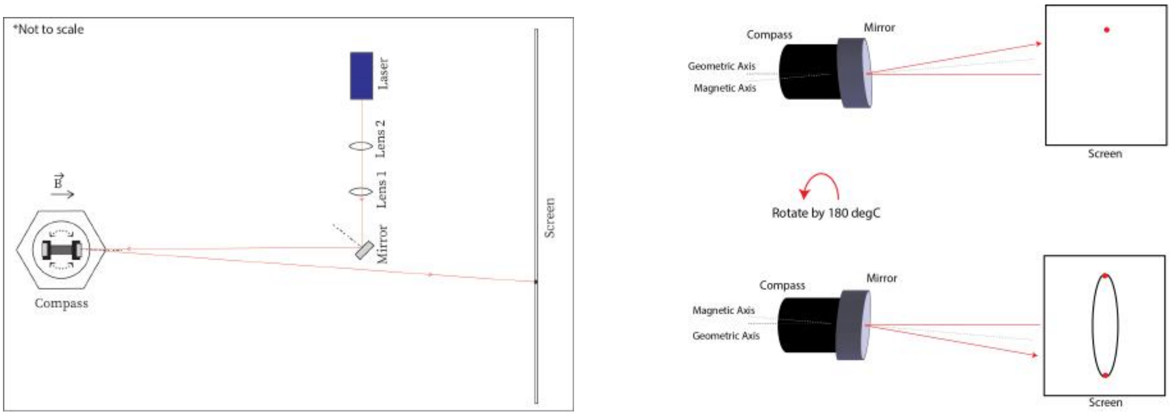
The horizontal compass (figure 1a) comprised of a cylindrical magnet mounted on a floating disk made of aluminum. The length and diameter of the neodymium magnetic were 2 inches and 0.25 inches respectively. The compass magnet was placed inside a V-groove. It was ideal for positioning a cylindrical magnet as it provided two lines of contact along the bottom and it allowed access to both ends of the cylinder.

The disk was floated by passing compressed nitrogen gas through an air inlet shown in the figure 1b). The bottom aluminum disk with the air-inlet was placed on three brass legs. The height of the compass was adjusted by adjusting the legs on the octagonal base plate. The compass magnet had two caps with circular scales and 1 mm deep pockets attached to both ends. Two mirrors (diamater 0.5 inches and 3 millimeter thick) were placed inside the pockets on top a circular neoprene sponge (0.5 inches diameter) with the help of three brass screws. The magnetic field direction was measured by reflecting a laser beam off the compass mirrors, aligned perpendicular to the magnetic axis of the magnet as precisely as possible. The circular scales had markings every 30° to reproduce the same angular position of the cylinder for every reading. The magnetic field direction was given by the surface normal of the compass mirror.

2 Compass Mirror Alignment

The misalignment of the surface normal of the compass mirrors and the unknown magnetic axis of the cylindrical magnet gave rise to a systematic errors in the magnetic field direction measurements. This error was minimized by aligning the surface normal of the compass mirrors parallel to the magnetic axis of the magnet with the help of the three brass screws and the neoprene sponge used as a spring material.

The compass mirror alignment was done in the target lab (figure 2a) at the Jefferson Lab before the field direction measurements in Hall C. The laser beam was reflected off the compass mirrors and

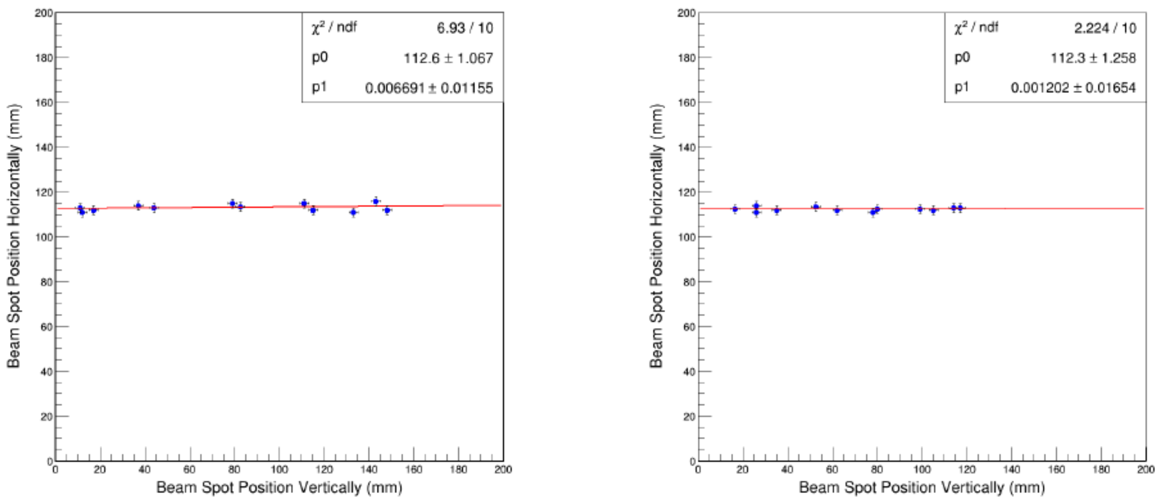


(a) Setup in the target lab

(b) Inscribed ellipse from the 360° scan of the compass magnet

Figure 2: Compass mirror alignment in the target lab

the reflected beam spot was monitored on a screen placed s=2 meter away from the compass.The laser pointer was fixed for the whole procedure so that the plane of incidence was always horizontal. Two lenses were used to get the beam spot diameter of 2 millimeters on the screen. To ensure the perfect flotation of the disk,a Starrett 98 series Mechanists' level was used to level the compass to the presision of 0.024°. The cylindrical magnet was rotated from the other end and the reflected beam spots were marked for all angular positions of the magnet. It inscribed an ellipse on the screen (figure 2b). This process was repeated until the minor axis of the ellipse was reduced to get a vertical straight line by fine tuning the angle of the mirrors attached.

