

Fit Residuals

2nd Supplemental Plots for 1st Generation of Fit Studies
(using electrons)

C. Yero
D. Mack

May 13, 2021

Original FIT Function

Fit Function:
(Norm. Yield)

$$f = A \cdot \frac{C \pm y/D}{C \pm y}$$

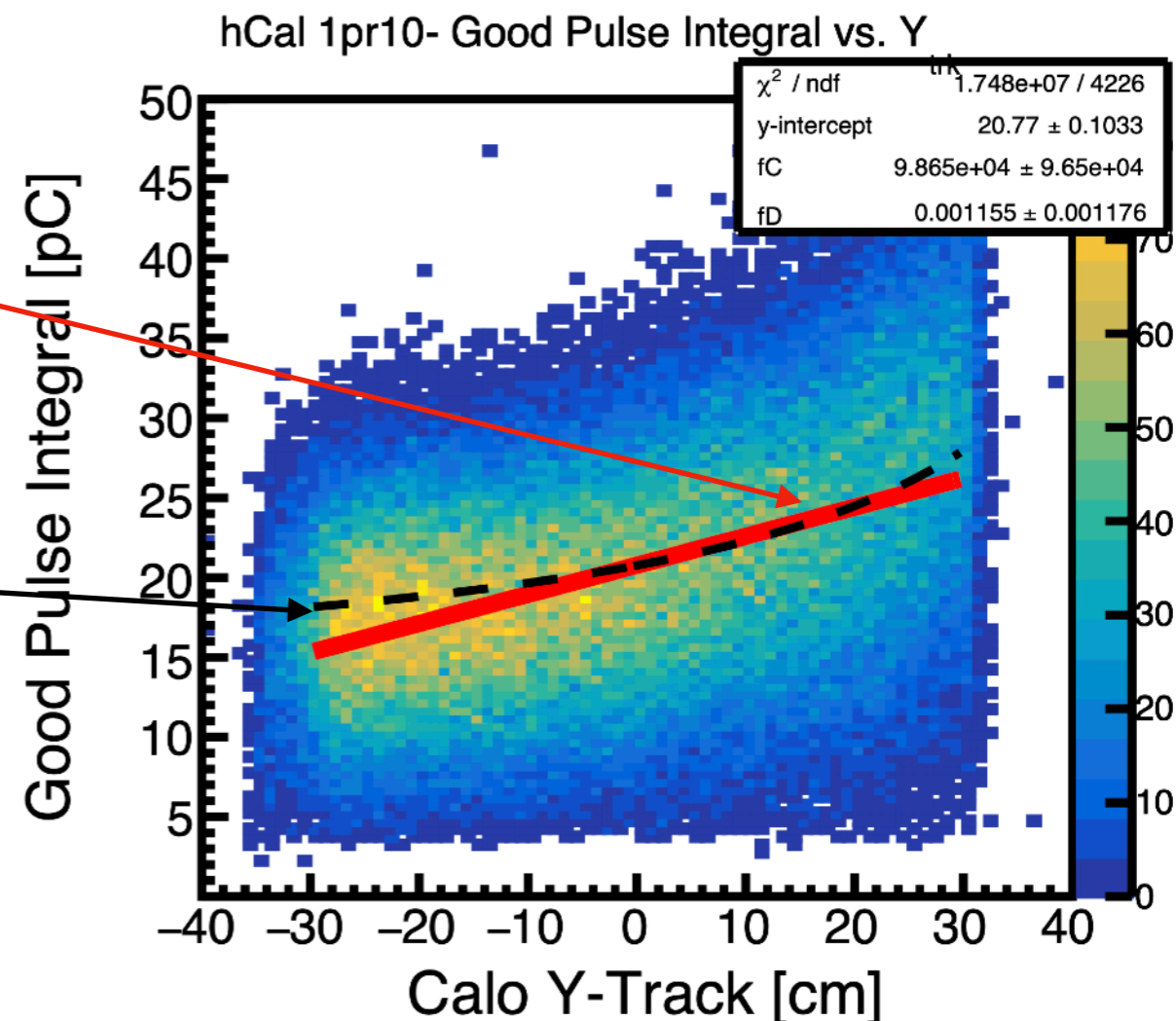
A : Vertical Offset
 C, D : Additional Parameters
 y : Y-Track Position [cm]

Data Fit:

- An initial guess of the fit parameters is made: $A = 15$ pC, $C = 64.36$ cm, $D = 1.66$
- The initial guess of C & D is EXACTLY the current parameters hardcoded in HCANA

HCANA "Model":

- Used the Fit Function (see above) with hardcoded parameters: $C = 64.36$ cm, $D = 1.66$
- For the offset parameter, I used the final offset determined from the **Data Fit**



The **Data Fit** uses the hardcoded parameters as an initial guess, and the ROOT Minuit Algorithm determines the “best” set of parameters to describe the 2D correlation. The same Fit Function used in HCANA is also plotted to make a comparison.

NEW FIT Function

Fit Function:
(Norm. Yield)

$$f = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$

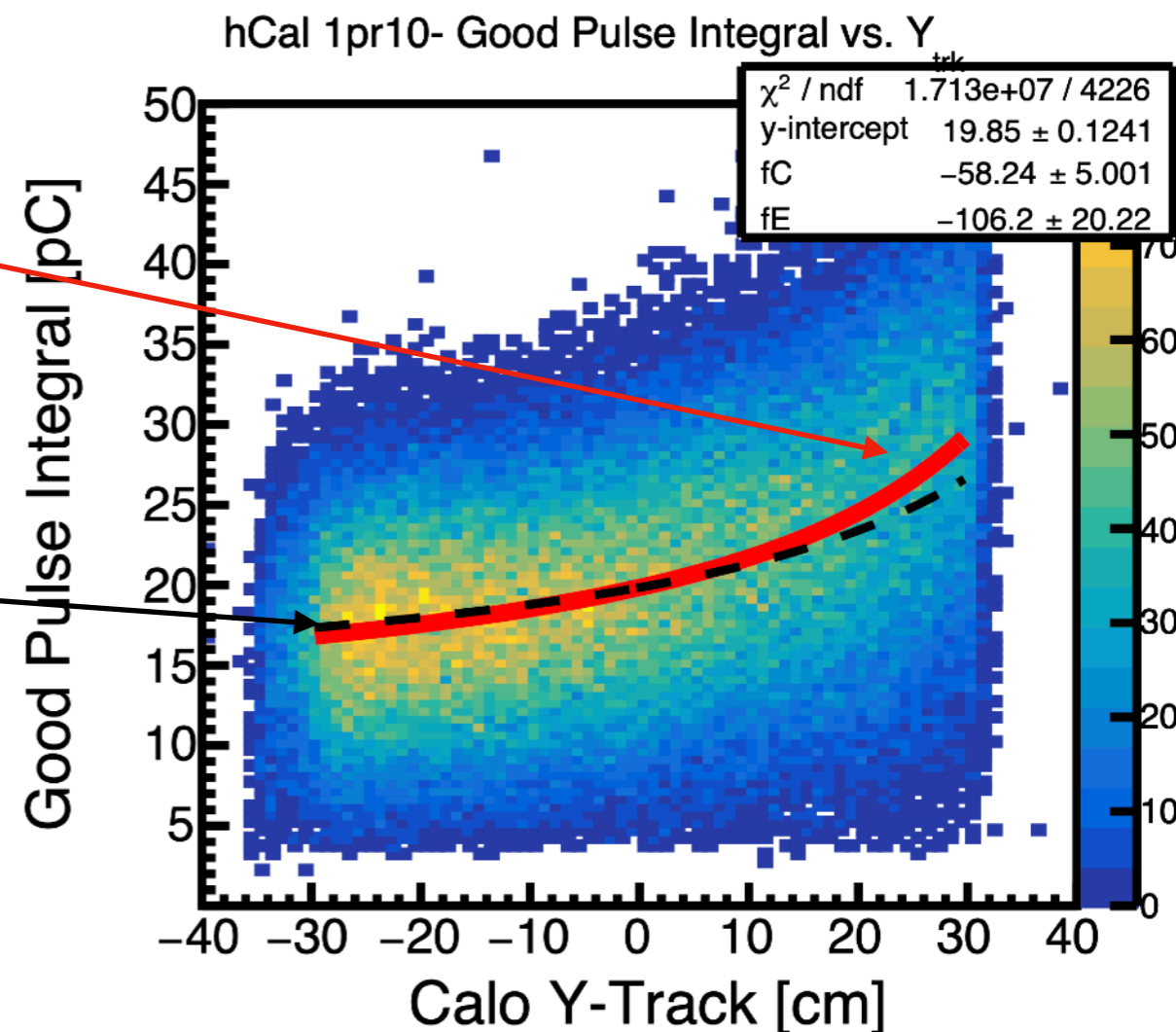
A : Vertical Offset
 C, E : Additional Parameters
 y : Y-Track Position [cm]

Data Fit:

- An initial guess of the fit parameters is made: $A = 15$ pC, $C = 64.36$ cm, $E = 106.83$ cm
- The initial guess of C is EXACTLY the current parameters hardcoded in HCANA
And $E = CD$ (product of two hardcoded HCANA params)

HCANA "Model":

- Used the Fit Function (see above) with hardcoded parameters: $C = 64.36$ cm, $E = 106.83$ cm
- For the offset parameter, I used the final offset determined from the **Data Fit**

