Fit Residuals
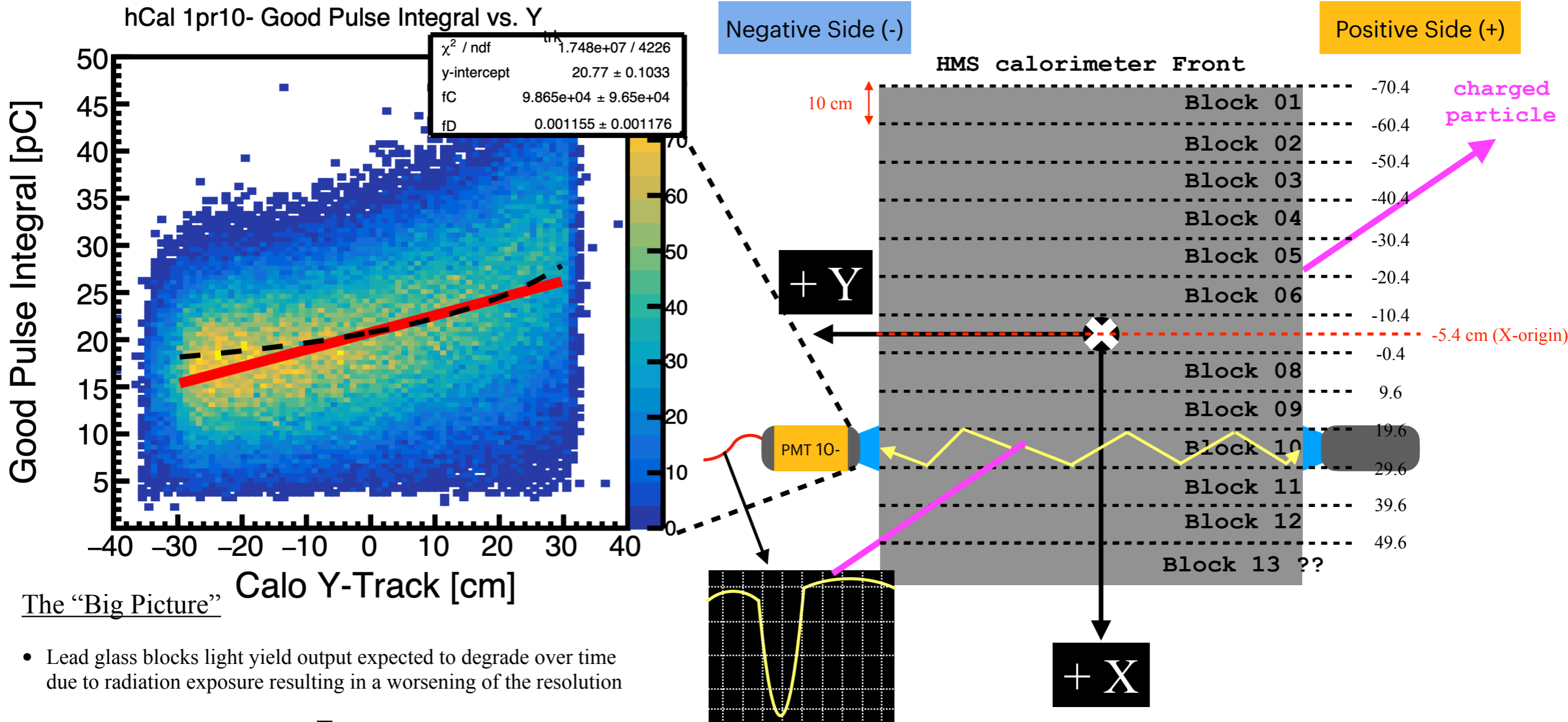
2nd Supplemental Plots for 1st Generation of Fit Studies
(using electrons)

C. Yero
D. Mack

May 13, 2021

1

hCal 1pr10- Good Pulse Integral vs. Y

χ^2 / ndf	1.748e+07 / 4226
y-intercept	20.77 ± 0.1033
fC	9.865e+04 ± 9.65e+04
fD	0.001155 ± 0.001176

Negative Side (-)

Positive Side (+)

HMS calorimeter Front

charged particle

10 cm

-70.4 Block 01
-60.4 Block 02
-50.4 Block 03
-40.4 Block 04
-30.4 Block 05
-20.4 Block 06
-10.4
-5.4 cm (X-origin)
-0.4 Block 08
9.6 Block 09
19.6 Block 10
29.6 Block 11
39.6 Block 12
49.6 Block 13 ??

+Y

PMT 10-

+X

The "Big Picture"

- Lead glass blocks light yield output expected to degrade over time due to radiation exposure resulting in a worsening of the resolution

- Expected resolution : $\sim 5\,\%/\sqrt{E}$ obtained from of width of gaussian fit to e- peak over various experiments
  - Main drivers for resolution: # of p.e. collected, non-uniformities in lead glass, small dead layers between blocks, etc.

- HCANA has hard-coded (per layer) parameters applied as a Y-coordinate correction factor (Ycor) to the total track energy
  Questions: *1. How well does HCANA fit works on recent (12 GeV era) data ?*
  2. How well does HCANA fit to data compare to our fit using (per pmt) optimized parameters ?
  3. Are the difference observed between HCANA and our fits significant enough to apply a PMT-basis correction ?
  4. By how much would a PMT-basis correction improve the resolution?

- We have focused on fitting only the Layers A and B (2-sided PMTs). We are currently working on Layers C and D (1-sided PMTs)

2

Original FIT Function

Fit Function:
(Norm. Light Yield)

$$f = A \cdot \frac{C \pm y/D}{C \pm y}$$

A : Vertical Offset
C, D : Additional Parameters

y : Y-Track Position [cm]

Data Fit:

- An initial guess of the fit parameters is made: $A = 15$ pC, $C = 64.36$ cm, $D = 1.66$

- The initial guess of C & D is EXACTLY the current parameters hardcoded in HCANA

HCANA "Model":

- Uses the inverse of the Fit Function (above) with hardcoded parameters: $C = 64.36$ cm, $D = 1.66$

- For the offset parameter, I used the final offset determined from the Data Fit
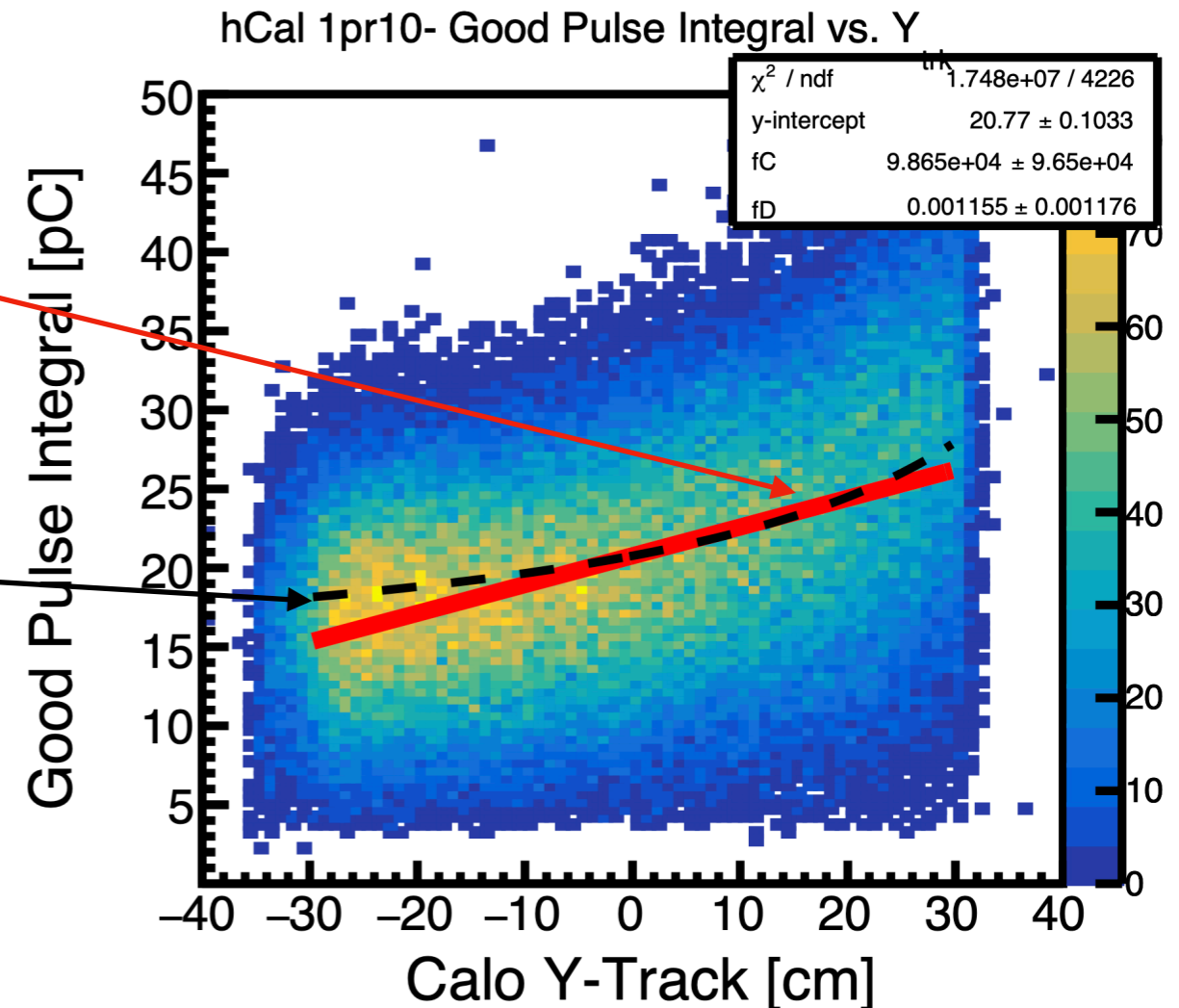
hCal 1pr10- Good Pulse Integral vs. Y

χ^2 / ndf	1.748e+07 / 4226
y-intercept	20.77 ± 0.1033
fC	9.865e+04 ± 9.65e+04
fD	0.001155 ± 0.001176

The Data Fit uses the hardcoded parameters as an initial guess, and the ROOT Minuit Algorithm determines the "best" set of parameters to describe the 2D correlation. The same Fit Function used in HCANA is also plotted to make a comparison.