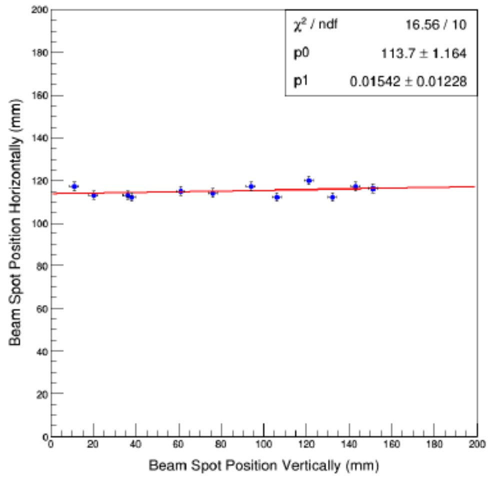


(a) Mirror 1

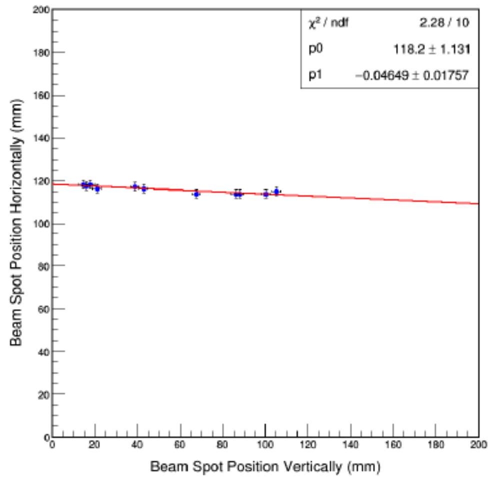
(b) Mirror 2

Figure 3: Straight line fits to the compass mirror alignment data for A_1^n experiment

The alignment data from both the compass mirrors were fitted to straight lines. Mirror 1 was used to measure the magnetic field directions in -X (beam right) and +Z (downstream) directions and Mirror 2 was used to measure the magnetic field directions in +X (beam left) and -Z (upstream) directions. In the figure 3 and figure 4 the straight lines were fitted after a 90° rotation of each data point so that the x and y axes correspond to the vertical and horizontal directions respectively. The y-spread of the data points about the fitted straight line determined the horizontal error in the magnetic field angle measurement generated from the compass mirror alignment. The total error in angle $\pm$ (θ_M $\pm$ dθ_M) was calculated from Dy $\pm$ dDy using the following equations.



(a) Mirror 1



(b) Mirror 2

Figure 4: Straight line fit to the compass mirror alignment data for d_2^n experiment

$$Dy = \frac{y_{max} - y_{min}}{2} \tag{1}$$

$$Dx = \frac{x_{max} - x_{min}}{2} \tag{2}$$

$$dDx = \sqrt{2x_{err}^2} \tag{3}$$

$$dDy = \sqrt{\left(\frac{\partial Dy}{\partial p_1}\right)^2 dp_1^2 + \left(\frac{\partial Dy}{\partial Dx}\right)^2 dDx^2 + \left(\frac{\partial Dy}{\partial p_0}\right)^2 dp_0^2} \tag{4}$$

$$\theta_M = \frac{Dy}{2s} \tag{5}$$

$$d\theta_M = \sqrt{\left(\frac{\partial \theta_M}{\partial Dy}\right)^2 dDy^2 + \left(\frac{\partial \theta_M}{\partial s}\right)^2 ds^2} \tag{6}$$

Where p_0 and p_1 are the fit parameters, dp_0 and dp_1 are the fit parameter errors, x_{err} was the error associated with each data point which was given by the laser beam spot diameter and Dy and dDy are the y-spread and error in y-spread rspectively. The results of the compass mirror alignment for both the A_1^n and d_2^n experiments are listed in table 1.

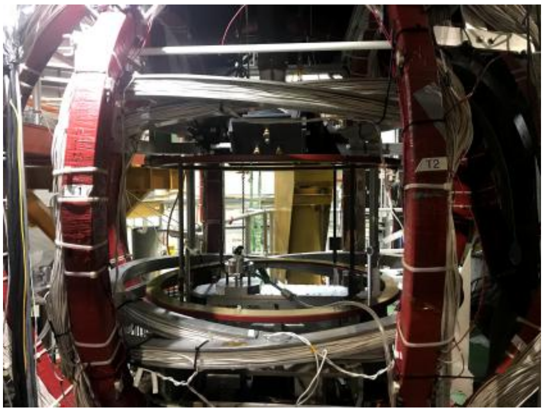
Experiment	Mirror	Dy (mm)	dDy (mm)	θ_M (°)	dθ_M (°)
A_1^n	1	0.4584	0.9541	0.0128	0.0266
	2	0.0604	1.0422	0.00179	0.0290
d_2^n	1	1.0563	1.0380	0.02942	0.0289
	2	2.1342	0.9794	0.0594	0.0273