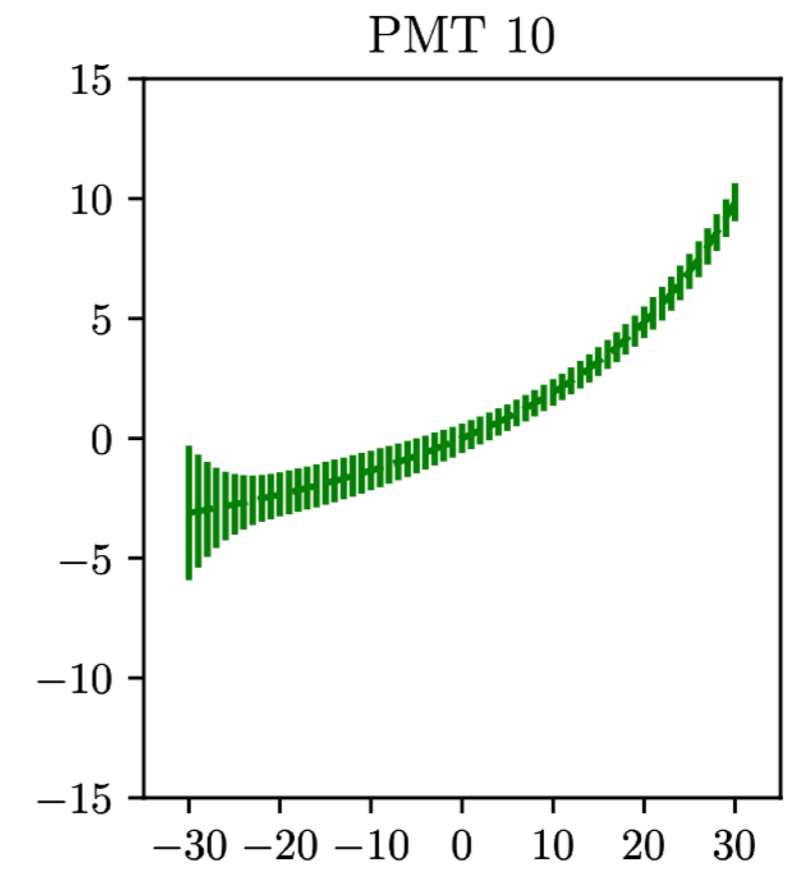
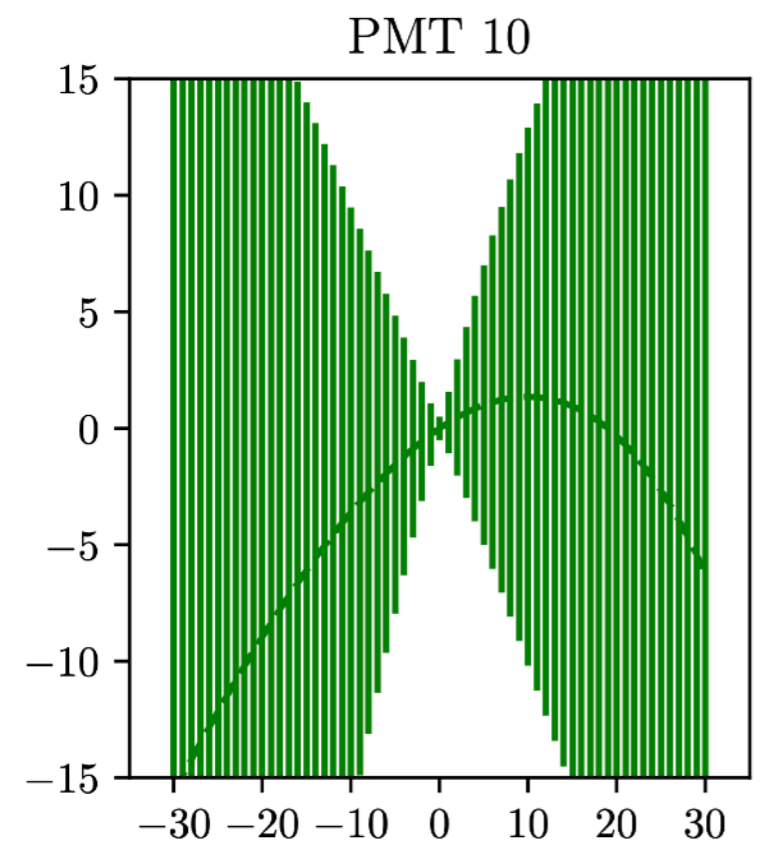
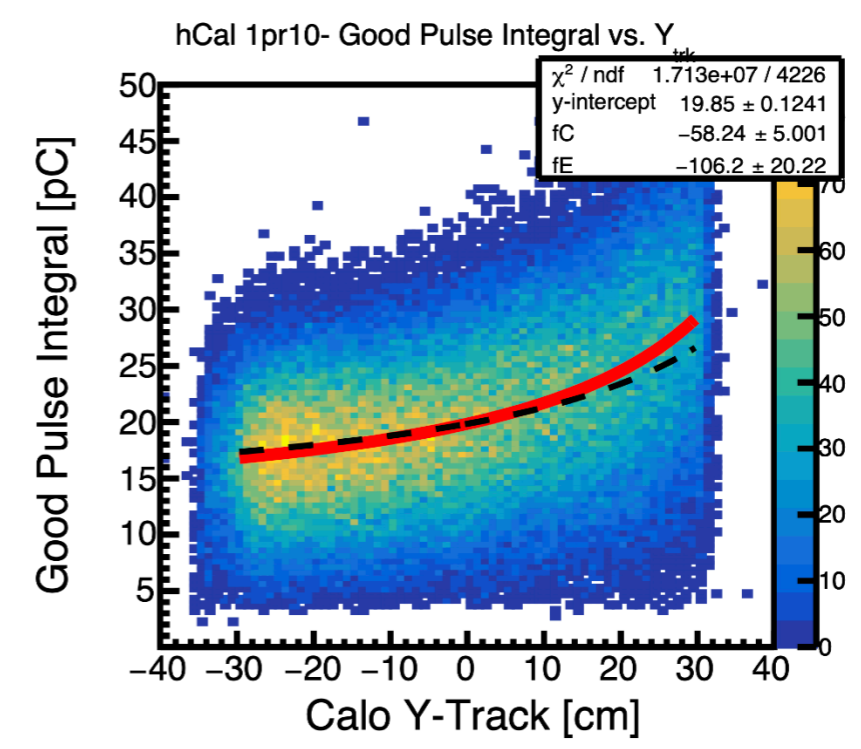
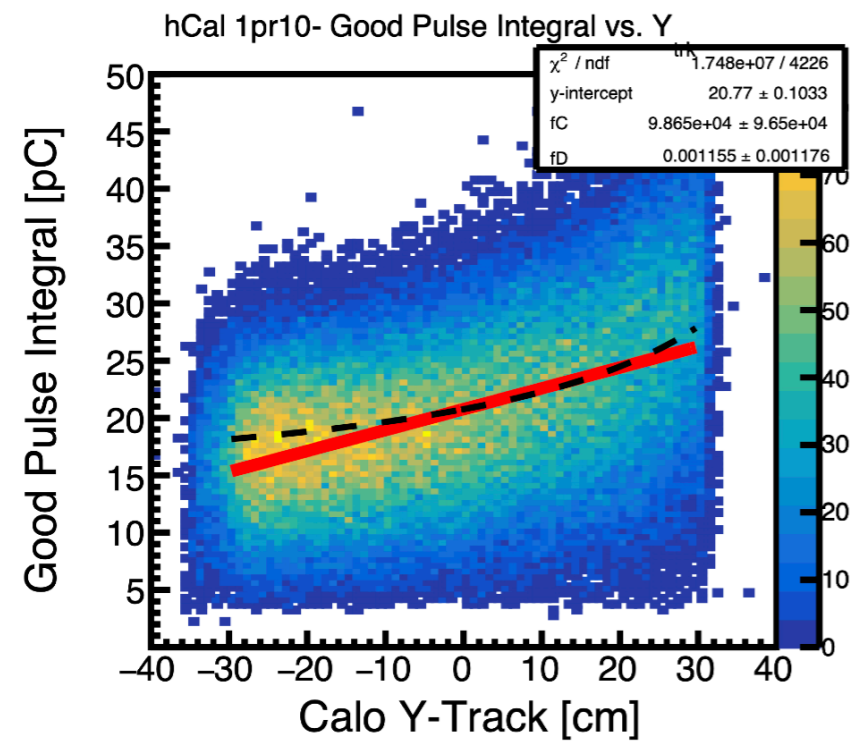


The **Data Fit** uses the hardcoded parameters as an initial guess, and the ROOT Minuit Algorithm determines the "best" set of parameters to describe the 2D correlation. The same Fit Function used in HCANA is also plotted to make a comparison.

Fit Function Comparison: (Showing Improvements)

$$f = A \cdot \frac{C \pm y/D}{C \pm y}$$

$$f = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$



Residuals: Original FIT Function

$$R = \frac{f_{\text{dataFit}} - f_{\text{HCANA}}}{f_{\text{HCANA}}}$$

$$\delta R = \frac{\delta f_{\text{dataFit}}}{f_{\text{HCANA}}}$$

$$f_{\text{dataFit}} = A \cdot \frac{C \pm y/D}{C \pm y}$$

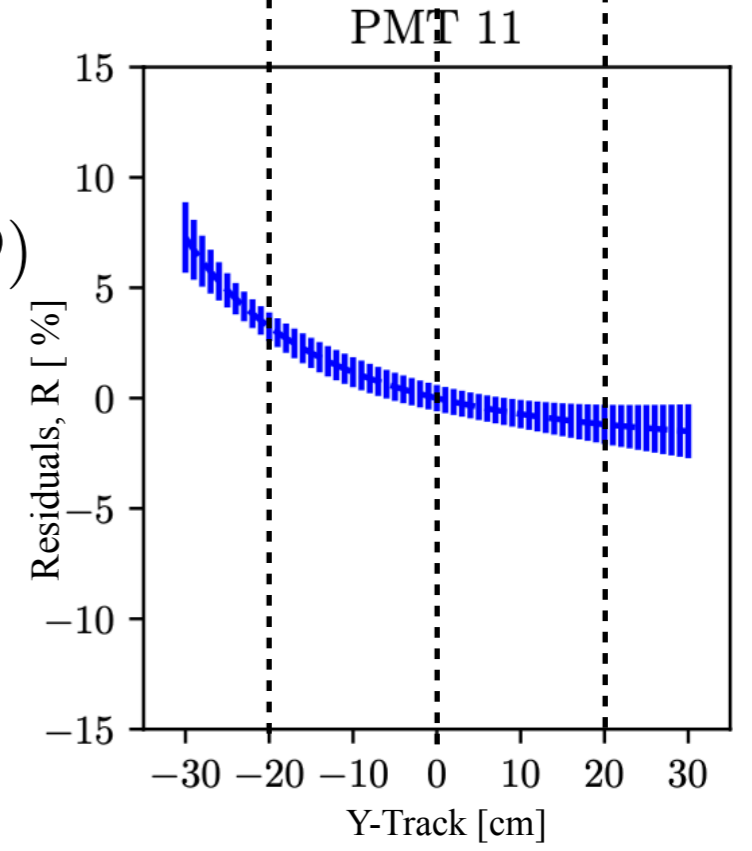
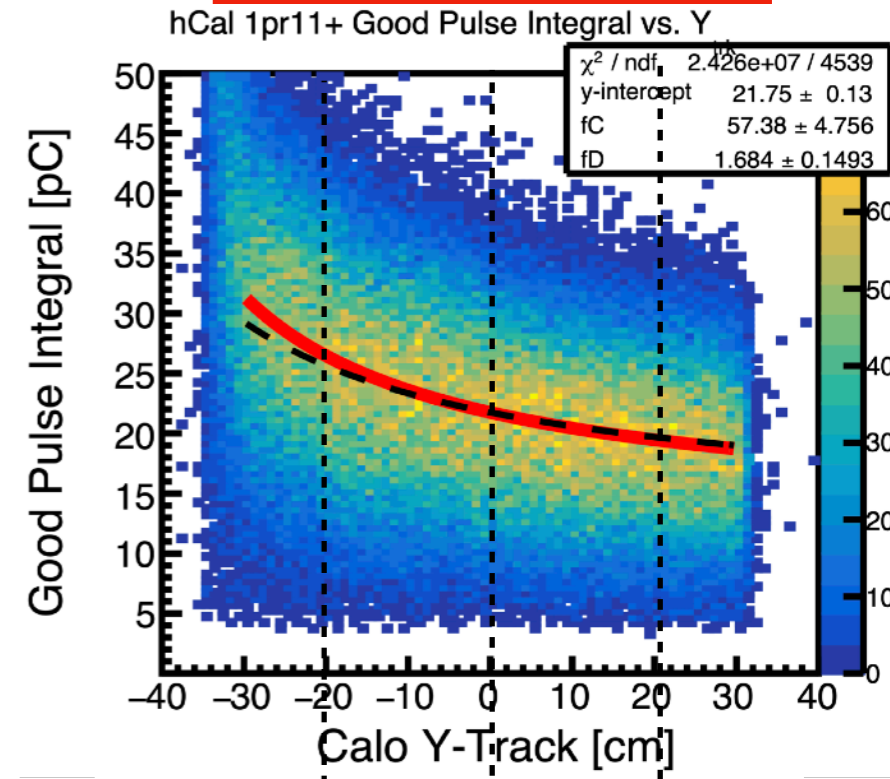
$$\delta f_{\text{dataFit}}^2 = \left(\frac{\partial f}{\partial A}\right)^2 \delta A^2 + \left(\frac{\partial f}{\partial C}\right)^2 \delta C^2 + \left(\frac{\partial f}{\partial D}\right)^2 \delta D^2 + 2 \frac{\partial^2 f}{\partial A \partial C} \text{cov}(A, C) + 2 \frac{\partial^2 f}{\partial A \partial D} \text{cov}(A, D) + 2 \frac{\partial^2 f}{\partial C \partial D} \text{cov}(C, D)$$

$$\left(\frac{\partial f}{\partial A}\right) = \frac{C + y/D}{C + y} \quad \left(\frac{\partial f}{\partial C}\right) = A \left(\frac{y(1 - 1/D)}{(C + y)^2}\right) \quad \left(\frac{\partial f}{\partial D}\right) = -\frac{Ay}{D^2(C + y)}$$