NEW FIT Function

Fit Function:
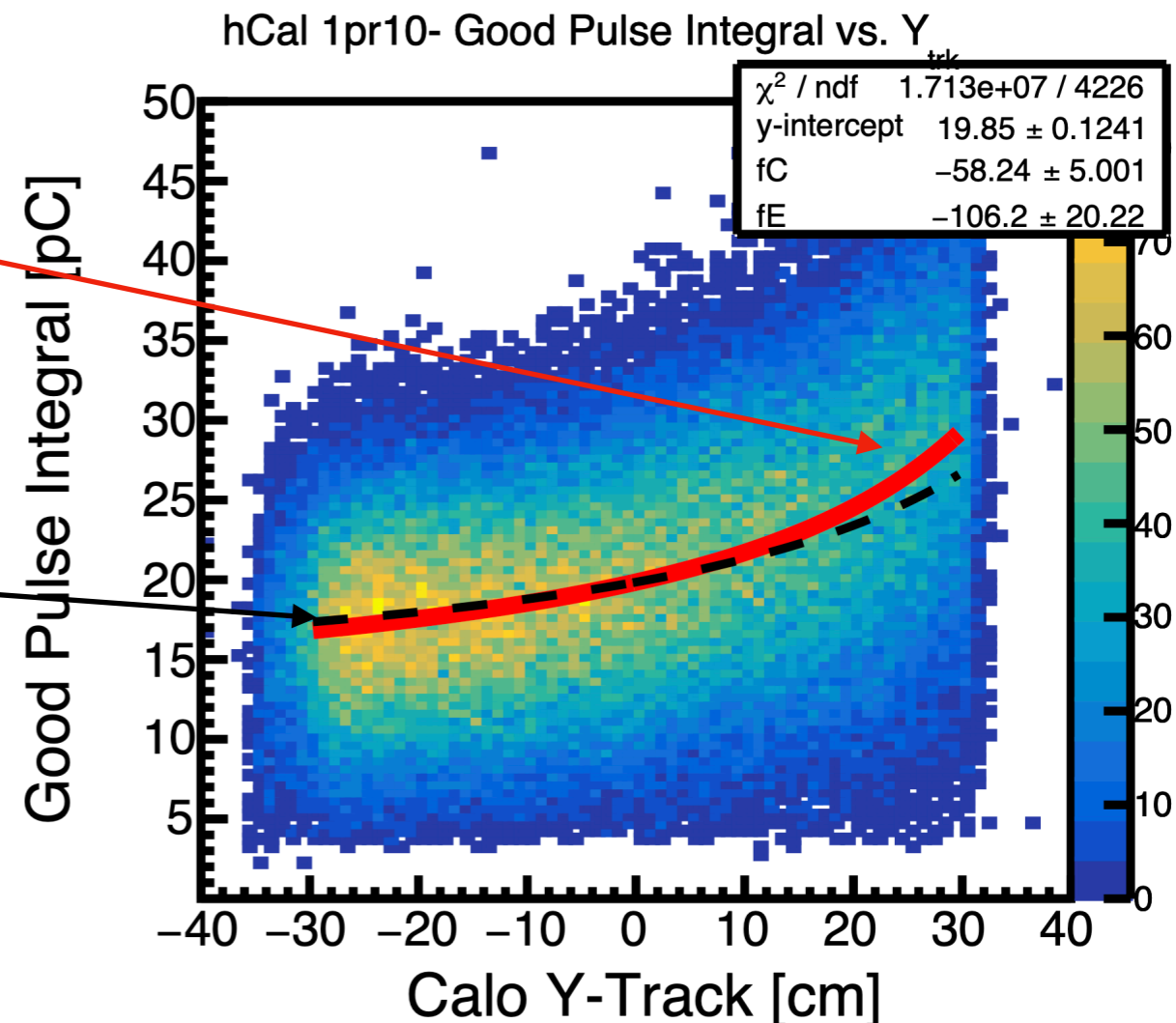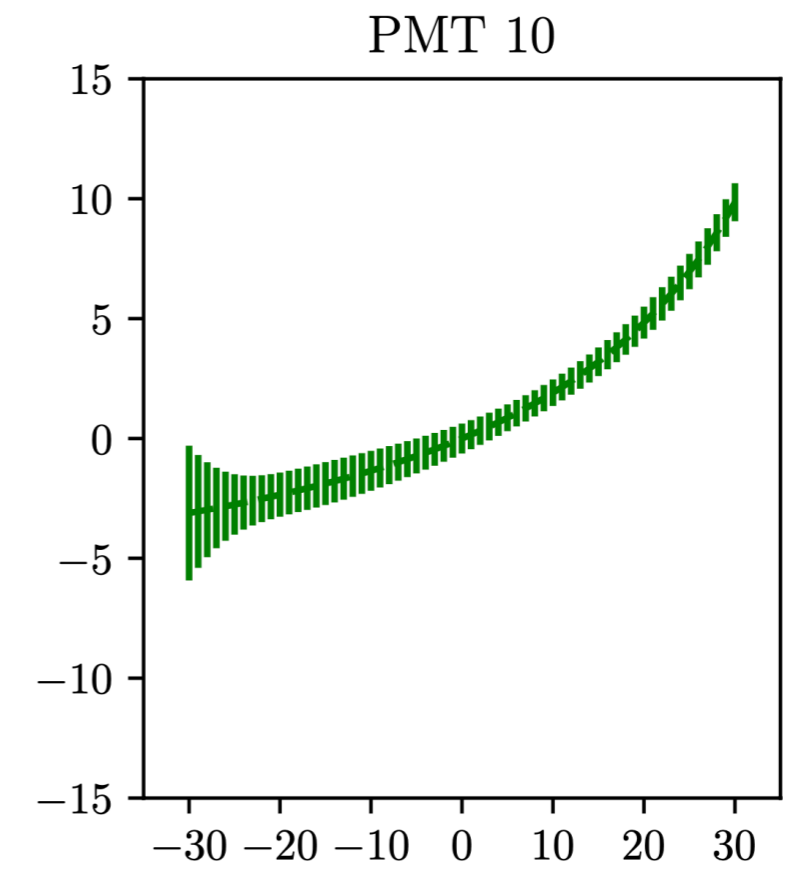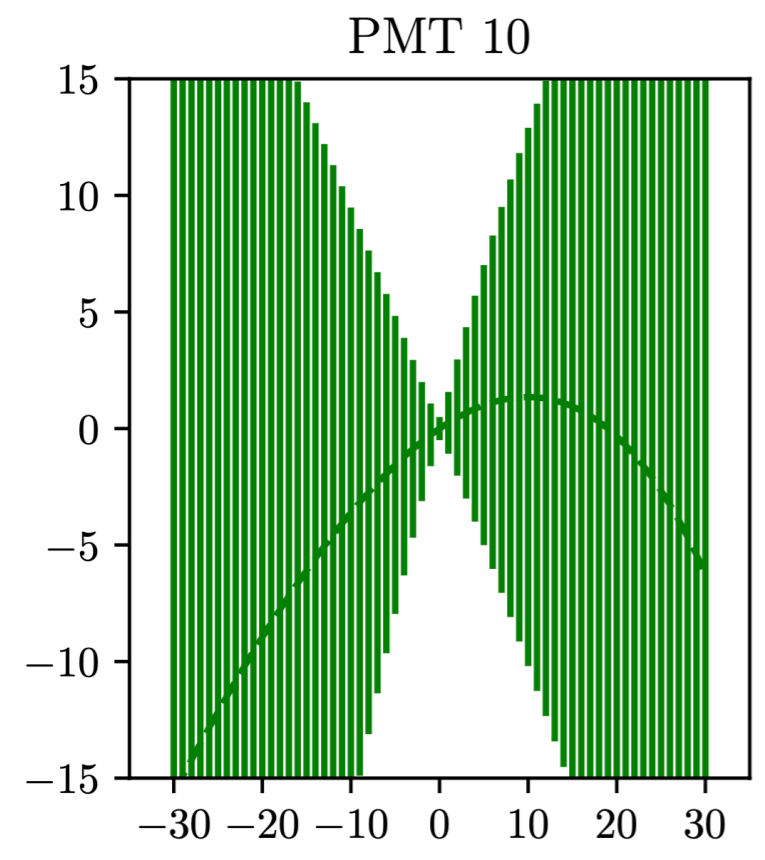(Norm. Yield)

$$f = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$

A : Vertical Offset
C, E : Additional Parameters

y : Y-Track Position [cm]

Data Fit:

- An initial guess of the fit parameters is made: $A = 15$ pC, $C = 64.36$ cm, $E = 106.83$ cm

- The initial guess of C is EXACTLY the current parameters hardcoded in HCANA And $E = CD$ (product of two hardcoded HCANA params)

HCANA "Model":

- Used the Fit Function (see above) with hardcoded parameters: $C = 64.36$ cm, $E = 106.83$ cm