Table 1: Systematic uncertainties from the compass mirror alignment

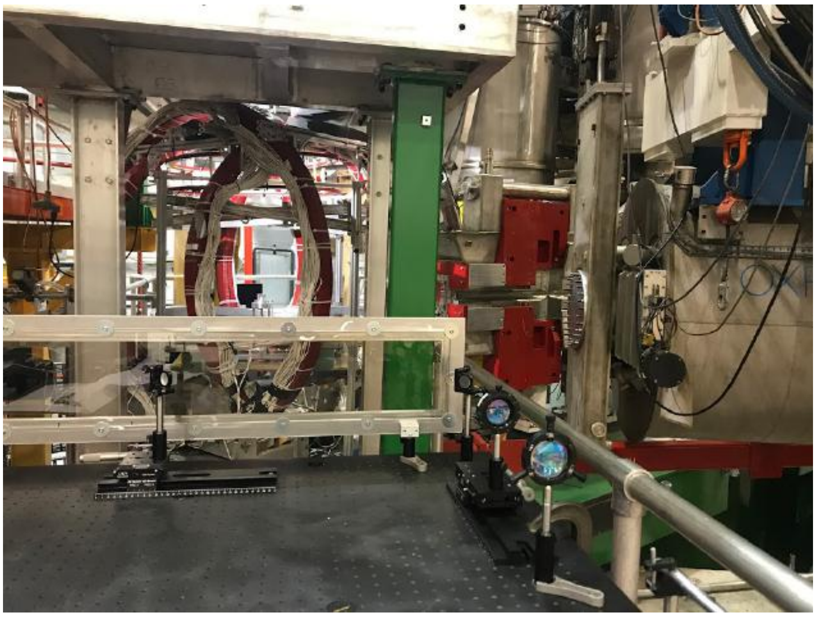
3 Compass Measurements in Hall C

The compass measurements were performed in Hall C in October, 2019 and March, 2020 for a couple of A_1^n and d_2^n kinematic settings respectively.The Helmholtz coil (MainL and MainS) and correction coil current settings (VL, VS, HL, HS) for all measured kinematic settings are listed in tabele 5 and table 3. The magnetic field directions were measured for all four polarization directions as shown in table 2. The compass was mounted on an aluminum fixture (figure 5a) with the help of two brass dowel pins. Both the transverse and longitudinal field directions were scanned in three different positions, 0 (target center), +12 cm and -12 cm along the beam direction. The beam direction was defined by two fiducials mounted on two posts (upstream and downstream) at the same height as the compass. by the alignemnt group. The three compass locations and the fiducials were surveyed by the Jefferson Lab alignment group before proceeding with the magnetic field direction measurements. All the optics and a transparent screen were installed (figure 5c) on an optics table, 2 meter away from the target region.

(a) Installation of the horizontal compass for the A_1^n experiment



(b) Installation of the horizontal compass for the d_2^n experiment



(c) Optics setup for compass measurements

Figure 5: The compass measurement setup in Hall C

Polarization	Direction	Angle with beamline
Transverse	Beam left: +X Beam right: -X	90° 270°
Longitudinal	Downstream: +Z Upstream: -Z	0° 180°

Table 2: Target polarization directions in Hall C

Kin. settings	Pol. directions	MainL current (A)	MainS current (A)	VL current (A)	VS current (A)	HL current (A)	HS current (A)
SHMS at 12.5°, 7.5 GeV/C	+X	+5.2306	+5.1632	8.7	6.7	-1.0	1.0
	-X	-5.2306	-5.1632	8.7	5.4	-1.0	1.0
	+Z	+5.2306	-5.1632	8.7	5.9	0.0	0.0
	-Z	-5.2306	+5.1632	8.7	5.9	0.0	0.0
SHMS at 30°, 3.4 GeV/C	+X	+5.2306	+5.1632	2.3	2.3	0.0	0.0
	-X	-5.2306	-5.1632	4.5	2.8	0.0	0.0
	+Z	+5.2306	-5.1632	4.5	3.5	0.0	0.0
	-Z	-5.2306	+5.1632	2.8	1.6	0.0	0.0

Table 3: Helmholtz coil and correction coil settings used for A_1^n kinematic settings with the Helmholtz coils at 45° w.r.t. electron beam direction.

Kin. settings	Pol. directions	MainL current (A)	MainS current (A)	VL current (A)	VS current (A)	HL current (A)	HS current (A)
SHMS at 18°, 5.6 GeV/C	+X	+5.2306	+5.1632	2.3	2.3	0.0	0.0
	-X	-5.2306	-5.1632	4.5	2.8	0.0	0.0
	+Z	+5.2306	-5.1632	4.5	3.5	-1.0	1.0
	-Z	-5.2306	+5.1632	2.8	1.6	0.0	0.0

Table 4: Helmholtz coil and correction coil settings used for d_2^n kinematic setting with the Helmholtz coils at 45° w.r.t. electron beam direction.

Kin. settings	Pol. directions	MainL current (A)	MainS current (A)	VL current (A)	VS current (A)	HL current (A)	HS current (A)
SHMS at 11°, 7.5 GeV/C	+X	7.407	0.137	4.0	6.0	0.0	0.0
	-X	-7.364	-0.237	4.1	6.0	0.0	0.0
	+Z	0.000	7.225	4.5	6.5	0.0	0.0
	-Z	0.000	-7.305	4.7	6.9	0.0	0.0
SHMS at 14.5°, 6.4 GeV/C	+X	7.406	0.139	4.3	6.1	0.0	0.0
	-X	-7.385	-0.239	4.2	6.1	0.0	0.0
	+Z	0.000	7.298	4.6	6.3	0.0	0.0
	-Z	0.000	-7.398	4.9	7.0	0.0	0.0