Standard Error Propagation of the Data Fit (errors in fit parameters are used)

NOTE: I dropped the subscript 'dataFit' for simplicity

Example: Layer 1pr, PMT 11+



Interpretation: In this example, there is up to ~ 7.5 % difference between our studies and HCANA on the lower one end of the block at ~ -30 cm

Residuals: NEW FIT Function

$$R = \frac{f_{\text{dataFit}} - f_{\text{HCANA}}}{f_{\text{HCANA}}}$$

$$f_{\text{dataFit}} = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$

$$\delta R = \frac{\delta f_{\text{dataFit}}}{f_{\text{HCANA}}}$$

$$\delta f_{\text{dataFit}}^2 = \left(\frac{\partial f}{\partial A} \right)^2 \delta A^2 + \left(\frac{\partial f}{\partial C} \right)^2 \delta C^2 + \left(\frac{\partial f}{\partial E} \right)^2 \delta E^2$$

$$+ 2 \frac{\partial^2 f}{\partial A \partial C} \text{cov}(A, C) + 2 \frac{\partial^2 f}{\partial A \partial E} \text{cov}(A, E) + 2 \frac{\partial^2 f}{\partial C \partial E} \text{cov}(C, E)$$

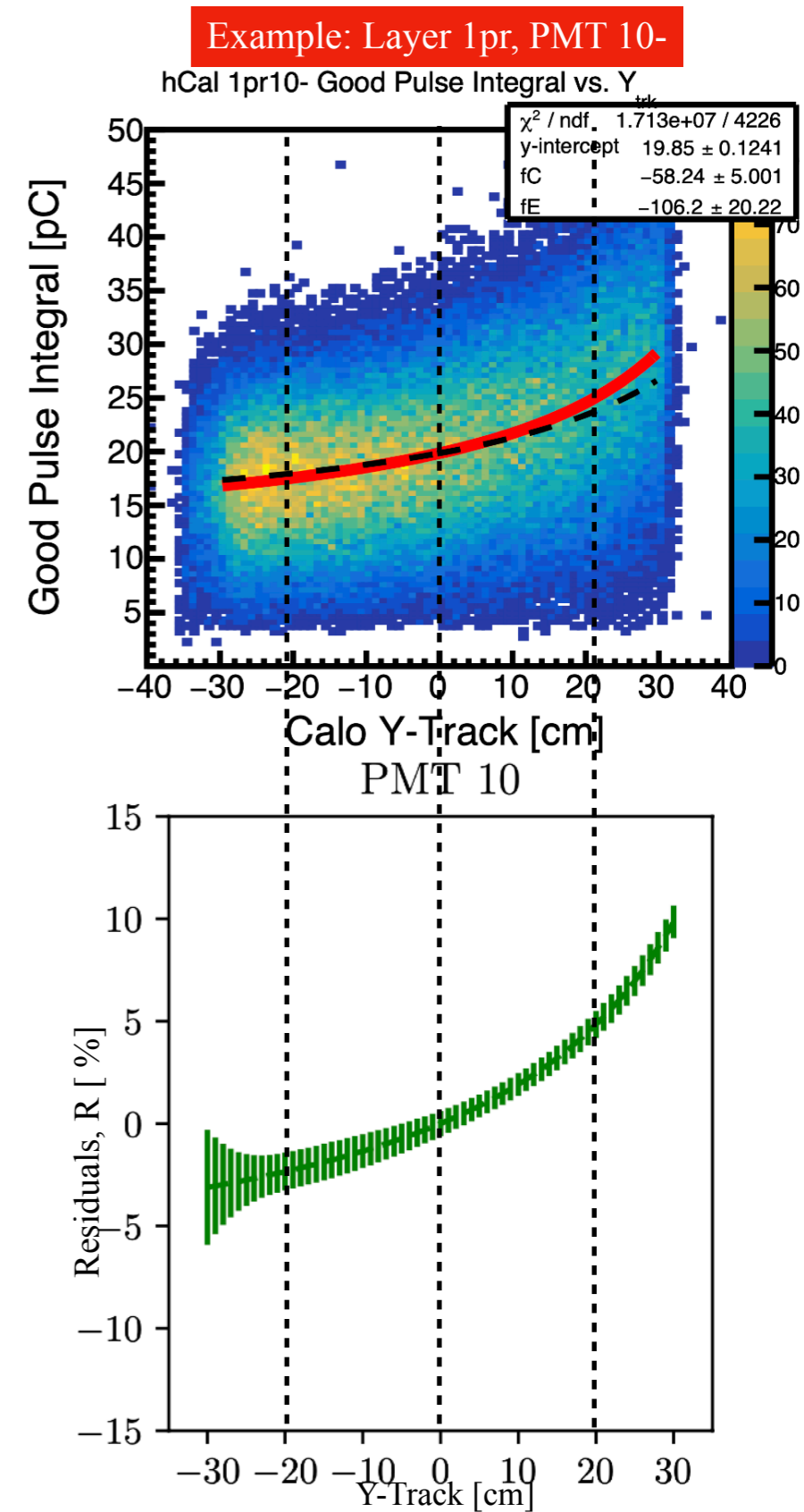
$$\left(\frac{\partial f}{\partial A} \right) = \frac{1 \pm y/E}{1 \pm y/C}$$

$$\left(\frac{\partial f}{\partial C} \right) = \pm A \cdot \frac{(1 \pm y/E)(y/C^2)}{(1 \pm y/C)^2}$$

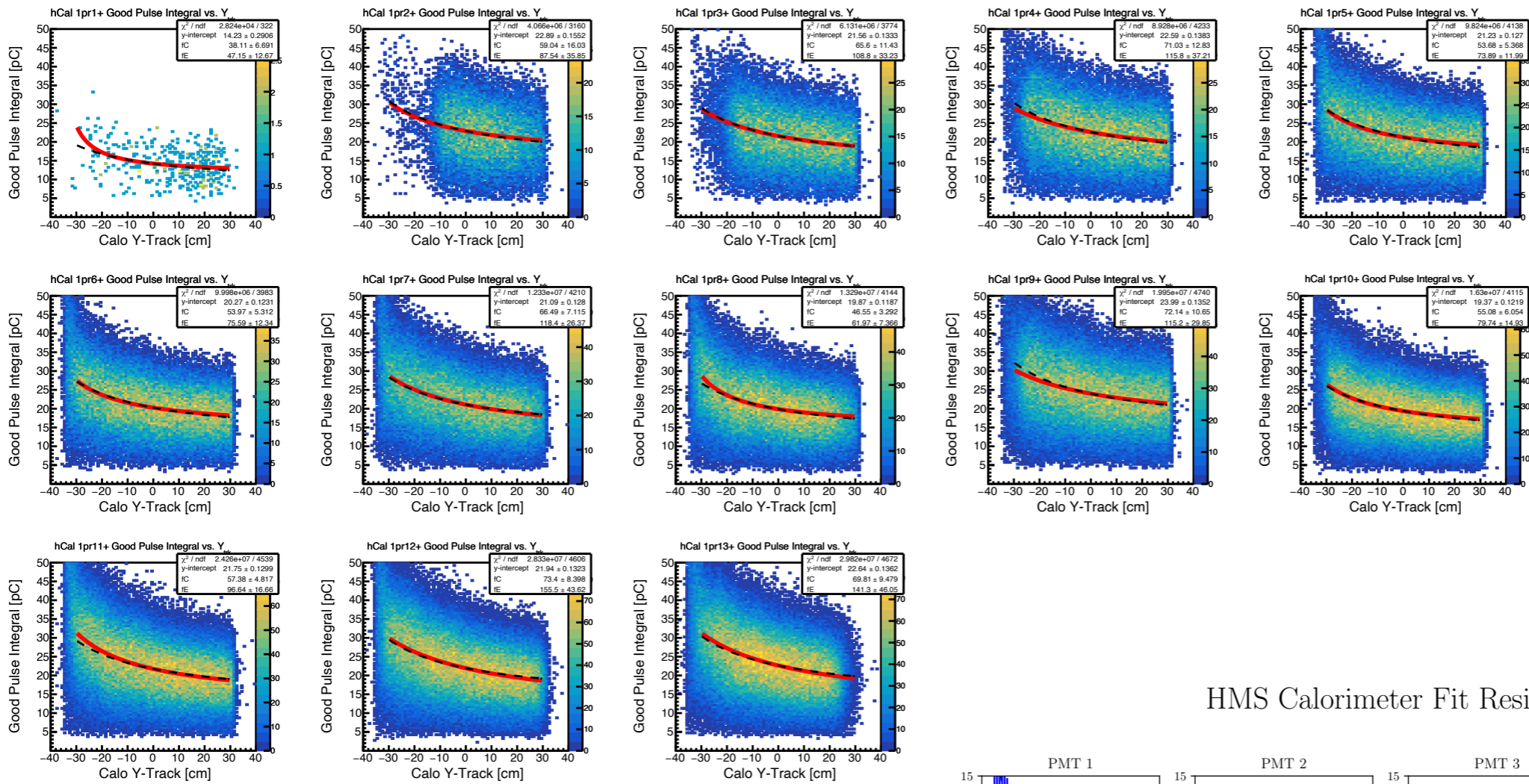
$$\left(\frac{\partial f}{\partial E} \right) = \pm A \cdot \frac{-y/E^2}{1 \pm y/C}$$

Standard Error Propagation of the Data Fit
(errors in fit parameters are used)