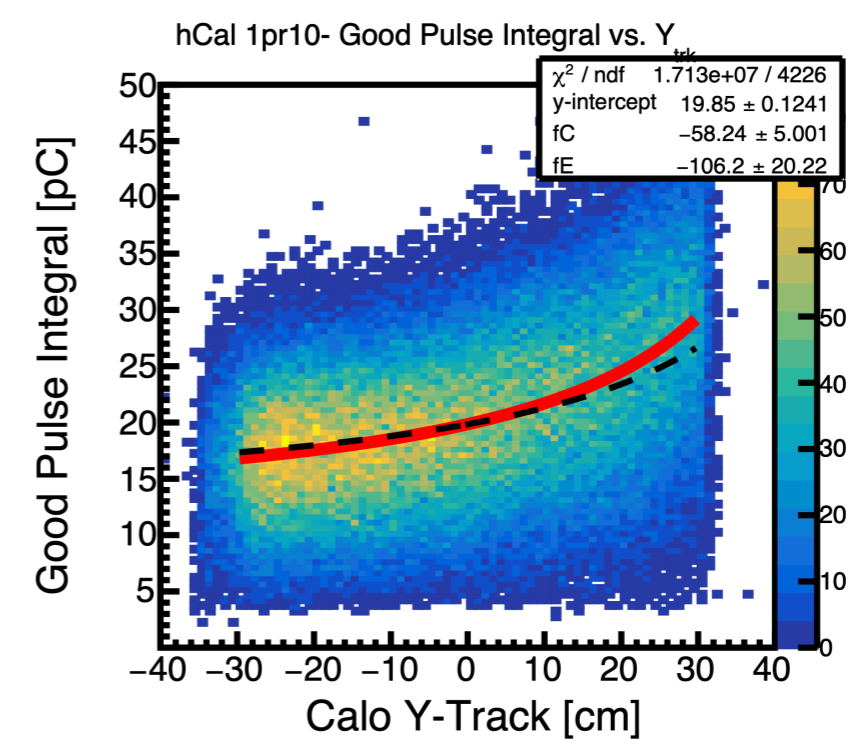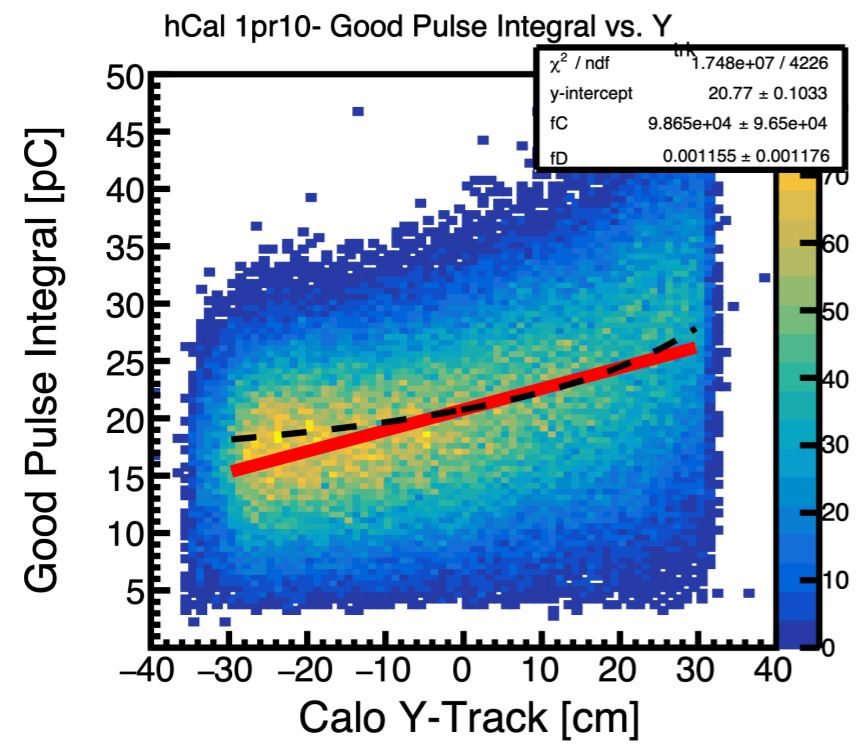- For the offset parameter, I used the final offset determined from the Data Fit

hCal 1pr10- Good Pulse Integral vs. Y$_{trk}$

χ^2 / ndf	1.713e+07 / 4226
y-intercept	19.85 ± 0.1241
fC	−58.24 ± 5.001
fE	−106.2 ± 20.22

The Data Fit uses the hardcoded parameters as an initial guess, and the ROOT Minuit Algorithm determines the "best" set of parameters to describe the 2D correlation. The same Fit Function used in HCANA is also plotted to make a comparison.

$$f = A \cdot \frac{C \pm y/D}{C \pm y}$$

$$f = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$

hCal 1pr10- Good Pulse Integral vs. Y

χ^2 / ndf	1.748e+07 / 4226
y-intercept	20.77 ± 0.1033
fC	9.865e+04 ± 9.65e+04
fD	0.001155 ± 0.001176

PMT 10

hCal 1pr10- Good Pulse Integral vs. Y

χ^2 / ndf	1.713e+07 / 4226
y-intercept	19.85 ± 0.1241
fC	−58.24 ± 5.001
fE	−106.2 ± 20.22

PMT 10

Residuals: Original FIT Function

$$R = \frac{f_{\text{dataFit}} - f_{\text{HCANA}}}{f_{\text{HCANA}}}$$

$$f_{\text{dataFit}} = A \cdot \frac{C \pm y/D}{C \pm y}$$

$$\delta R = \frac{\delta f_{\text{dataFit}}}{f_{\text{HCANA}}}$$

hCal 1pr11+ Good Pulse Integral vs. Y

χ^2 / ndf	2.426e+07 / 4539
y-intercept	21.75 ± 0.13
fC	57.38 ± 4.756
fD	1.684 ± 0.1493

$$\delta f^2_{\text{dataFit}} = \left(\frac{\partial f}{\partial A}\right)^2 \delta A^2 + \left(\frac{\partial f}{\partial C}\right)^2 \delta C^2 + \left(\frac{\partial f}{\partial D}\right)^2 \delta D^2$$

$$+ 2\frac{\partial^2 f}{\partial A \partial C} cov(A,C) + 2\frac{\partial^2 f}{\partial A \partial D} cov(A,D) + + 2\frac{\partial^2 f}{\partial C \partial D} cov(C,D)$$
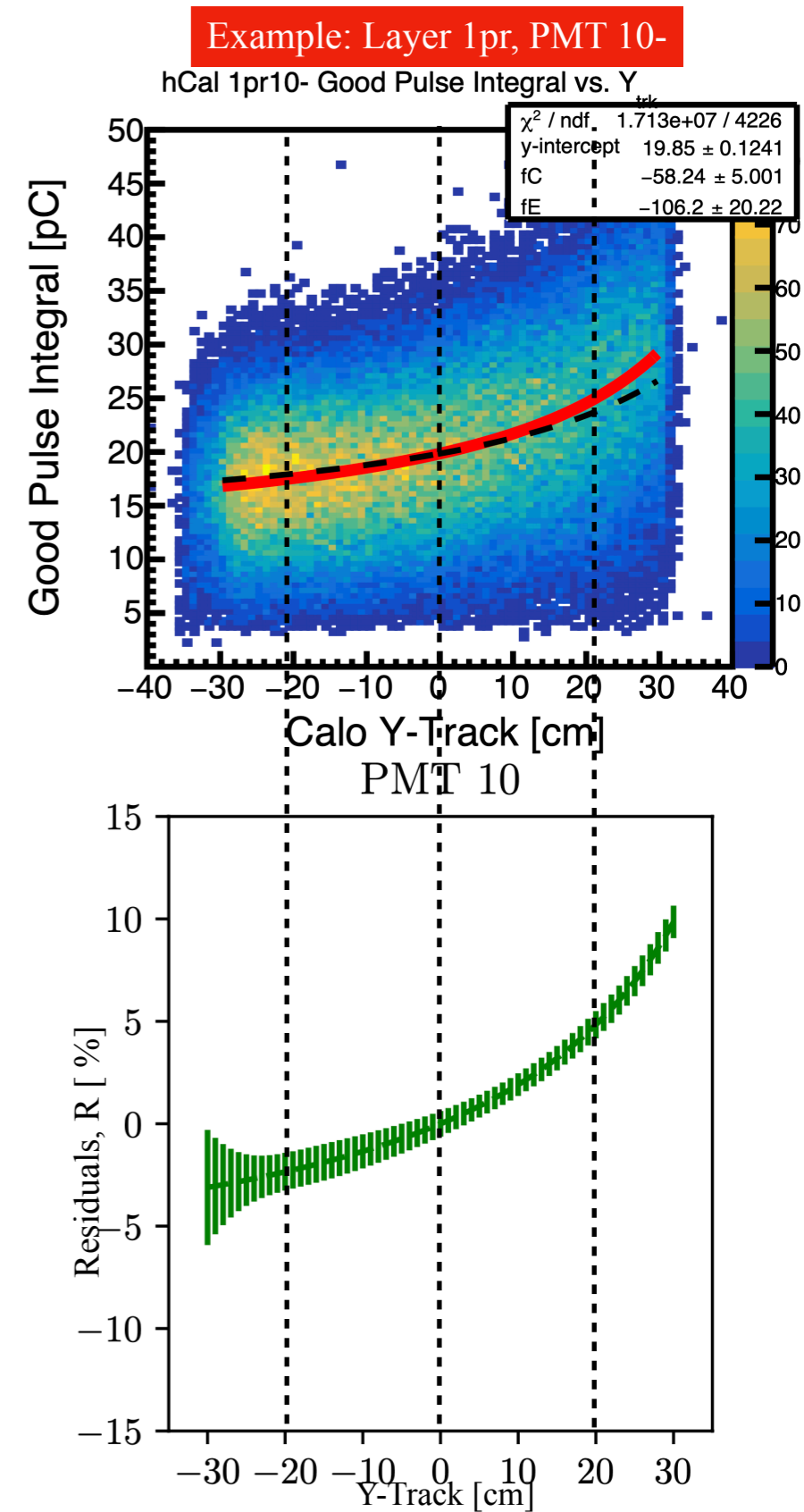
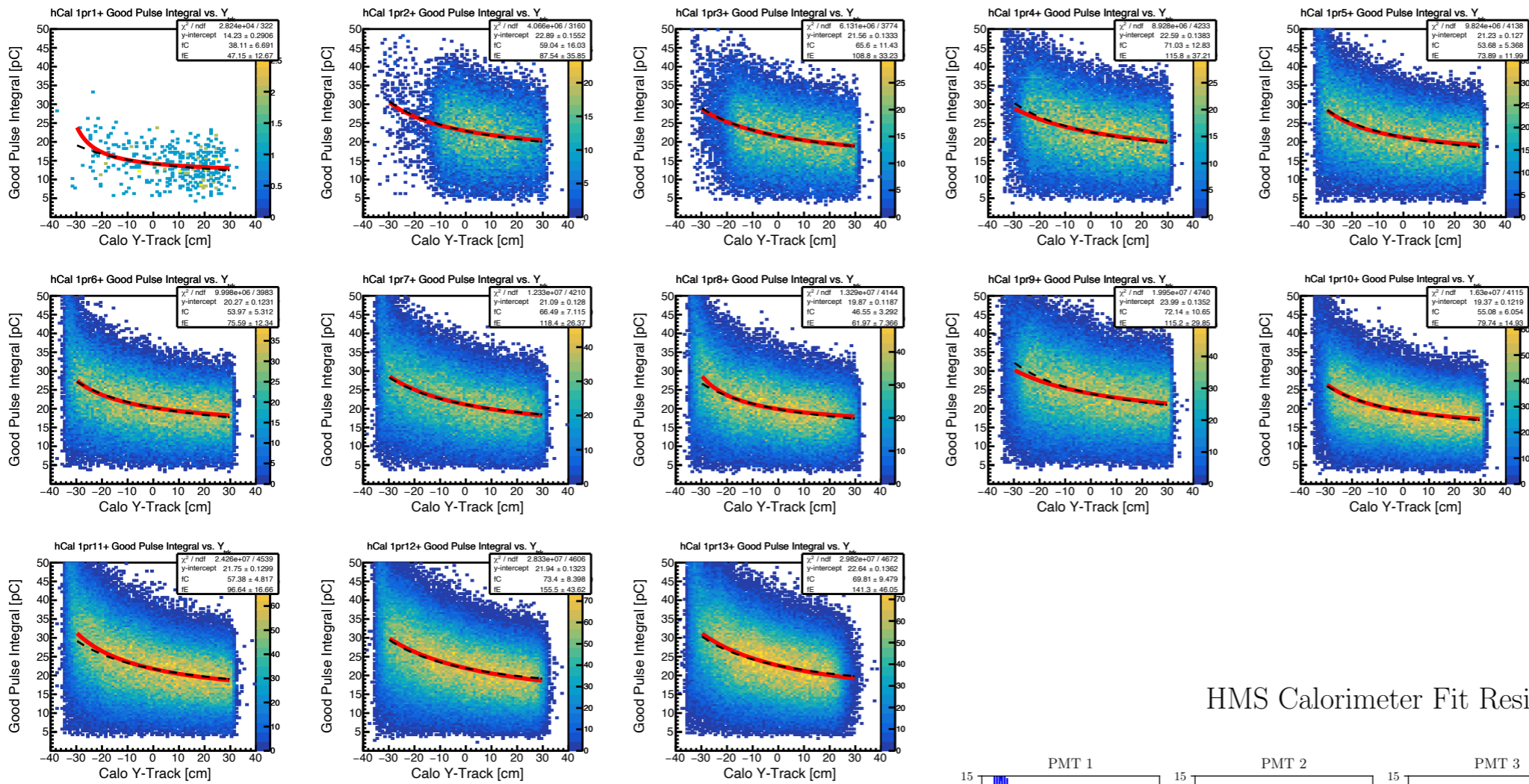$$\left(\frac{\partial f}{\partial A}\right) = \frac{C + y/D}{C + y} \qquad \left(\frac{\partial f}{\partial C}\right) = A\left(\frac{y(1 - 1/D)}{(C + y)^2}\right) \qquad \left(\frac{\partial f}{\partial D}\right) = -\frac{Ay}{D^2(C + y)}$$