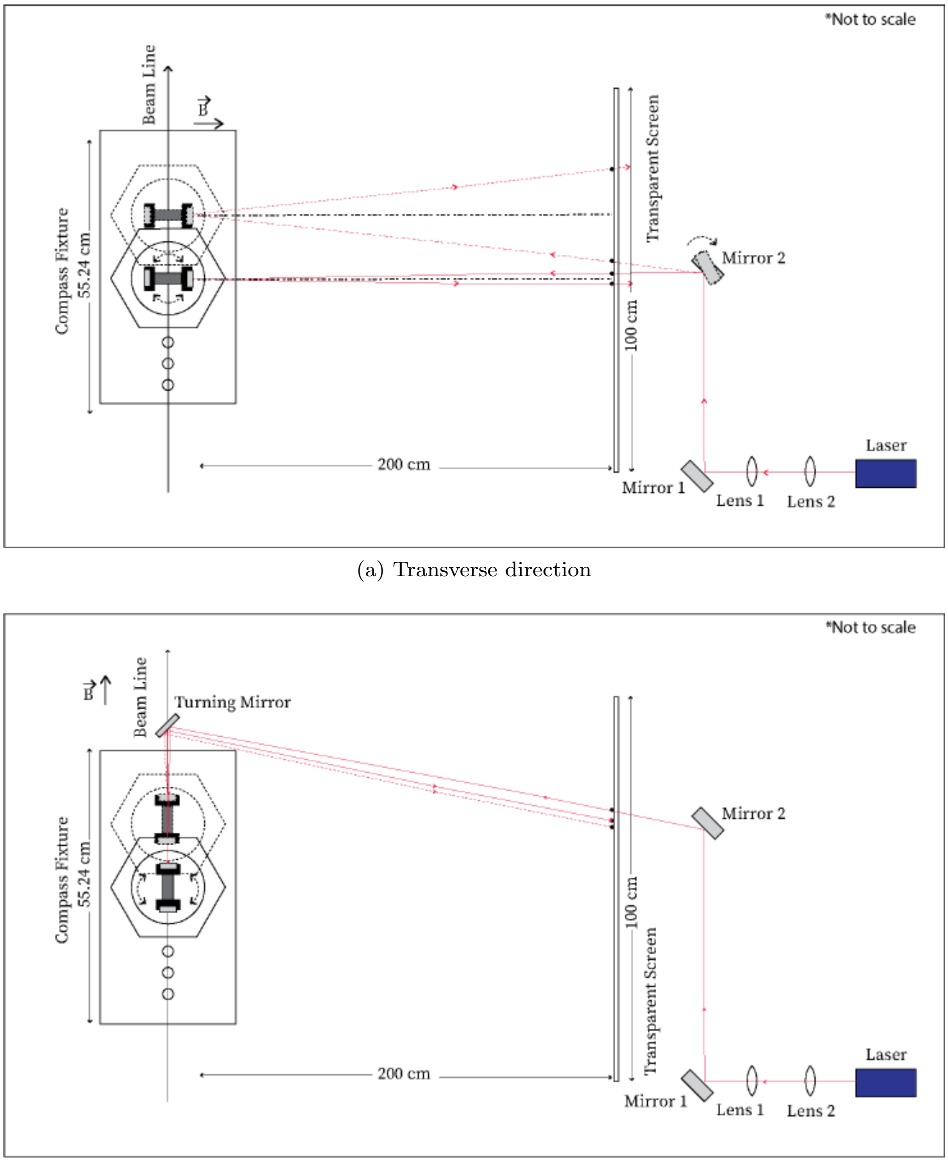
Table 5: Helmholtz coil and correction coil settings used for d_2^n kinematic settings with the Helmholtz coils at 0° w.r.t. electron beam direction.*Note: VL and VS are small and large vertical correction coils respectectively unlike previous measurements.

3.1 Transverse Magnetic Field Direction Measurement Procedure

The laser beam was incident at the center of the compass mirror from the beam right direction (figure 6a). The mirror 1 and mirror 2 were facing the beam right direction when the polarization directions were -X and +X respectively. While the compass was floating, the incident and reflected beam spots were marked on the transparent screen and they were surveyed by the alignment group after all the measurements were completed. This procedure was repeated for all three locations of the compass along the target length. The angular bisector of the incident and reflected laser beams determined the transverse magnetic field direction in absolute Hall C coordianate system.

3.2 Longitudinal Magnetic Field Direction Measurement Procedure

In order to measure the longitudinal magnetic field direction, a turning mirror with 1 inch diameter was mounted on the same aluminum fixture (figure 5a) as the compass for the A_1^n kinematics. The downstream fiducial was replaced with the turning mirror (figure 5b) during the compass measurements for the d_2^n kinematics. Before begining the longitudinal field direction measurement the angle of the



(a) Transverse direction

(b) Longitudinal direction

Figure 6: The magnetic field direction measurement procedures in Hall C

turning mirror was aligned to define the beamline and it was fixed at that orientation which helped in determining the longitudinal field direction from the surveyed points.The turning mirror alignment was checked periodically during the whole longitudinal field direction measurement procedure.The incident laser beam was reflected from the turning mirror to hit the center of the compass mirror and the reflected laser beam was reflected back to the transparent screen for all three compass locations (figure 6b). The mirror 1 and mirror 2 were facing the turning mirror when the polarization directions were +Z and -Z respectively. The angular bisector of the incident and reflected laser beams determined the longitudinal magnetic field direction.

4 Survey Data Analysis and Results

The three compass locations and the upstream and downstream fiducials were surveyed in the beginning of the compass measurements. The incident and reflected points on the transparent screen for all kinematic settings were surveyed after the magnetic field direction measurements were completed. The survey reports were analyzed and the angles between the magnetic field directions and the electron beam direction were calculated for all the kinematic settings.

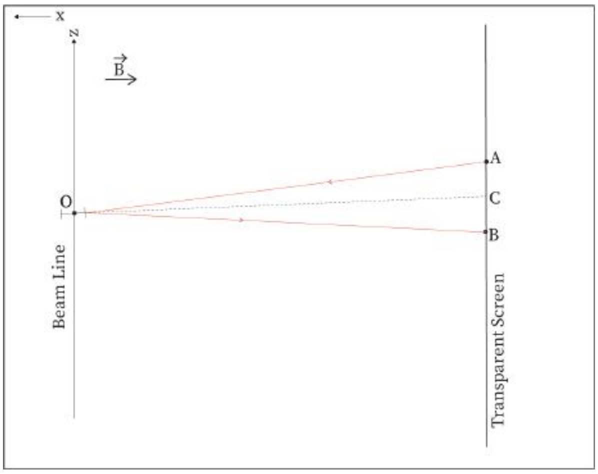


Figure 7: Determining the transverse magnetic field direction

4.1 Determining the Transverse Magnetic Field Direction

As shown in figure 7, the coordinates of the compass center (O), the incident (A) and the reflected (B) points on the transparent screen were surveyed in the absolute Hall C coordinate system. The transverse field direction was given by $\overrightarrow{OC}$, where C was the midpoint of A and B. The angle made by the transverse magnetic field with the beam line (w.r.t. +Z direction) was calculated as,

$$\alpha = \arccos\left(\frac{z_c - z_0}{\sqrt{(x_c - x_0)^2 + (z_c - z_0)^2}}\right) \tag{7}$$