NOTE: I dropped the subscript 'dataFit'
for simplicity



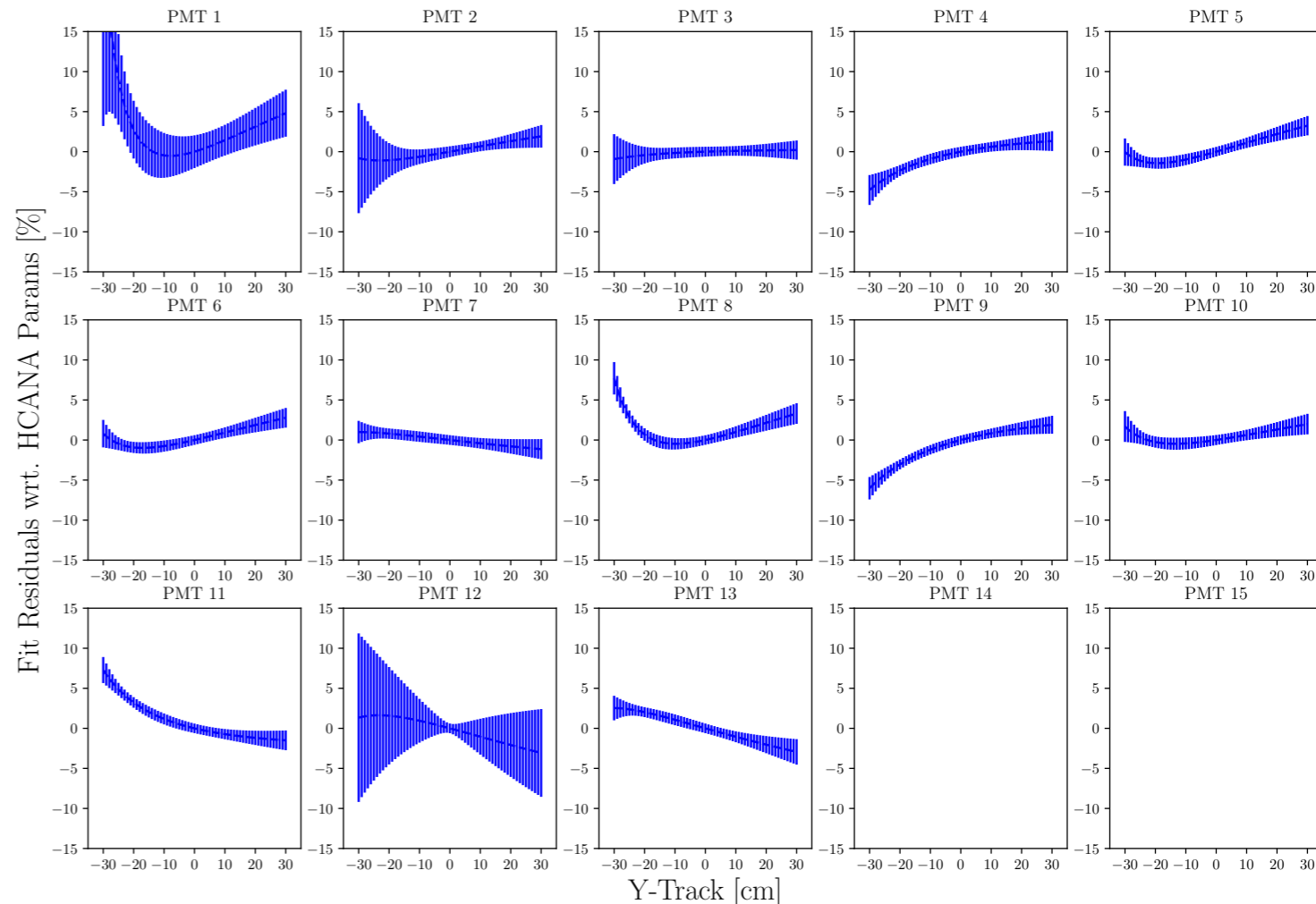
Interpretation: In this example, there is up to
~ 10 % difference between our studies and HCANA
on the upper end of the block at ~ +30 cm

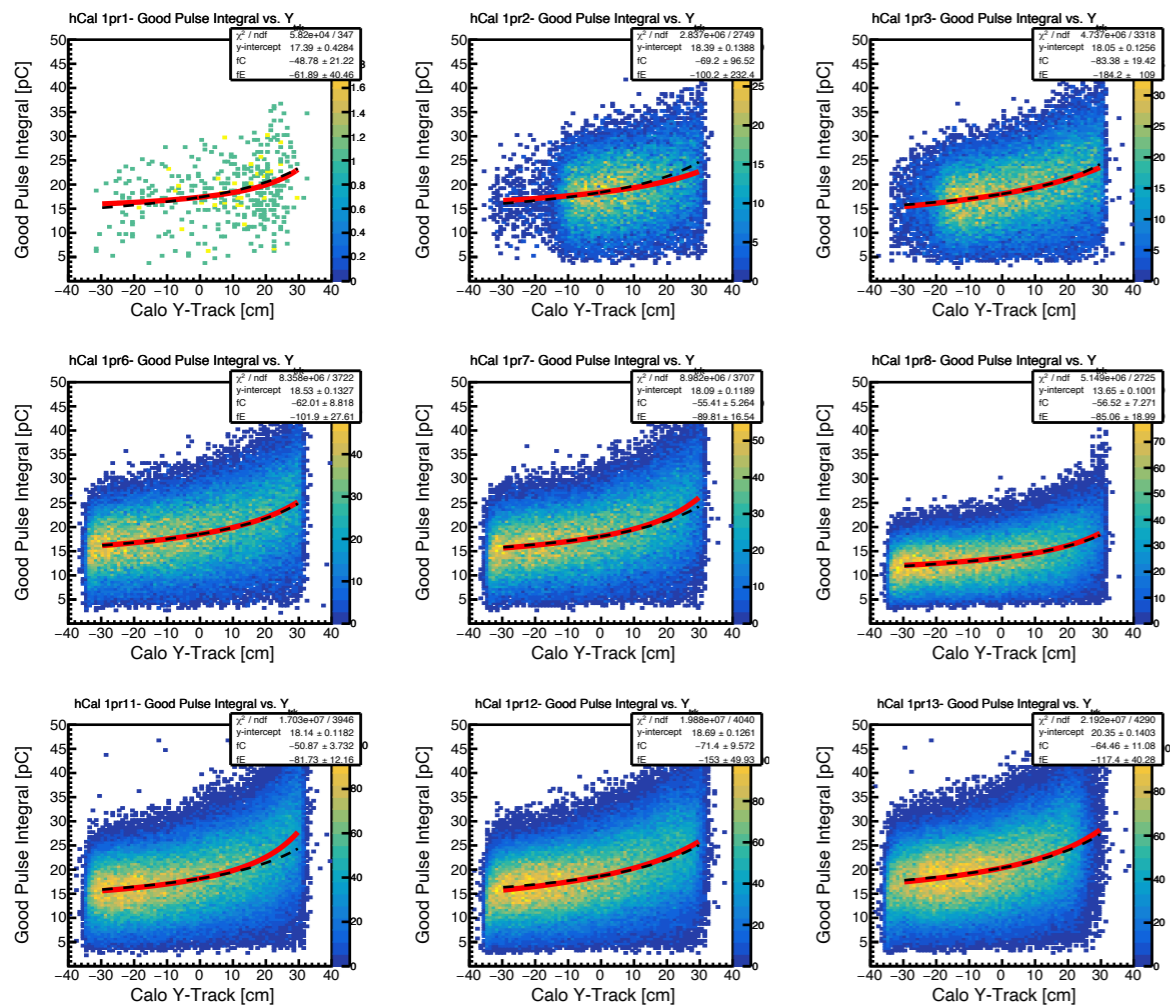


HMS Calorimeter Fit Residuals: Layer 1pr+

Layer 1pr +

- Correlated errors are included
- X-axis range set to (-35, 35) cm
- Y-axis range set to (-15, 15) %