Standard Error Propagation of the Data Fit
(errors in fit parameters are used)

NOTE: I dropped the subscript 'dataFit'
for simplicity

PMT 11

Interpretation: In this example, there is up to ~ 7.5 % difference between our studies and HCANA on the lower one end of the block at ~ -30 cm

Residuals: NEW FIT Function

$$R = \frac{f_{\text{dataFit}} - f_{\text{HCANA}}}{f_{\text{HCANA}}}$$

$$f_{\text{dataFit}} = A \cdot \frac{1 \pm y/E}{1 \pm y/C}$$

$$\delta R = \frac{\delta f_{\text{dataFit}}}{f_{\text{HCANA}}}$$

$$\delta f^2_{\text{dataFit}} = \left(\frac{\partial f}{\partial A}\right)^2 \delta A^2 + \left(\frac{\partial f}{\partial C}\right)^2 \delta C^2 + \left(\frac{\partial f}{\partial E}\right)^2 \delta E^2$$

$$+ 2\frac{\partial^2 f}{\partial A \partial C} cov(A,C) + 2\frac{\partial^2 f}{\partial A \partial E} cov(A,E) + 2\frac{\partial^2 f}{\partial C \partial E} cov(C,E)$$

$$\left(\frac{\partial f}{\partial A}\right) = \frac{1 \pm y/E}{1 \pm y/C} \qquad \left(\frac{\partial f}{\partial C}\right) = \pm A \cdot \frac{(1 \pm y/E)(y/C^2)}{(1 \pm y/C)^2} \qquad \left(\frac{\partial f}{\partial E}\right) = \pm A \cdot \frac{-y/E^2}{1 \pm y/C}$$

Standard Error Propagation of the Data Fit
(errors in fit parameters are used)

NOTE: I dropped the subscript 'dataFit'
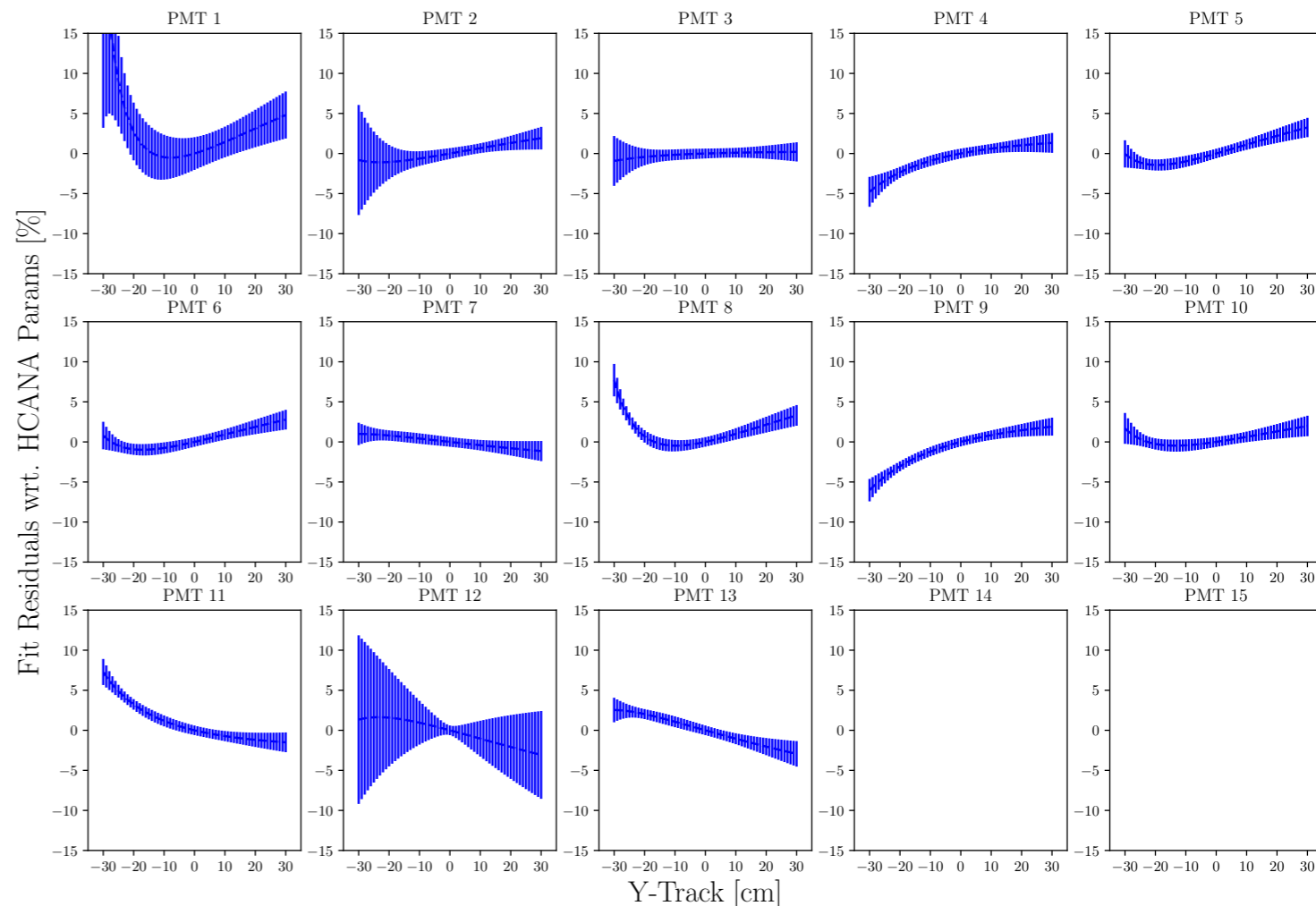for simplicity

Example: Layer 1pr, PMT 10-

hCal 1pr10- Good Pulse Integral vs. Y$_{trk}$

χ^2 / ndf	1.713e+07 / 4226
y-intercept	19.85 ± 0.1241
fC	−58.24 ± 5.001
fE	−106.2 ± 20.22