4.2 Determining the Longitudinal Magnetic Field Direction

The center of the turning mirror coincided with the center of the fiducial I_2 as shown in figure 8a. The fiducials I_1 and I_2 were aligned to define electron beam direction. The turning mirror was aligned such that the laser beam coming from I_3 followed the beamline after reflecting off the turning mirror. The coordinates of the points I_1, I_2 and I_3 were surveyed to determine the surface normal of the turning mirror, which was given by the angular bisector of $\overrightarrow{I_1I_2}$ and $\overrightarrow{I_2I_3}$. The surveyed incident and reflected beamspots on the screen were A and B respectively. The point M_R on the turning mirror corresponded to P_s (midpoint of A and B) on the transparent screen (figure 8b). The longitudinal magnetic field direction was given by $\overrightarrow{OM_R}$.

Geometric analysis (figure 8c) was used to determine the coordinate of M_R from all the information available for each case and the angle β that $\overrightarrow{OM_R}$ made with positive Z direction was calculated. All calculations were done in x-z plane as the compass was only sensitive to the horizontal angle. Perpendicular line to the angular bisector of $\overrightarrow{I_1I_2}$ and $\overrightarrow{I_2I_3}$ passing through I_2 lying in x-z plane was $\overrightarrow{N_1N_2}$. The equation of $\overrightarrow{N_1N_2}$:

$$(z - z_{I_2}) = \left(\frac{-1}{m}\right)(x - x_{I_2}) \tag{8}$$

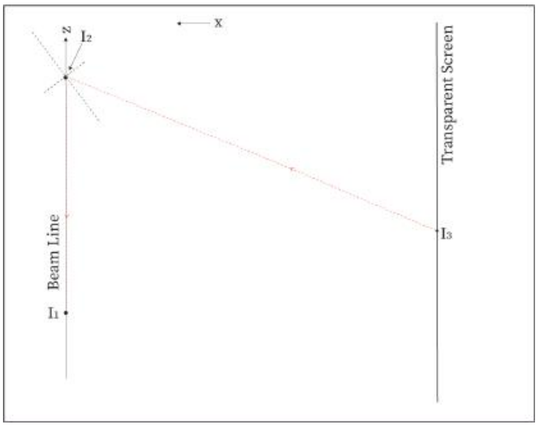
Where the slope of the angular bisector,

$$m = \tan\left(\frac{\arctan\left(\frac{z_{I_2}-z_{I_1}}{x_{I_2}-x_{I_1}}\right) + \arctan\left(\frac{z_{I_2}-z_{I_3}}{x_{I_2}-x_{I_3}}\right)}{2}\right) \tag{9}$$

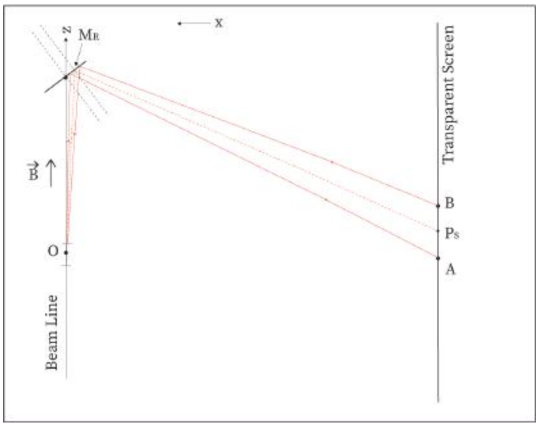
The equations of $\overrightarrow{ON_1}$ and $\overrightarrow{P_sN_2}$ (perpendiculars drawn to $\overrightarrow{N_1N_2}$ from O and P_s respectively) were,

$$z - z_0 = m(x - x_0) \tag{10}$$

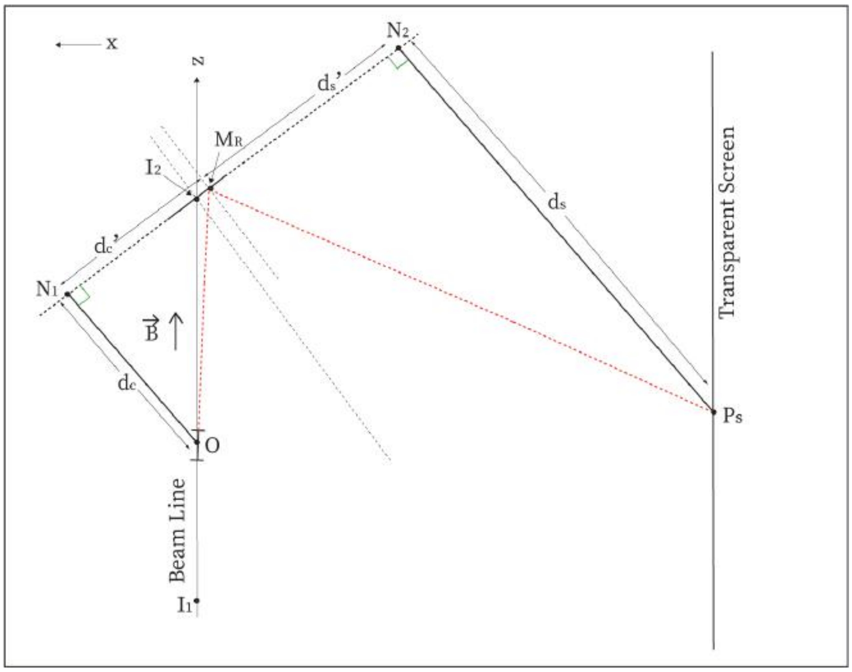
$$z - z_{P_s} = m(x - x_{P_s}) \tag{11}$$