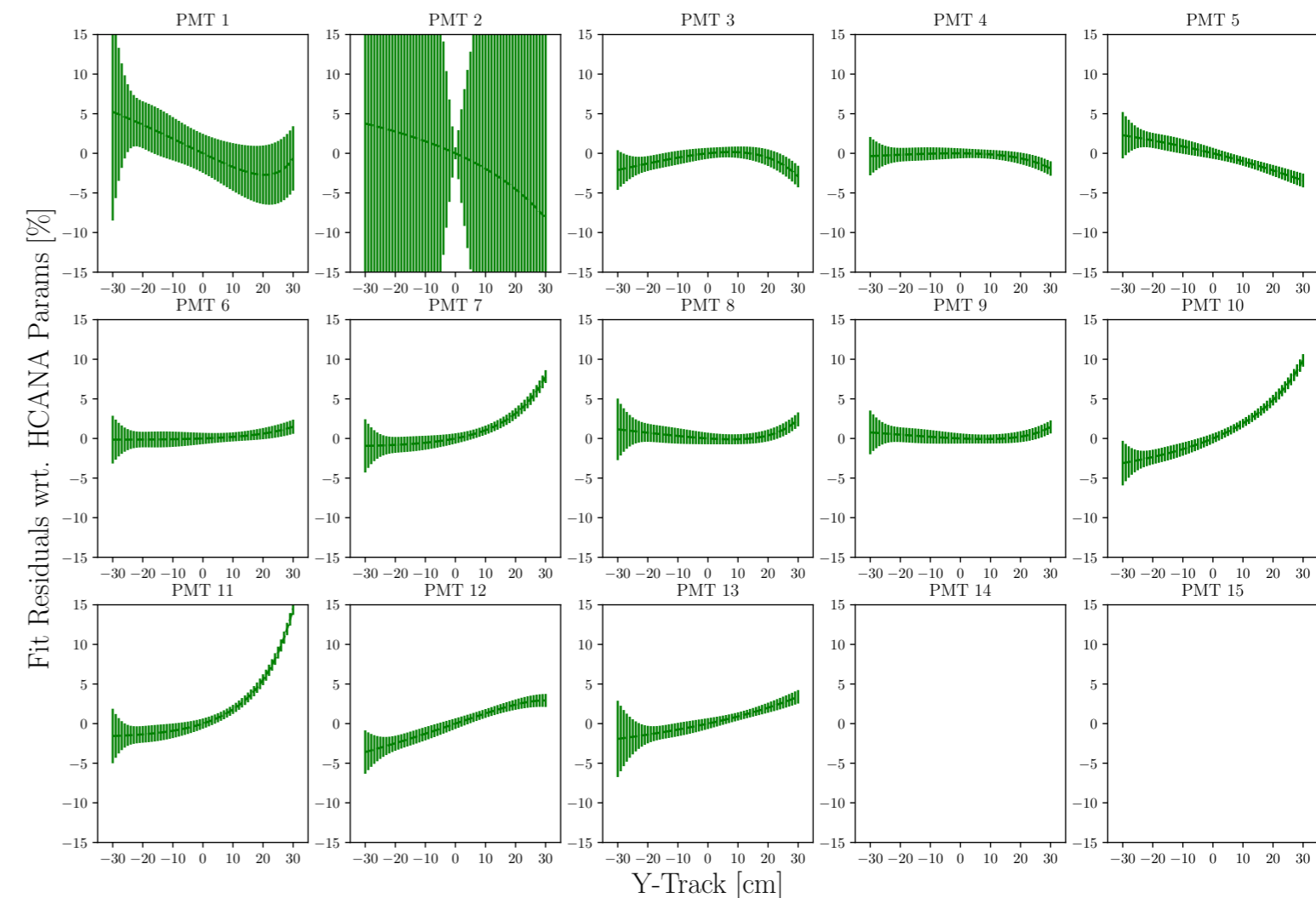




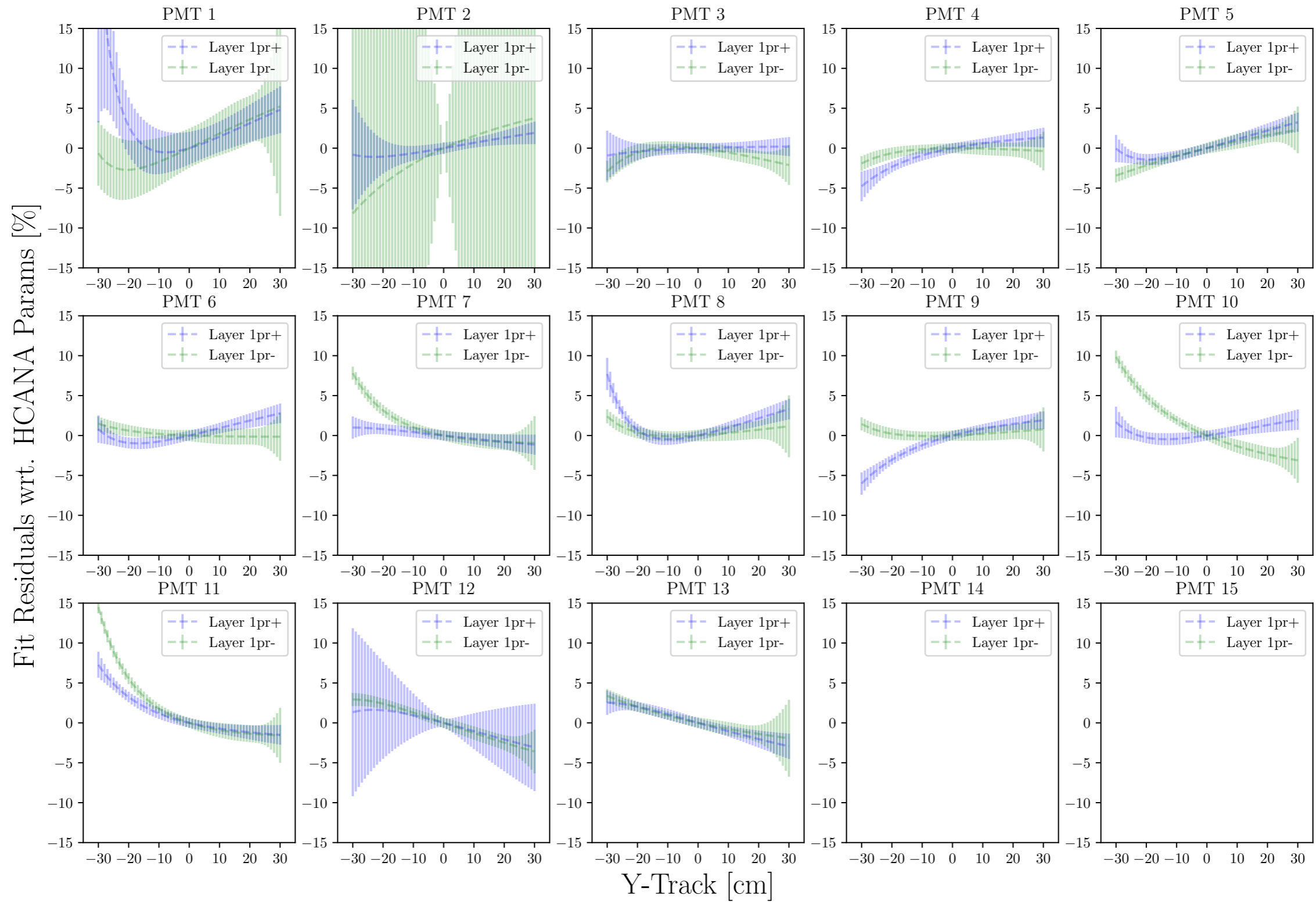
HMS Calorimeter Fit Residuals: Layer 1pr-

Layer 1pr -

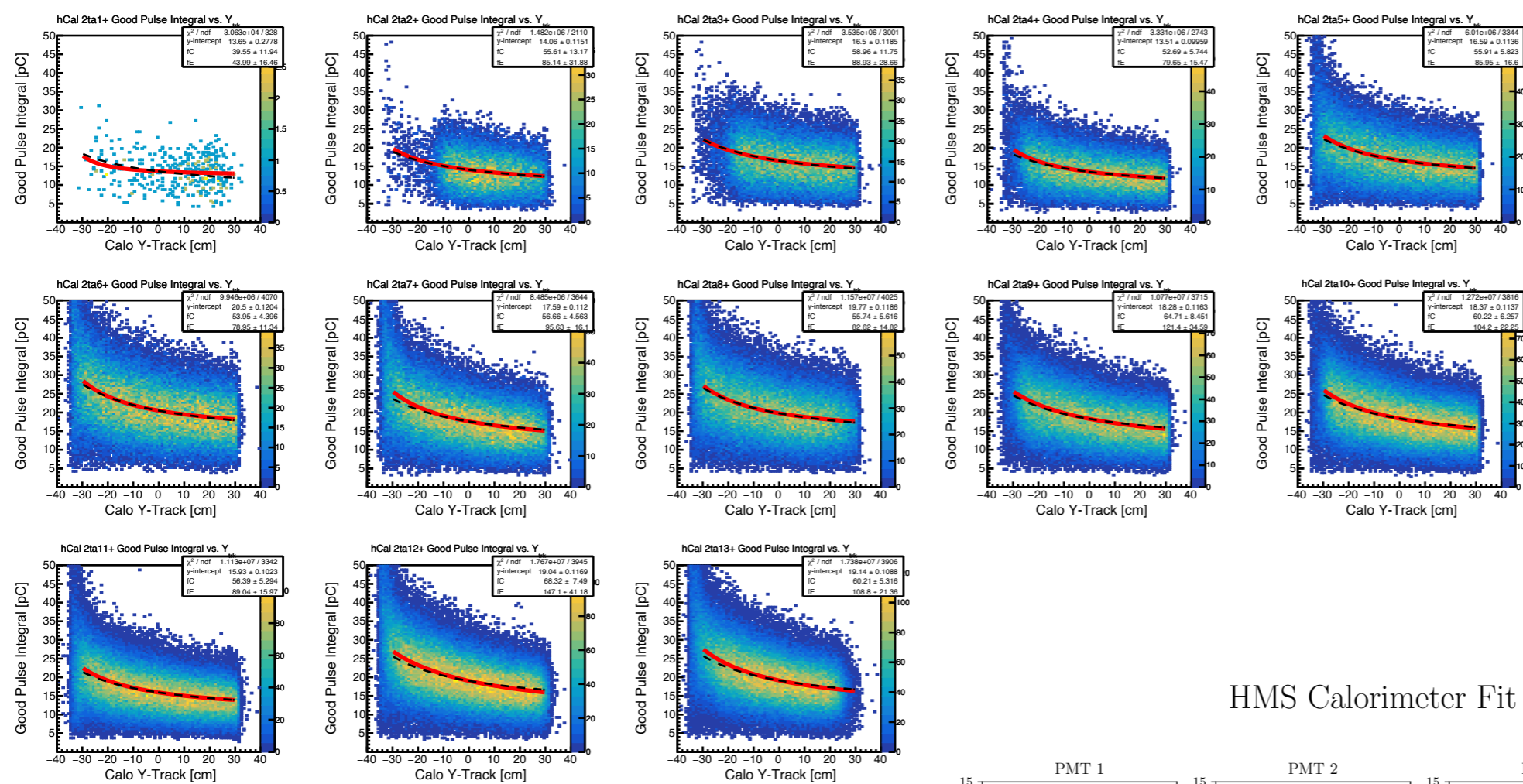
- Correlated errors are included
- X-axis range set to (-35, 35) cm
- Y-axis range set to (-15, 15) %



HMS Calorimeter Fit Residuals Overlay: Layer 1pr



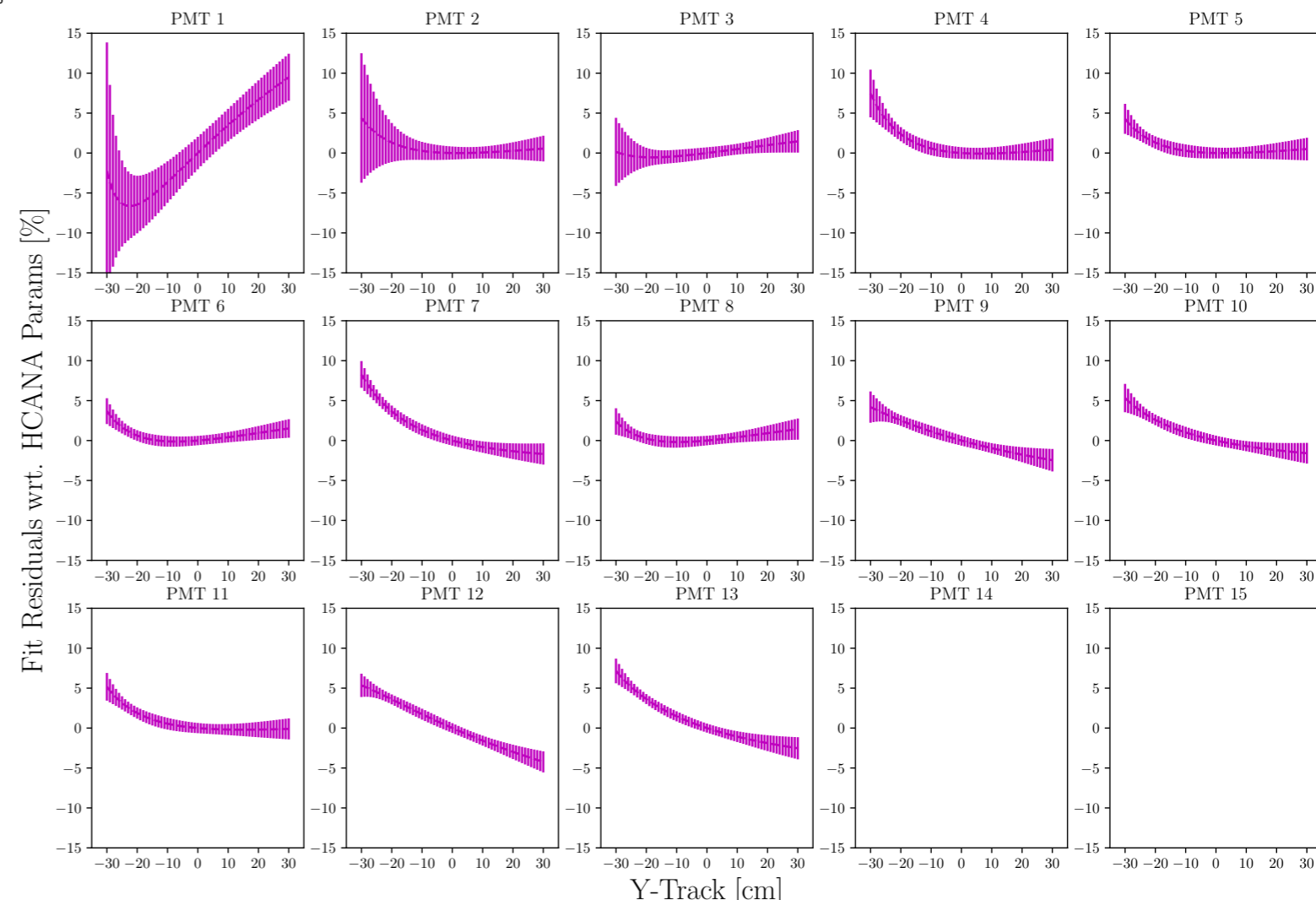
- Fit Residuals for Layer 1pr- have been reversed wrt. x-axis for a direct comparison of between both sides of the layer