Good Pulse Integral [pC] vs. Calo Y-Track [cm]

PMT 10

Residuals, R [%] vs. Y-Track [cm]

Interpretation: In this example, there is up to ~ 10 % difference between our studies and HCANA on the upper end of the block at ~ +30 cm

Layer 1pr +

- Correlated errors are included
- X-axis range set to (-35, 35) cm
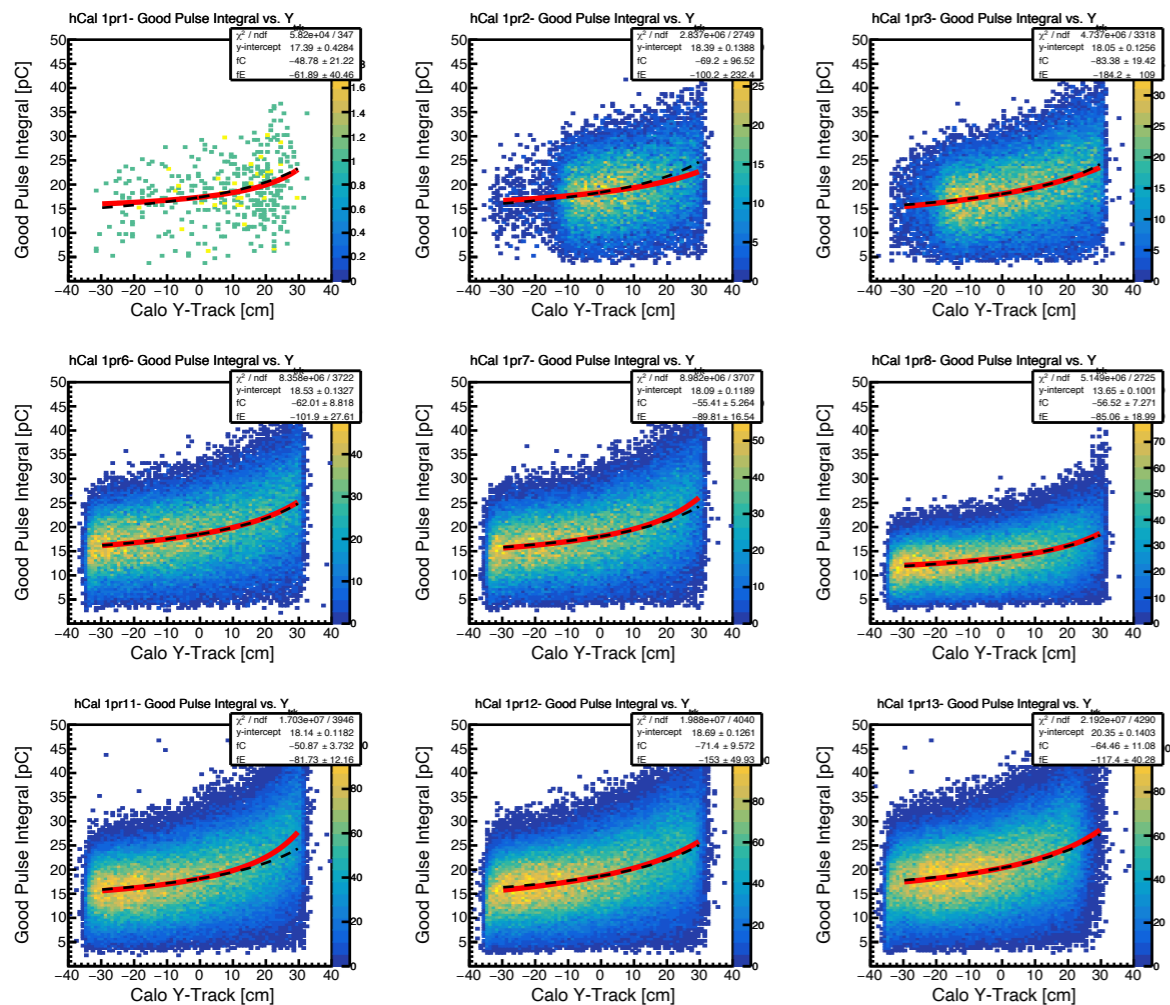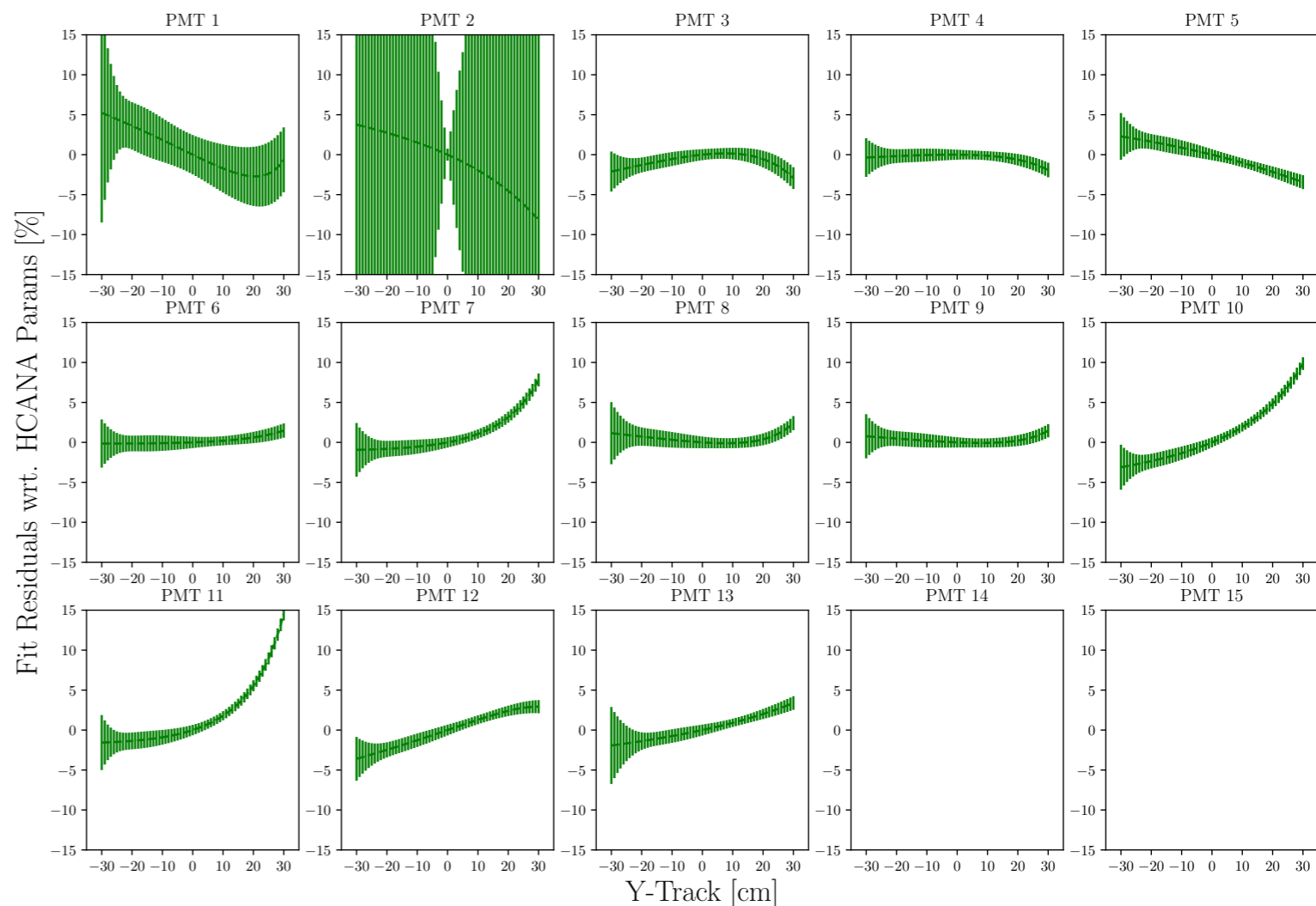- Y-axis range set to (-15, 15) %