(a) Turning mirror alignment



(b) Mapping surveyed points on the turining mirror



(c) Geometric analysis

Figure 8: Determining the longitudinal magnetic field direction

The coordinates of N_1 and N_2 were determined by solving ($\overrightarrow{ON_1}$, $\overrightarrow{N_1N_2}$) and($\overrightarrow{P_sN_2}$, $\overrightarrow{N_1N_2}$) respectively.
Length of $\overrightarrow{ON_1}$:

$$d_c = \sqrt{(x_0 - x_{N_1})^2 + (z_0 - z_{N_1})^2} \tag{12}$$

Length of $\overrightarrow{P_sN_2}$:

$$d_s = \sqrt{(x_{P_s} - x_{N_2})^2 + (z_{P_s} - z_{N_2})^2} \tag{13}$$

Length of $\overrightarrow{N_1N_2}$:

$$L = \sqrt{(x_{N_1} - x_{N_2})^2 + (z_{N_1} - z_{N_2})^2} \tag{14}$$

The following equations were solved to determine the coordinates of M_R.

$$d'_c = \sqrt{(x_{N_1} - x_{M_R})^2 + (z_{N_1} - z_{M_R})^2} = L - d'_s \tag{15}$$

$$d'_s = \sqrt{(x_{N_2} - x_{M_R})^2 + (z_{N_2} - z_{M_R})^2} = \frac{L}{1 + \frac{d_c}{d_s}} \tag{16}$$

The angle made by the longitudinal magnetic field with the beam line (w.r.t. +Z direction) was calculated as,

$$\beta = \arccos\left(\frac{(z_{M_R} - z_0)}{\sqrt{(x_{M_R} - x_0)^2 + (z_{M_R} - z_0)^2}}\right) \tag{17}$$

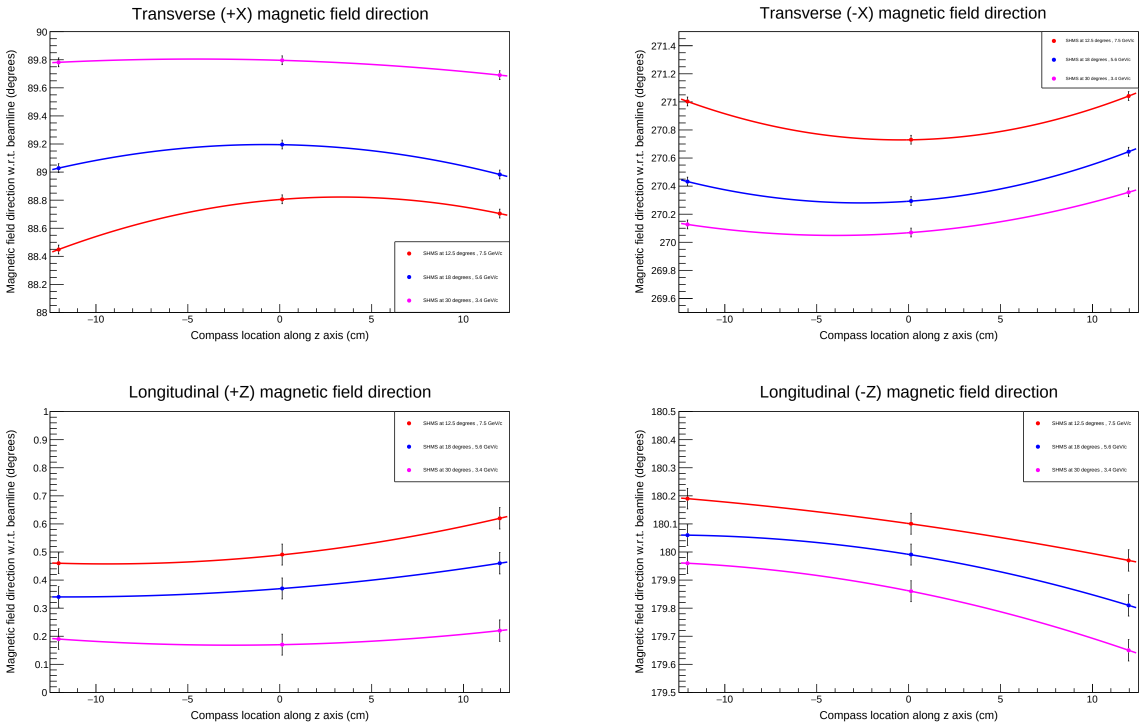


Figure 9: The magnetic field direction measurement results for the A_1^n experiment. All angles were calculated w.r.t. +Z direction in absolute Hall C coordinate system. The results from one of the d_2^n kinematic settings (SHMS at 18°, 5.6 GeV/C) were added to this plot because of the similar Helmholtz coil orientation in Hall C.

4.3 Results

The angles between the electron beam direction and the magnetic field directions were determined and plotted along z-axis for all measured kinematic settings of both A_1^n and d_2^n experiments.The results are shown is figure and figure.The field direction at different locations along target cell deviated from that at the center due to presence of steel structures close to the target region and effect of the Horizontal Bender magnet. All systematic uncertainties were propagated or added in quadrature to generate the error bars on the data points. The total uncertainty in the magnetic field direction was within ± 0.1° satisfying the requirement of the experiments.

5 Sources of Systematic Uncertainties

Several systematic uncertainties contributed to generate the total error in the compass measurements and they are listed below.