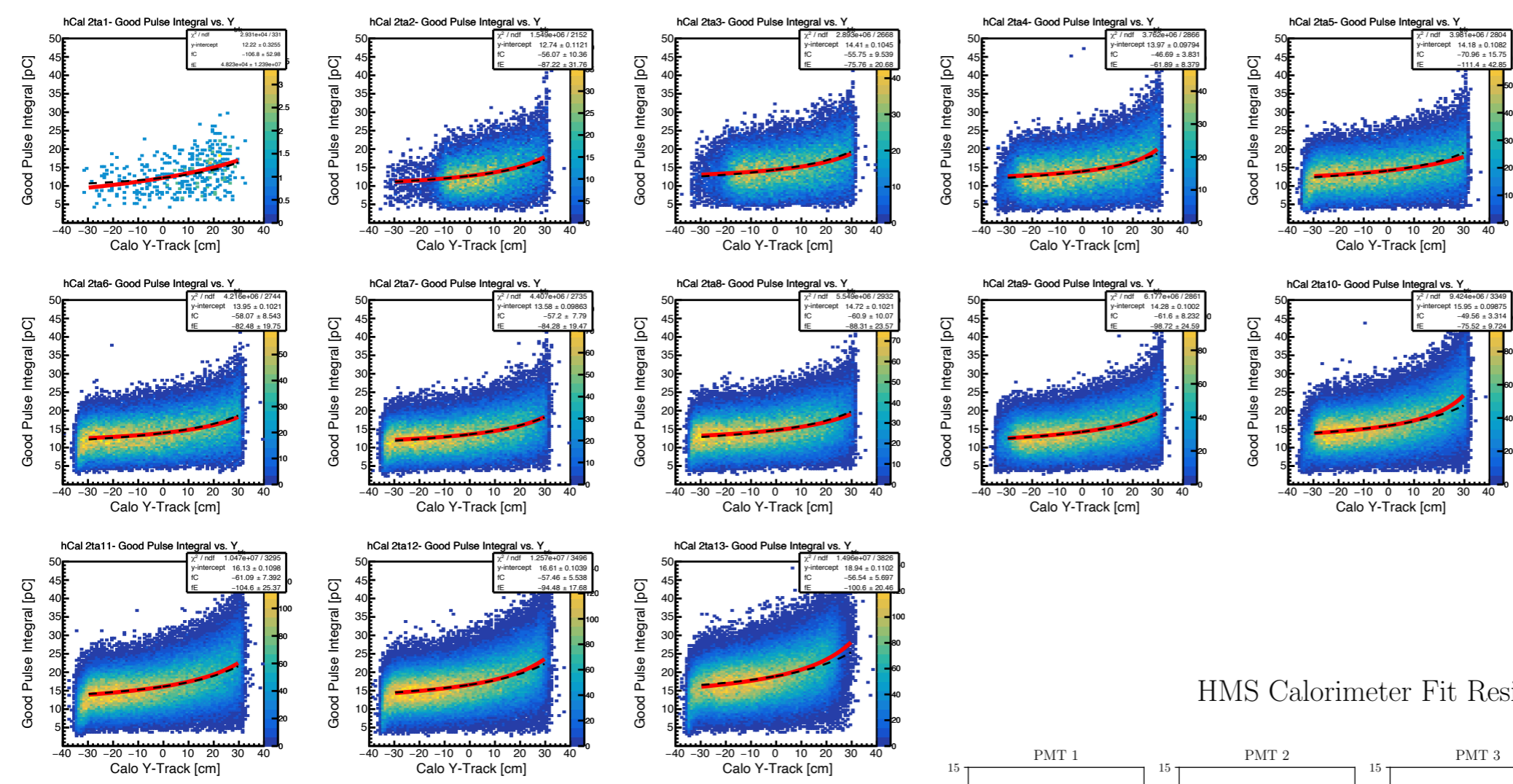


HMS Calorimeter Fit Residuals: Layer 2ta+

Layer 2ta+

- Correlated errors are included
- X-axis range set to (-35, 35) cm
- Y-axis range set to (-15, 15) %

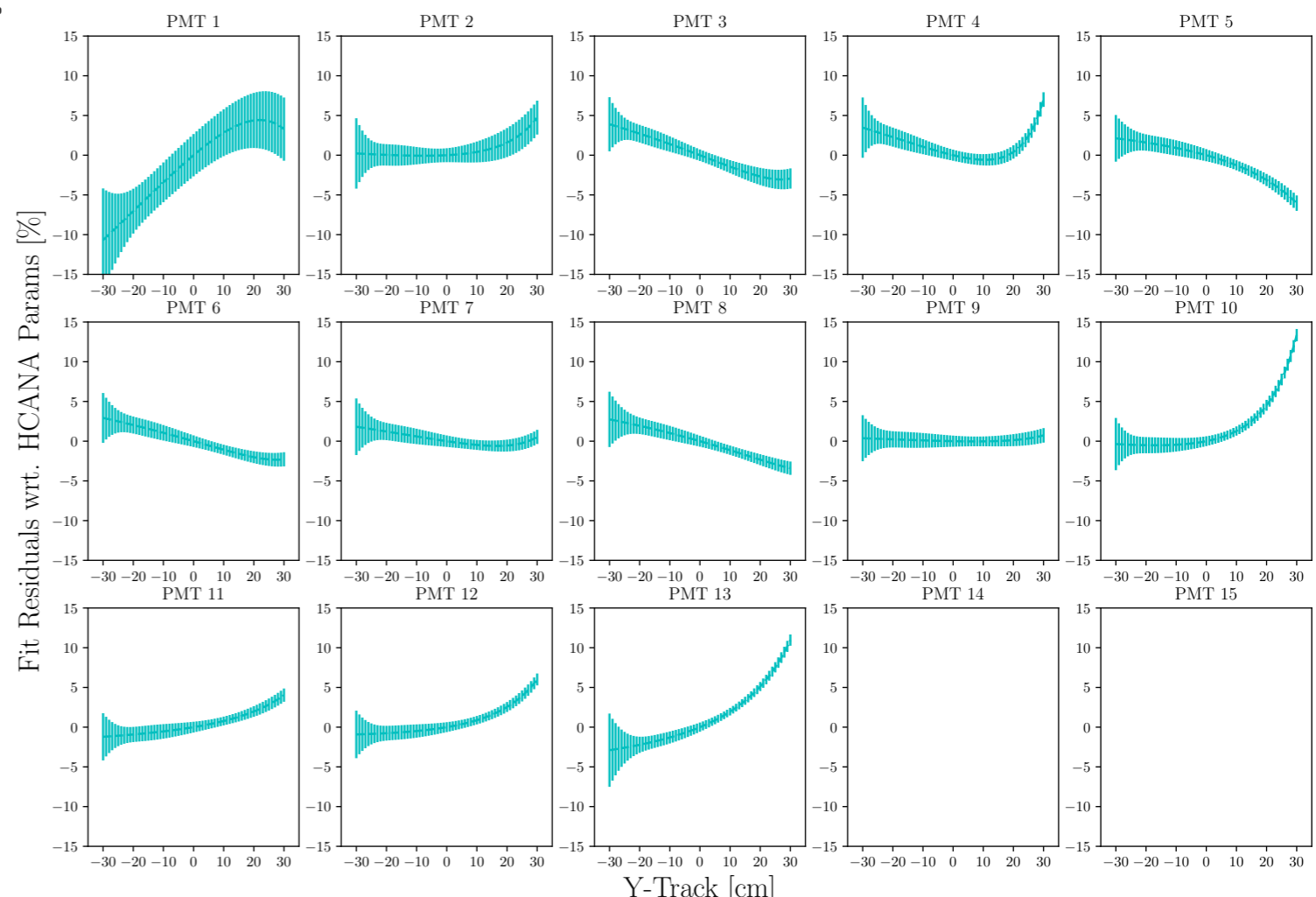




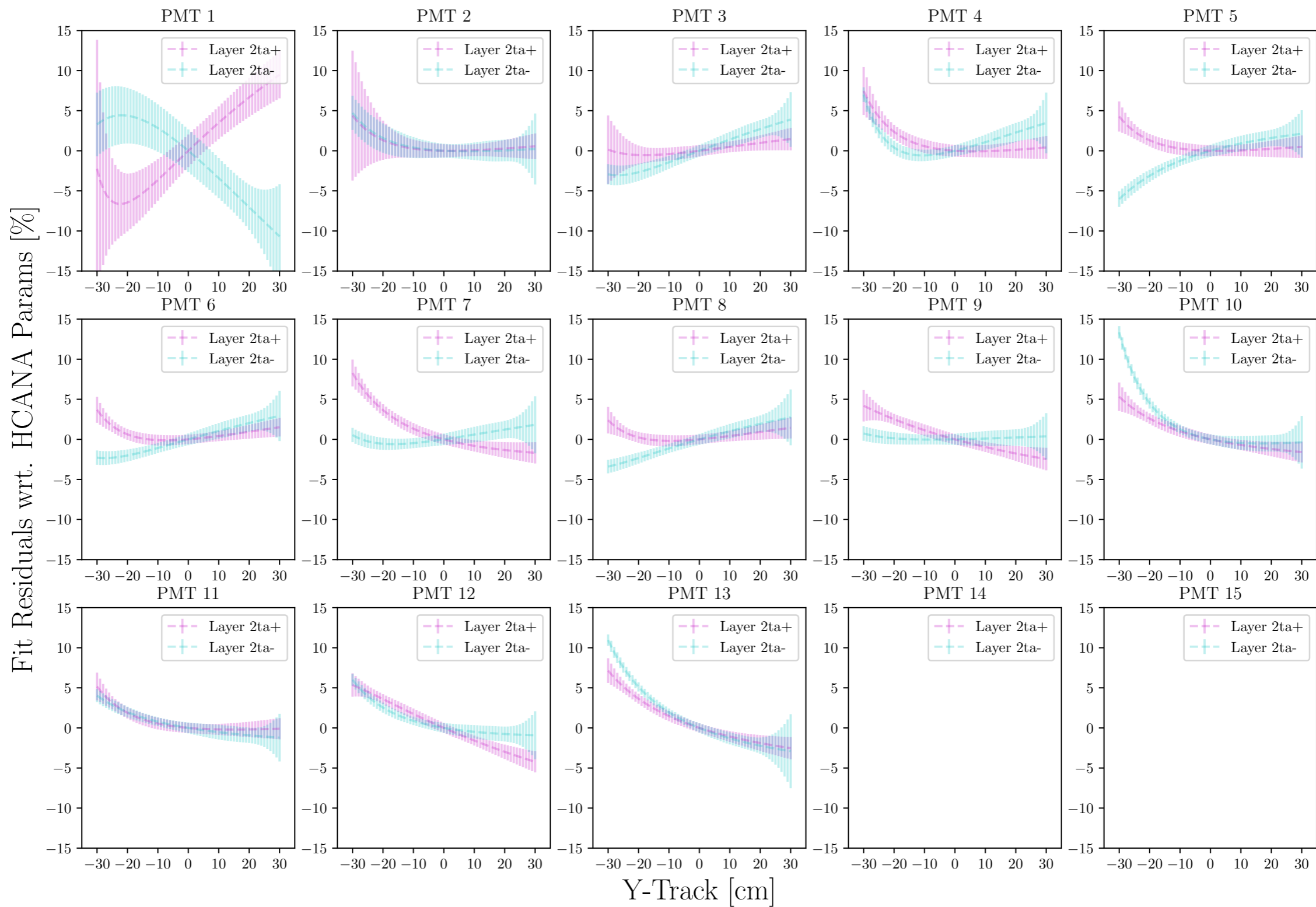
HMS Calorimeter Fit Residuals: Layer 2ta-

Layer 2ta-

- Correlated errors are included
- X-axis range set to (-35, 35) cm
- Y-axis range set to (-15, 15) %