HMS Calorimeter Fit Residuals: Layer 1pr+

Layer 1pr -

- Correlated errors are included
- X-axis range set to (-35, 35) cm
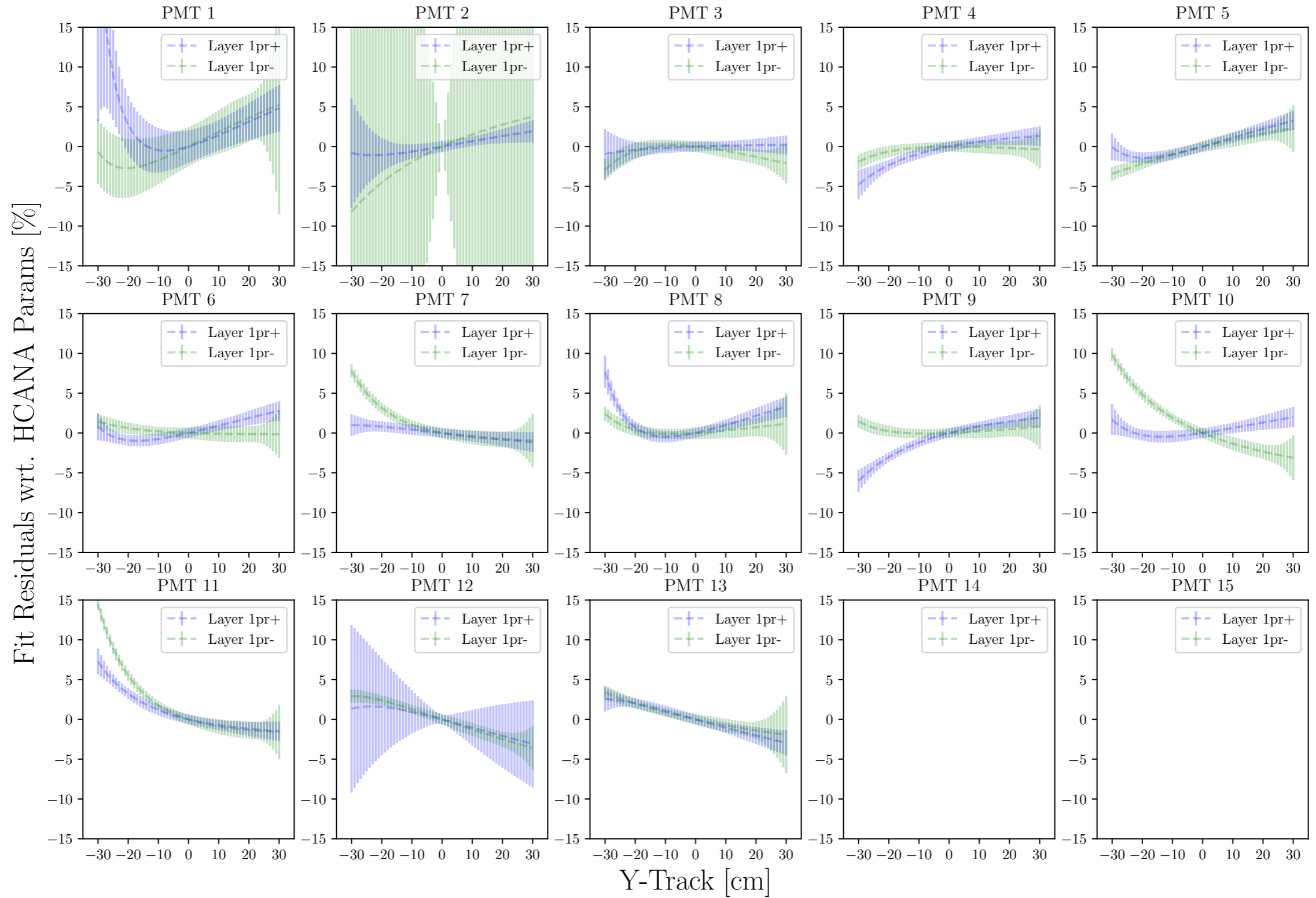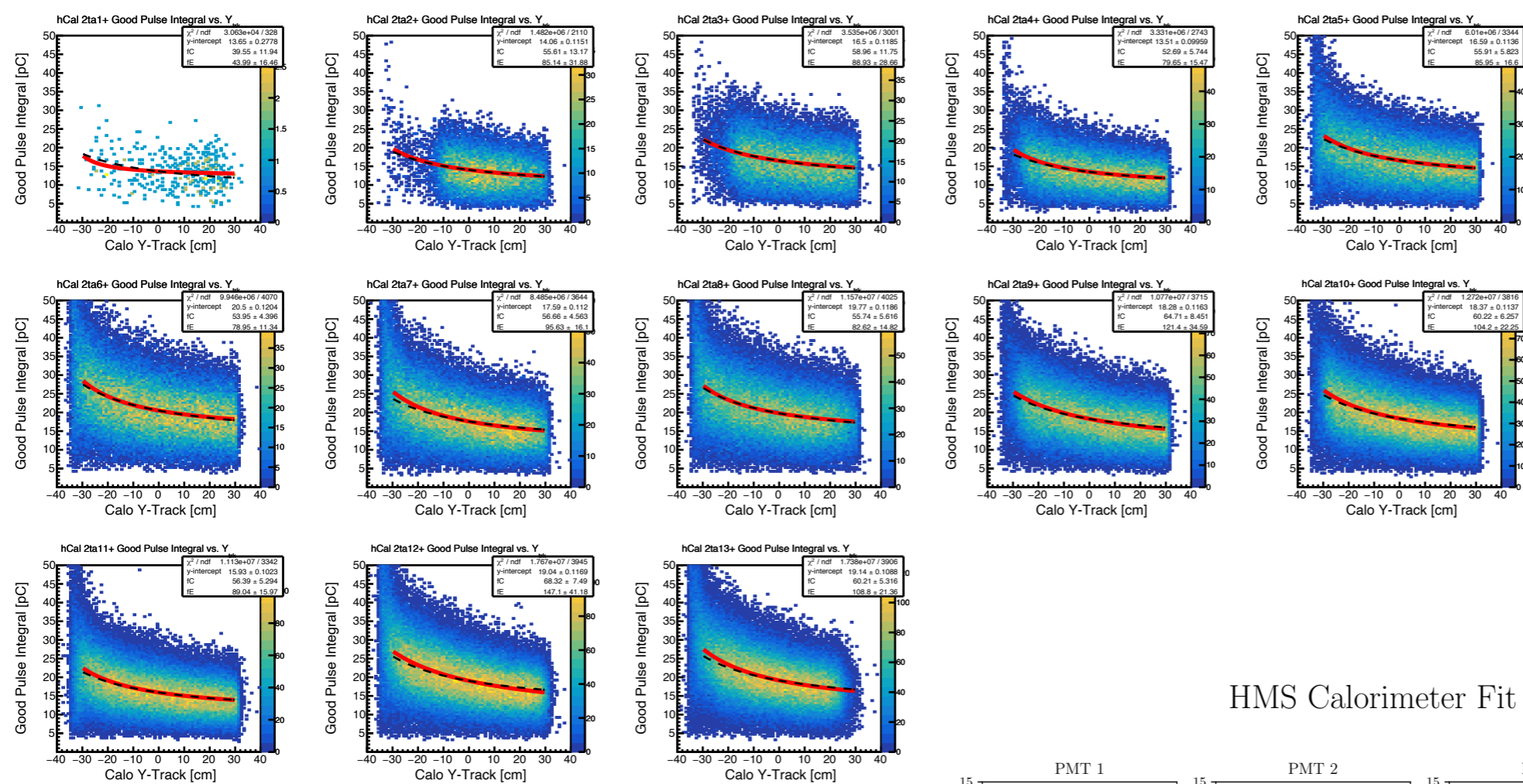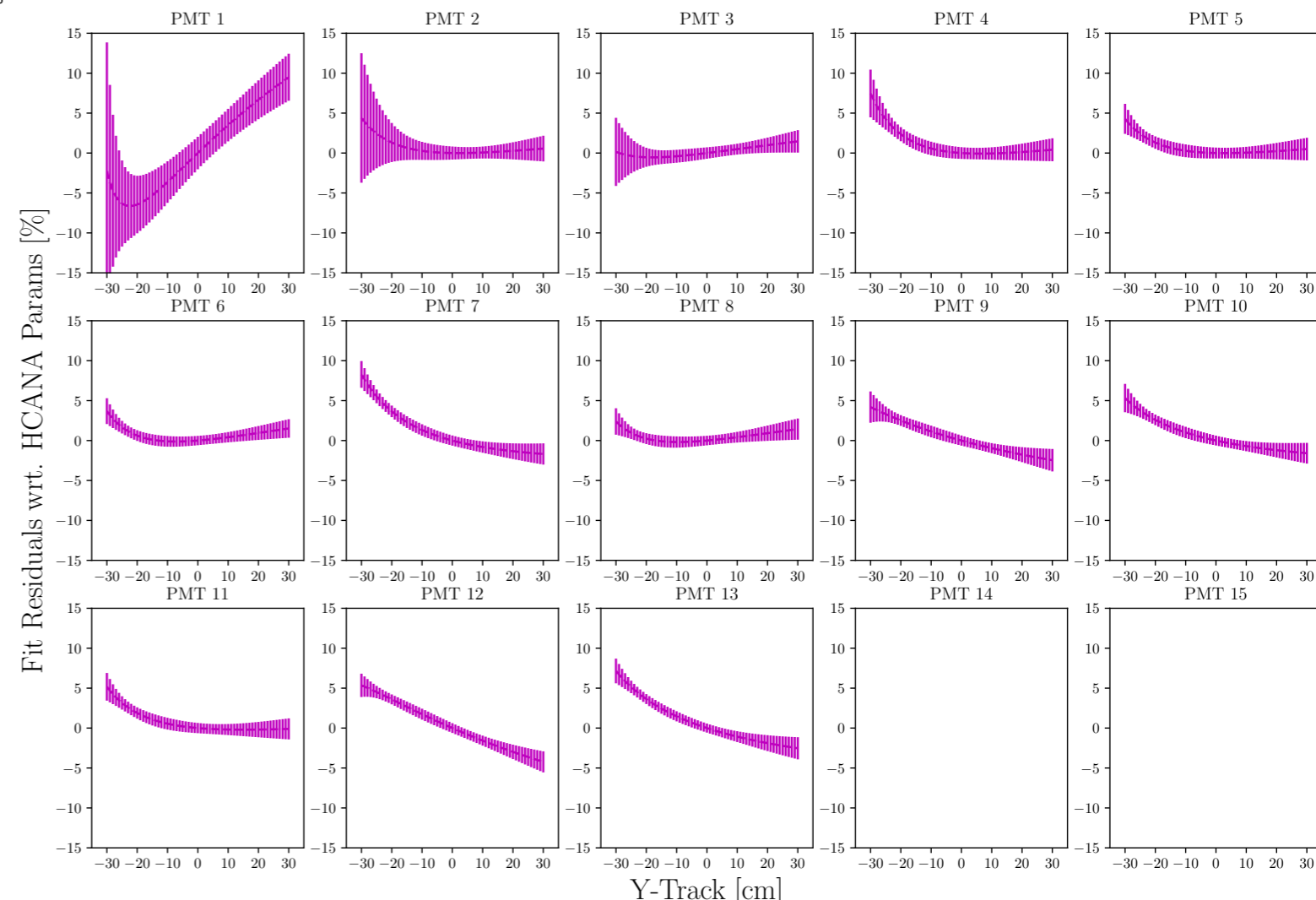- Y-axis range set to (-15, 15) %