1. **Error propagation from the surveyed points** (Error_{Survey}): The uncertainty in surveying the compass center (O) and the locations of the two fiducials (I_1 and I_2) was ±0.28 millimeters and that in the mapped points (I_3, A and B) on the transparent screen was ±0.50 millimeters. The error propagation for the transverse ($d\alpha$) and longitudinal ($d\beta$) field direction measurements were calculated using the following formulas.

Kinematic settings	Polarization direction	Compass locations along z axis (cm)	Magnetic field direction (degrees)	± $Error_{Survey}$ (degrees)	± $Error_{Mirror}$ (degrees)	± $Error_{Spotsize}$ (degrees)	± $Error_{Center}$ (degrees)	± $Error_{Total}$ (degrees)
SHMS at 12.5°, 7.5 GeV/c	+X	11.965	88.705	0.012	0.029	0.006	0.01	0.034
		0.135	88.806	0.012	0.029	0.006	0.01	0.034
		-12.028	88.449	0.012	0.029	0.006	0.01	0.034
	-X	11.965	271.041	0.012	0.029	0.006	0.01	0.034
		0.135	270.73	0.012	0.029	0.006	0.01	0.034
		-12.028	271.003	0.012	0.029	0.006	0.01	0.034
	+Z	11.965	0.62	0.025	0.029	0.006	0.01	0.04
		0.135	0.49	0.023	0.029	0.006	0.01	0.039
		-12.028	0.46	0.022	0.029	0.006	0.01	0.038
	-Z	11.965	179.97	0.025	0.029	0.006	0.01	0.04
		0.135	180.1	0.023	0.029	0.006	0.01	0.039
		-12.028	180.19	0.022	0.029	0.006	0.01	0.038
SHMS at 30°, 3.4 GeV/c	+X	11.965	89.691	0.012	0.029	0.006	0.01	0.034
		0.135	89.796	0.012	0.029	0.006	0.01	0.034
		-12.028	89.782	0.012	0.029	0.006	0.01	0.034
	-X	11.965	270.356	0.012	0.029	0.006	0.01	0.034
		0.135	270.069	0.012	0.029	0.006	0.01	0.034
		-12.028	270.127	0.012	0.029	0.006	0.01	0.034
	+Z	11.965	0.22	0.025	0.029	0.006	0.01	0.04
		0.135	0.17	0.023	0.029	0.006	0.01	0.039
		-12.028	0.19	0.022	0.029	0.006	0.01	0.038
	-Z	11.965	179.65	0.025	0.029	0.006	0.01	0.04
		0.135	179.86	0.023	0.029	0.006	0.01	0.039
		-12.028	179.96	0.022	0.029	0.006	0.01	0.038

(a) A_1^n kinematic settings

Kinematic settings	Polarization direction	Compass locations along z axis (cm)	Magnetic field direction (degrees)	± $Error_{Survey}$ (degrees)	± $Error_{Mirror}$ (degrees)	± $Error_{Spotsize}$ (degrees)	± $Error_{Center}$ (degrees)	± $Error_{Total}$ (degrees)
SHMS at 18°, 5.6 GeV/c	+X	11.965	88.983	0.012	0.029	0.006	0.01	0.034
		0.135	89.195	0.012	0.029	0.006	0.01	0.034
		-12.028	89.028	0.012	0.029	0.006	0.01	0.034
	-X	11.965	270.645	0.012	0.029	0.006	0.01	0.034
		0.135	270.293	0.012	0.029	0.006	0.01	0.034
		-12.028	270.432	0.012	0.029	0.006	0.01	0.034
	+Z	11.965	0.46	0.025	0.029	0.006	0.01	0.04
		0.135	0.37	0.023	0.029	0.006	0.01	0.039
		-12.028	0.34	0.022	0.029	0.006	0.01	0.038
	-Z	11.965	179.81	0.025	0.029	0.006	0.01	0.04
		0.135	179.99	0.023	0.029	0.006	0.01	0.039
		-12.028	180.06	0.022	0.029	0.006	0.01	0.038

(b) d_2^n kinematic setting with Helmholtz coil at 45° w.r.t. beamline

Kinematic settings	Polarization direction	Compass locations along z axis (cm)	Magnetic field direction (degrees)	± $Error_{Survey}$ (degrees)	± $Error_{Mirror}$ (degrees)	± $Error_{Spotsize}$ (degrees)	± $Error_{Center}$ (degrees)	± $Error_{Total}$ (degrees)
SHMS at 11°, 7.5 GeV/c	+X	11.965	91.513	0.012	0.065	0.006	0.01	0.067
		-0.14	91.387	0.012	0.065	0.006	0.01	0.067
		-12.017	91.533	0.012	0.065	0.006	0.01	0.067
	-X	11.965	271.191	0.012	0.041	0.006	0.01	0.044
		-0.14	271.298	0.012	0.041	0.006	0.01	0.044
		-12.017	271.117	0.012	0.041	0.006	0.01	0.044
	+Z	11.965	0.166	0.036	0.041	0.006	0.01	0.056
		-0.14	0.182	0.034	0.041	0.006	0.01	0.055
		-12.017	-0.23	0.032	0.041	0.006	0.01	0.054
	-Z	11.965	180.054	0.036	0.065	0.006	0.01	0.075
		-0.14	180.202	0.034	0.065	0.006	0.01	0.075
		-12.017	179.779	0.032	0.065	0.006	0.01	0.074
SHMS at 14.5°, 6.4 GeV/c	+X	11.965	91.457	0.012	0.065	0.006	0.01	0.067
		-0.14	91.353	0.012	0.065	0.006	0.01	0.067
		-12.017	91.53	0.012	0.065	0.006	0.01	0.067
	-X	11.965	270.653	0.012	0.041	0.006	0.01	0.044
		-0.14	271.214	0.012	0.041	0.006	0.01	0.044
		-12.017	271.116	0.012	0.041	0.006	0.01	0.044
	+Z	11.965	0.572	0.036	0.041	0.006	0.01	0.056
		-0.14	0.338	0.034	0.041	0.006	0.01	0.055
		-12.017	0.223	0.032	0.041	0.006	0.01	0.054
	-Z	11.965	180.17	0.036	0.065	0.006	0.01	0.075
		-0.14	180.129	0.034	0.065	0.006	0.01	0.075
		-12.017	180.19	0.032	0.065	0.006	0.01	0.074