HMS Calorimeter Fit Residuals Overlay: Layer 2ta

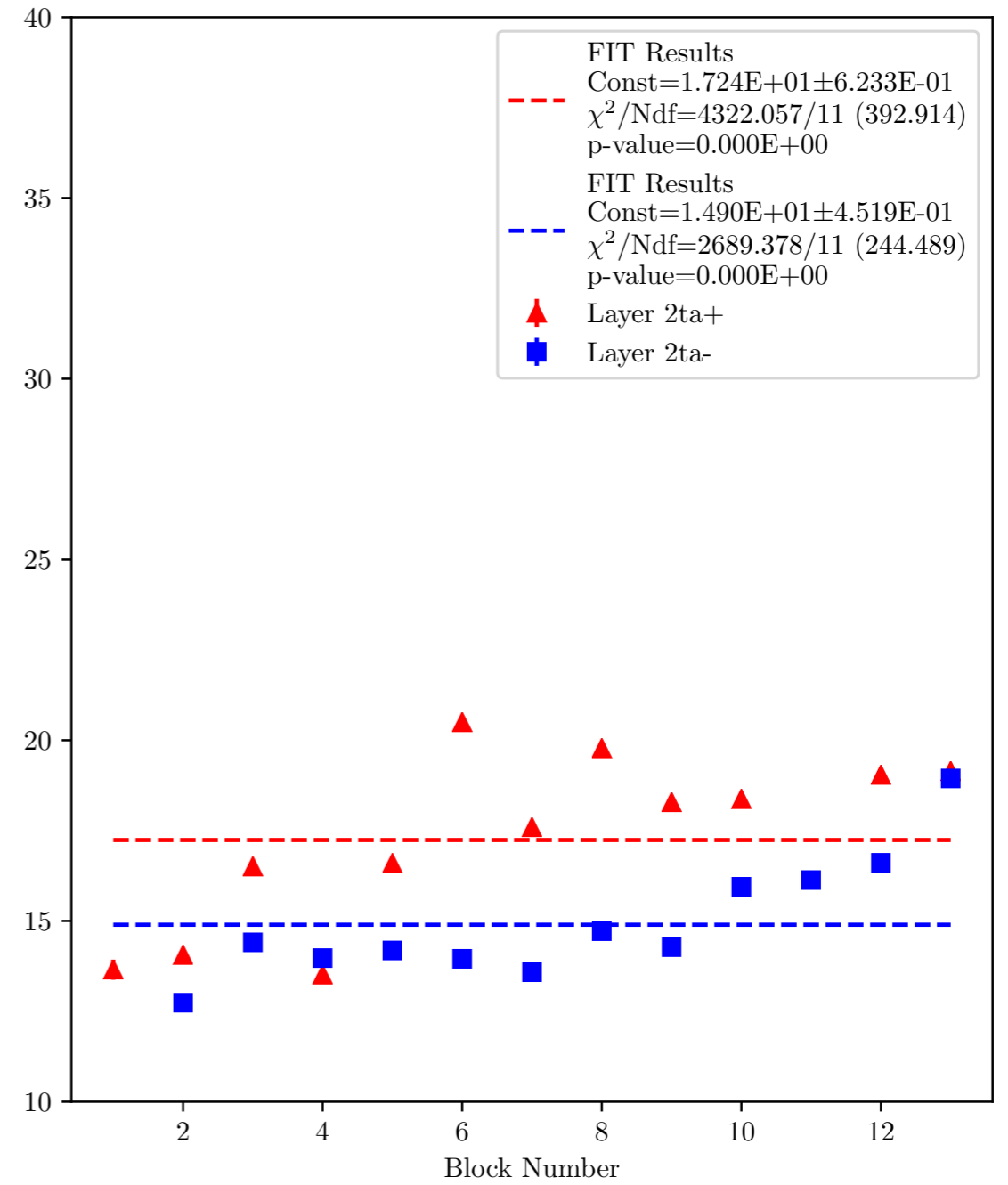
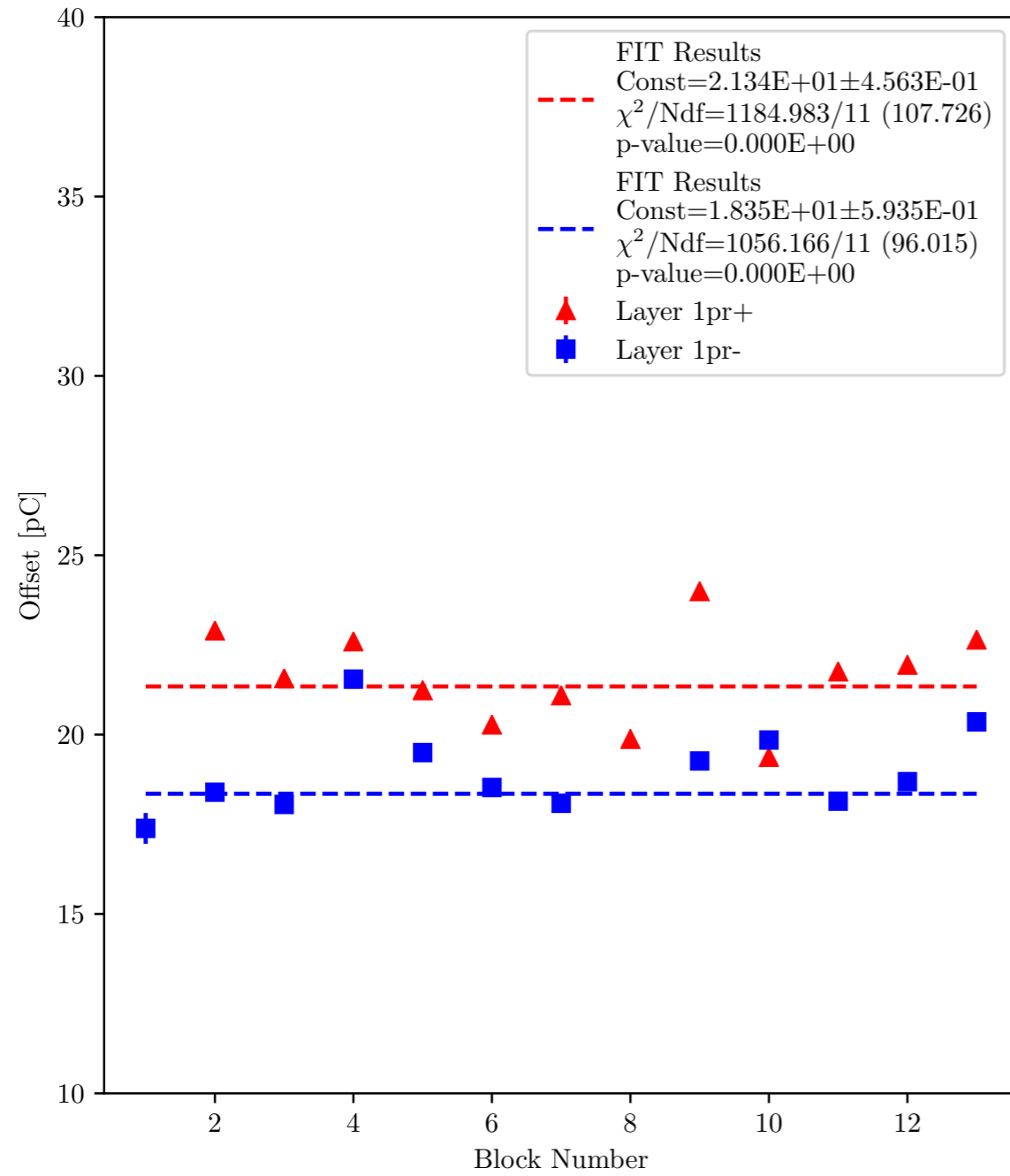


- Fit Residuals for Layer 2ta- have been reversed wrt. x-axis for a direct comparison of between both sides of the layer

FIT Results Using New Function

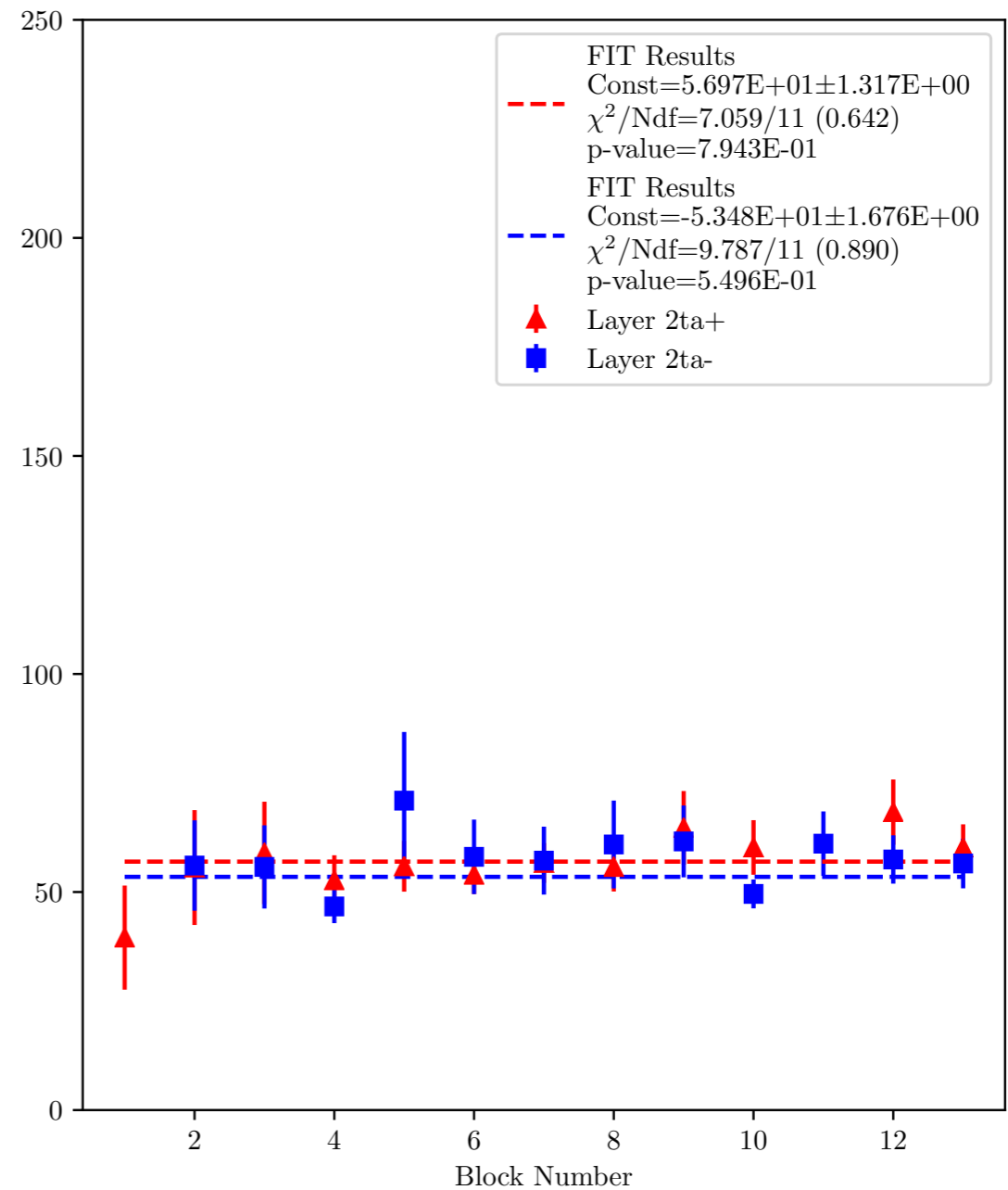
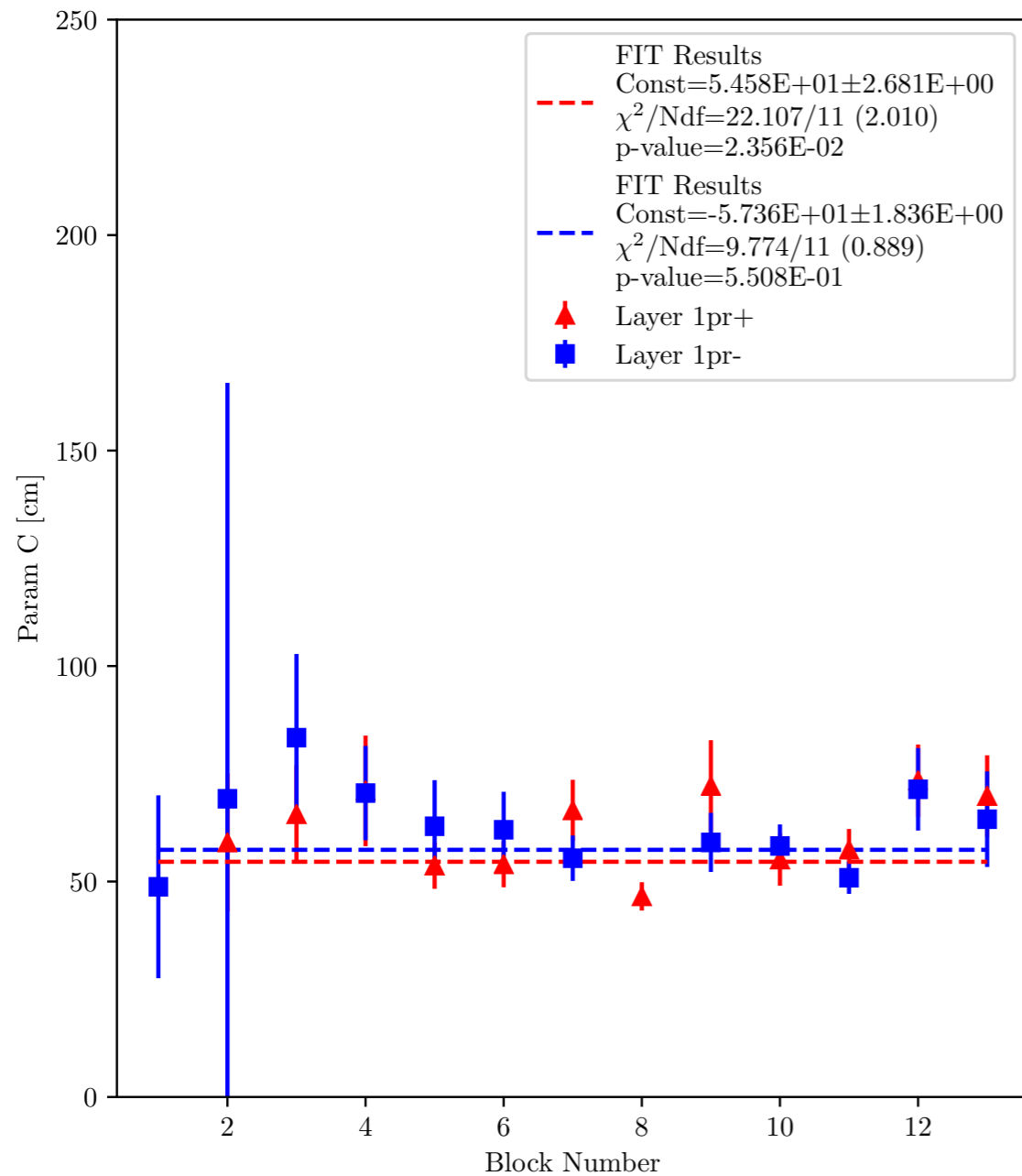
- The same PMTs as in the pion analysis have been MASKED
 - ◆ Layer 1+: 1
 - ◆ Layer 1-: 8
 - ◆ Layer 2+: 11
 - ◆ Layer 2-: 1