HMS Calorimeter Fit Residuals: Layer 1pr-

HMS Calorimeter Fit Residuals Overlay: Layer 1pr

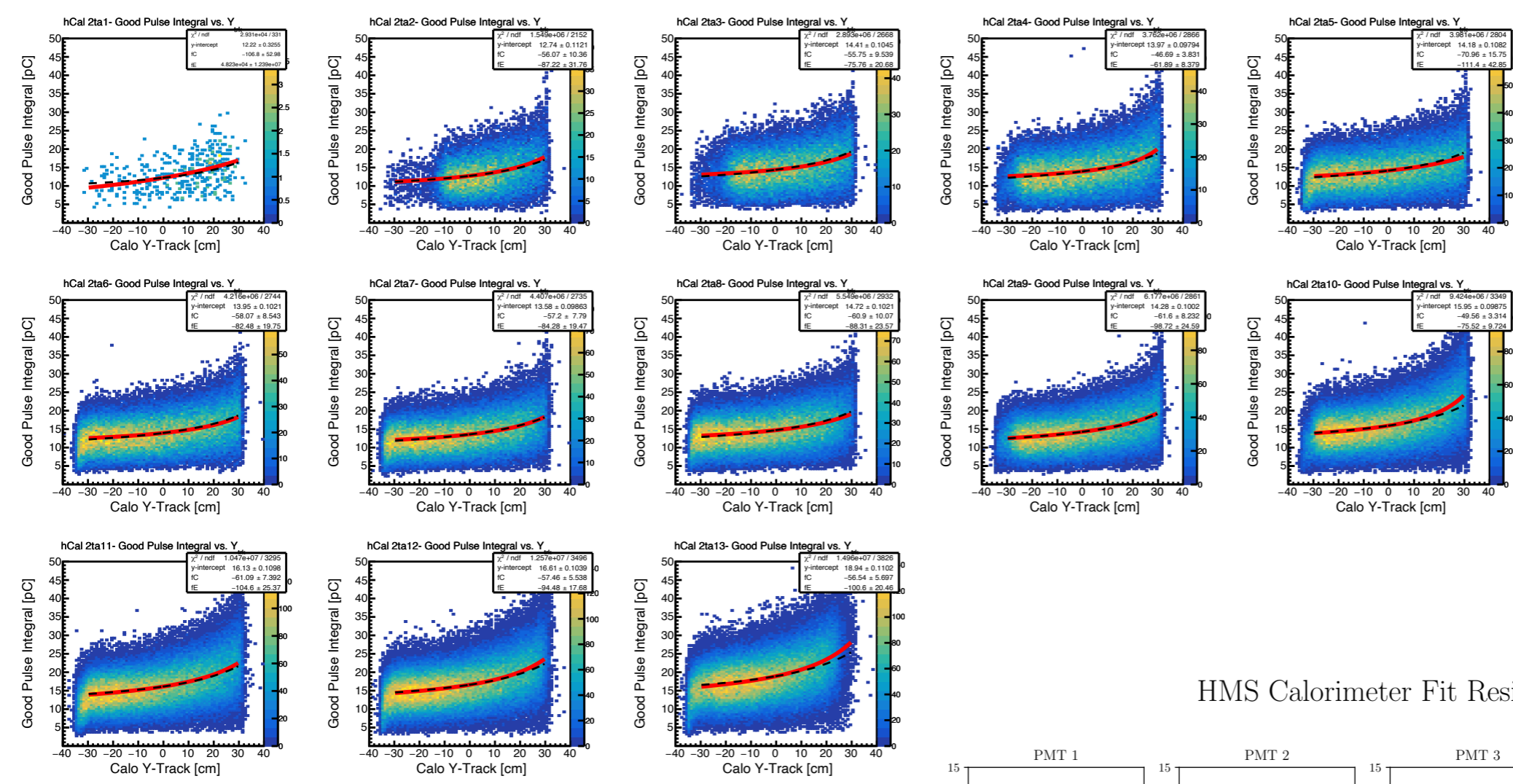Fit Residuals for Layer 1pr- have been reversed wrt. x-axis for a direct comparison of between both sides of the layer

Layer 2ta+

- ◦ Correlated errors are included
- ◦ X-axis range set to (-35, 35) cm
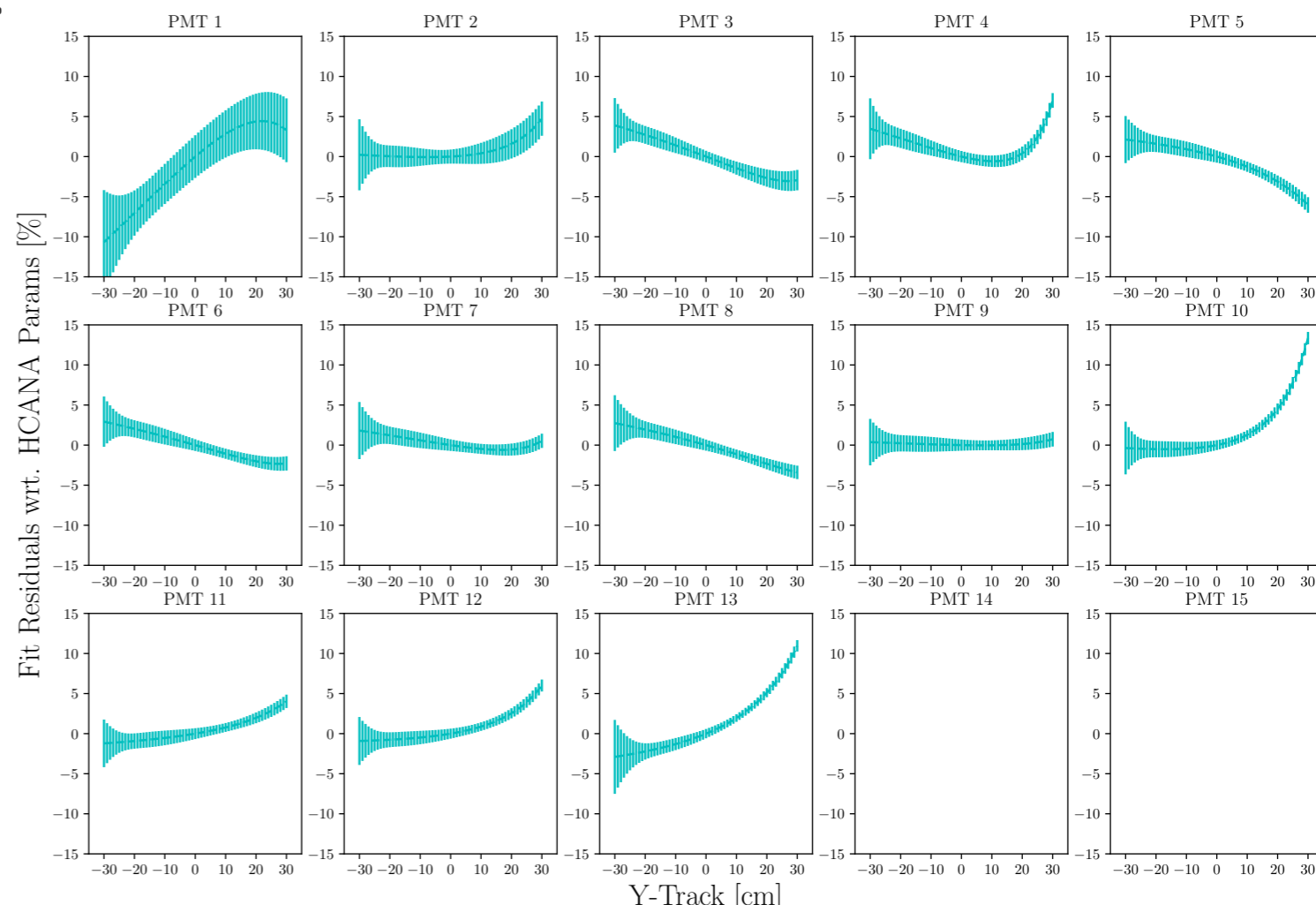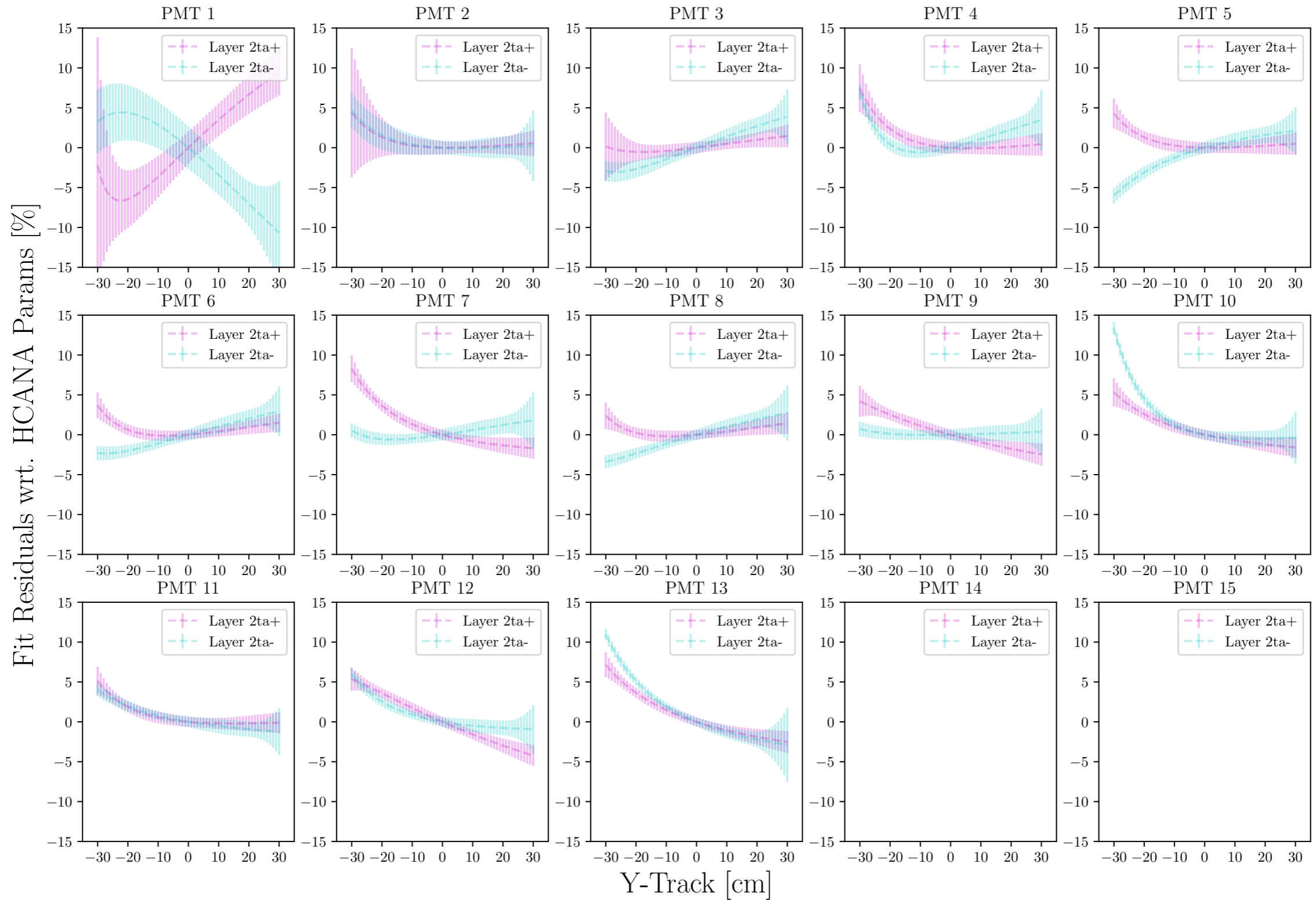- ◦ Y-axis range set to (-15, 15) %