(c) d_2^n kinematic settings

Table 6: The uncertainties in the magnetic field direction measurements

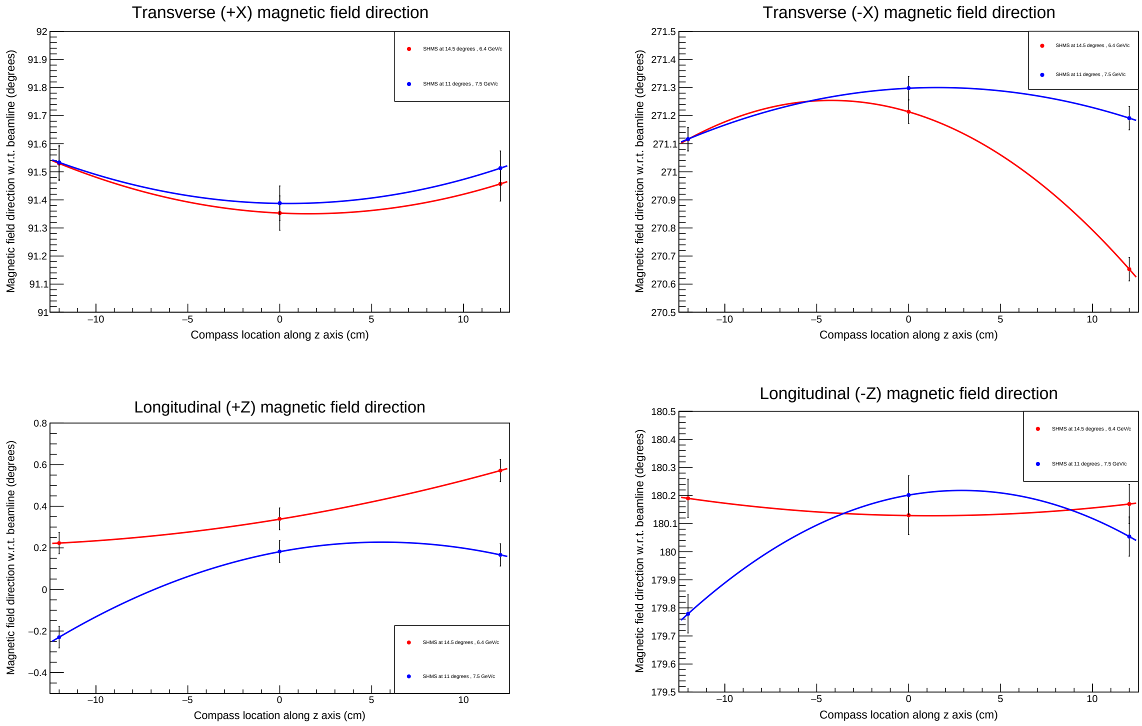


Figure 10: The magnetic field direction measurement results for the d_2^n experiment. All angles were calculated w.r.t. +Z direction in absolute Hall C coordinate system.

(a) Transverse direction:

$$d\alpha^2 = \left(\frac{\partial\alpha}{\partial x_0}\right)^2 dx_0^2 + \left(\frac{\partial\alpha}{\partial z_0}\right)^2 dz_0^2 + \left(\frac{\partial\alpha}{\partial x_A}\right)^2 dx_A^2 + \left(\frac{\partial\alpha}{\partial z_A}\right)^2 dz_A^2 + \left(\frac{\partial\alpha}{\partial x_B}\right)^2 dx_B^2 + \left(\frac{\partial\alpha}{\partial z_B}\right)^2 dz_B^2 \tag{18}$$

(b) Longitudinal direction:

$$d\beta = \sqrt{\sum_j \left(\frac{\partial\beta}{\partial x_j}\right)^2 dx_j^2} \tag{19}$$

Where $x_j = x_0, z_0, x_A, z_A, x_B, z_B, x_{I_1}, z_{I_1}, x_{I_2}, z_{I_2}, x_{I_3}, z_{I_3}$ were the surveyed coordinates and dx_j was the survey error associated with the particular coordinate.

2. **Compass mirror alignment** (Error_{Mirror}): The misalignment between magnetic axis of the cylindrical magnet and compass mirror generated additional errors as described in section 2 .

   (a) Projection of the fitted straight line on the horizontal axis.

   (b) Fit parameter errors.

3. **Laser beam spot size** ($\text{Error}_{Spotsize}$): The laser beam spot size was reduced to 2 millimeters diameter with the help of a pair of lenses. This beam spot size contributed ±0.006° systematic uncertainty in the horizontal angle measurements.

4. **Position of the incident laser beam on the compass mirror** (Error_{Center}): The laser beam was always reflected off the center of the compass mirror within ±0.5 millimeter uncertainty that generated ±0.01° uncertainty in the horizontal angle measurements.

6 Conclusion

The compass measurements in Hall C for the A_1^n and d_2^n experiments were completed in October, 2019 and March, 2020 respectively with the help of Jefferson Lab alignment group. The magnetic field direction was scanned at three locations along the target length for all four polarization directions and kinematic settings with optimum Helmholtz coil and correction coil settings determined from the field mapping performed in Hall C. The angle of the magnetic field w.r.t. electron beam direction was calculated in absolute Hall C coordinate system with less than $\pm 0.01°$ uncertainty satisfying the requirement of the experiments.