Good Pulse Integral vs. Y track: Offset Param Fit Results



○ First FIT Iteration (using new function)

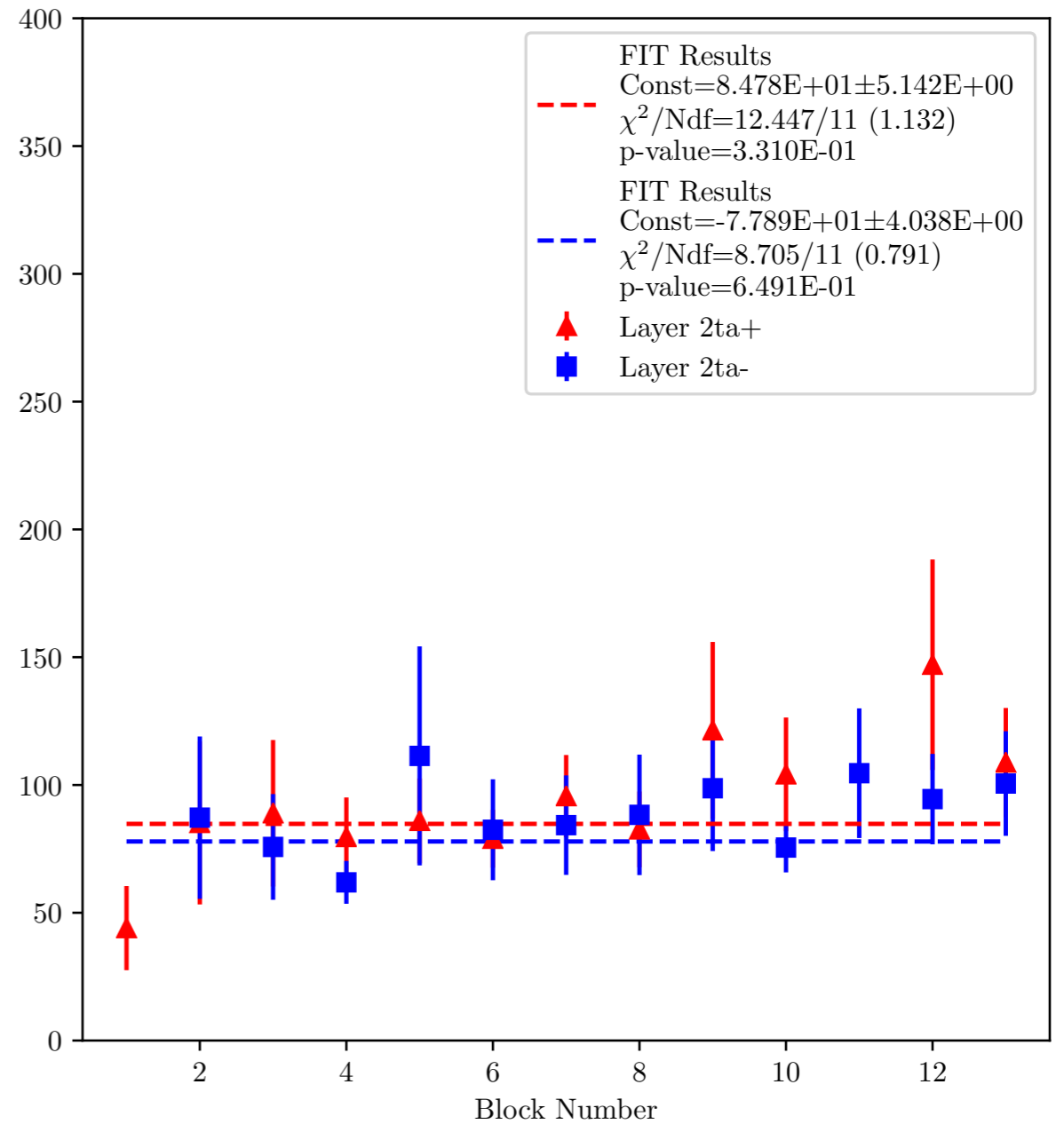
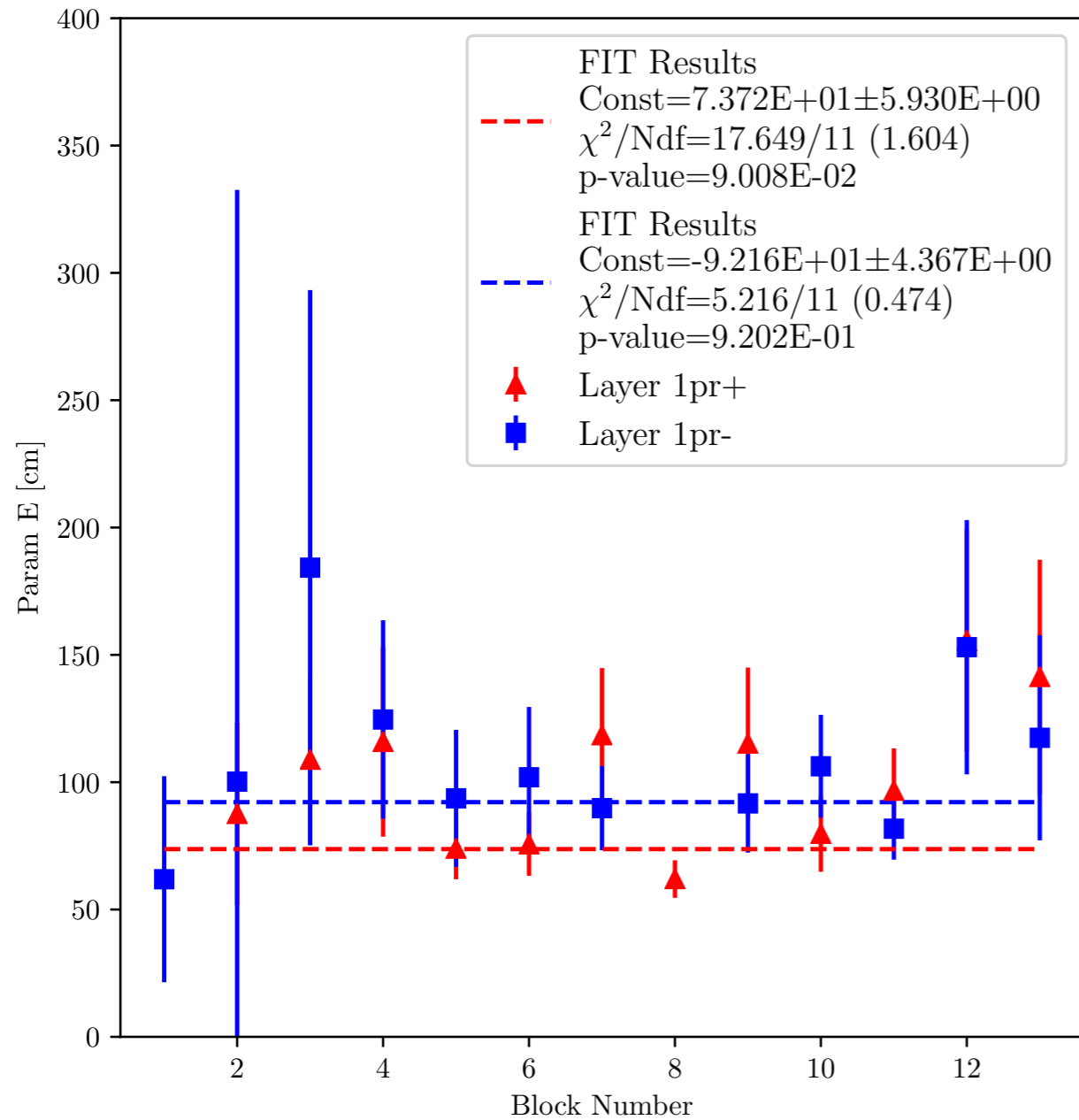
Good Pulse Integral vs. Y track: Param C Fit Results



○ First FIT Iteration (using new function)

◆ (negative layer sign is flipped for ease of comparison)

Good Pulse Integral vs. Y track: Param E Fit Results



○ First FIT Iteration (using new function)

◆ (negative layer sign is flipped for ease of comparison)