HMS Calorimeter Fit Residuals: Layer 2ta-

Layer 2ta-

- Correlated errors are included
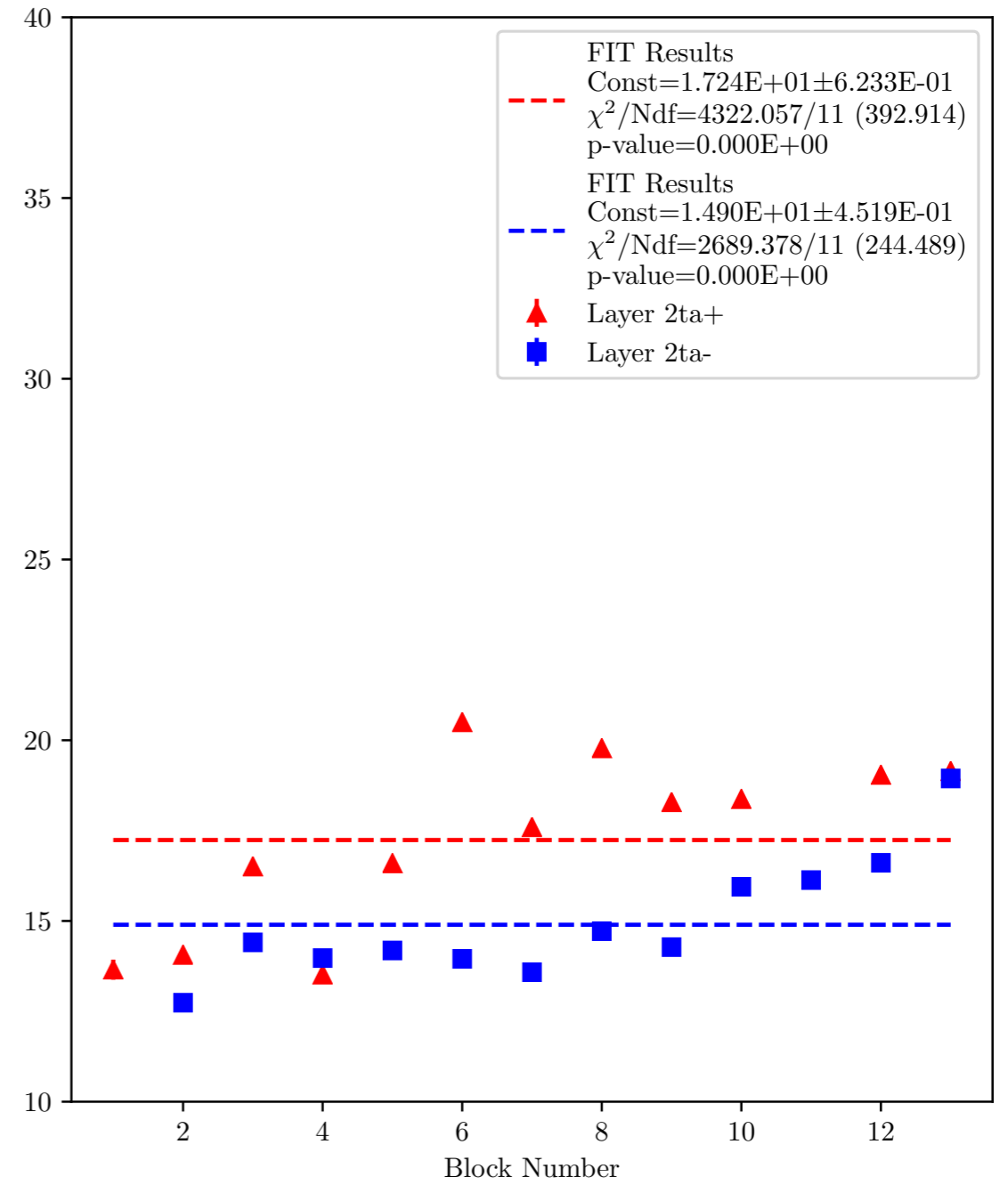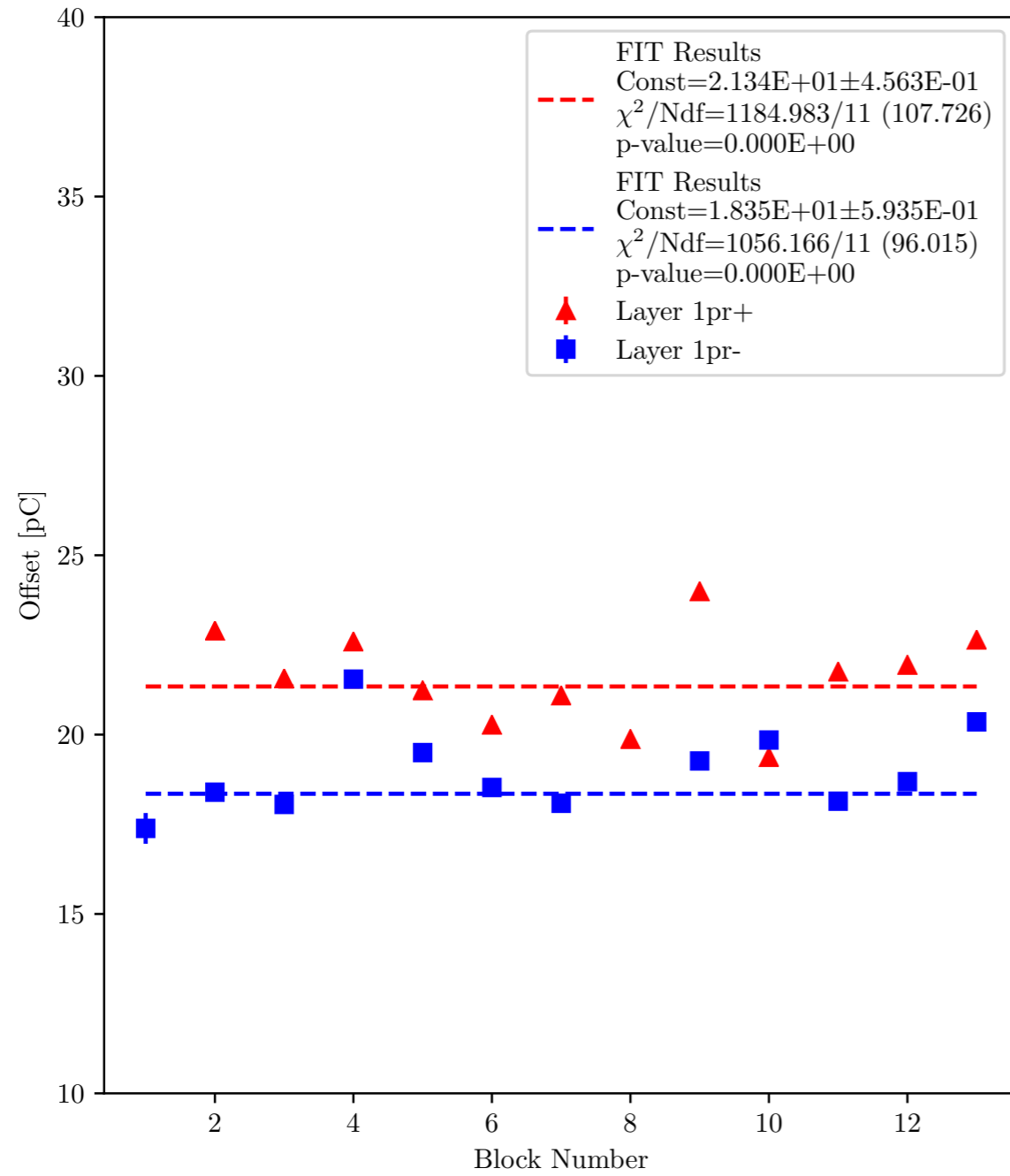- X-axis range set to (-35, 35) cm
- Y-axis range set to (-15, 15) %

HMS Calorimeter Fit Residuals Overlay: Layer 2ta

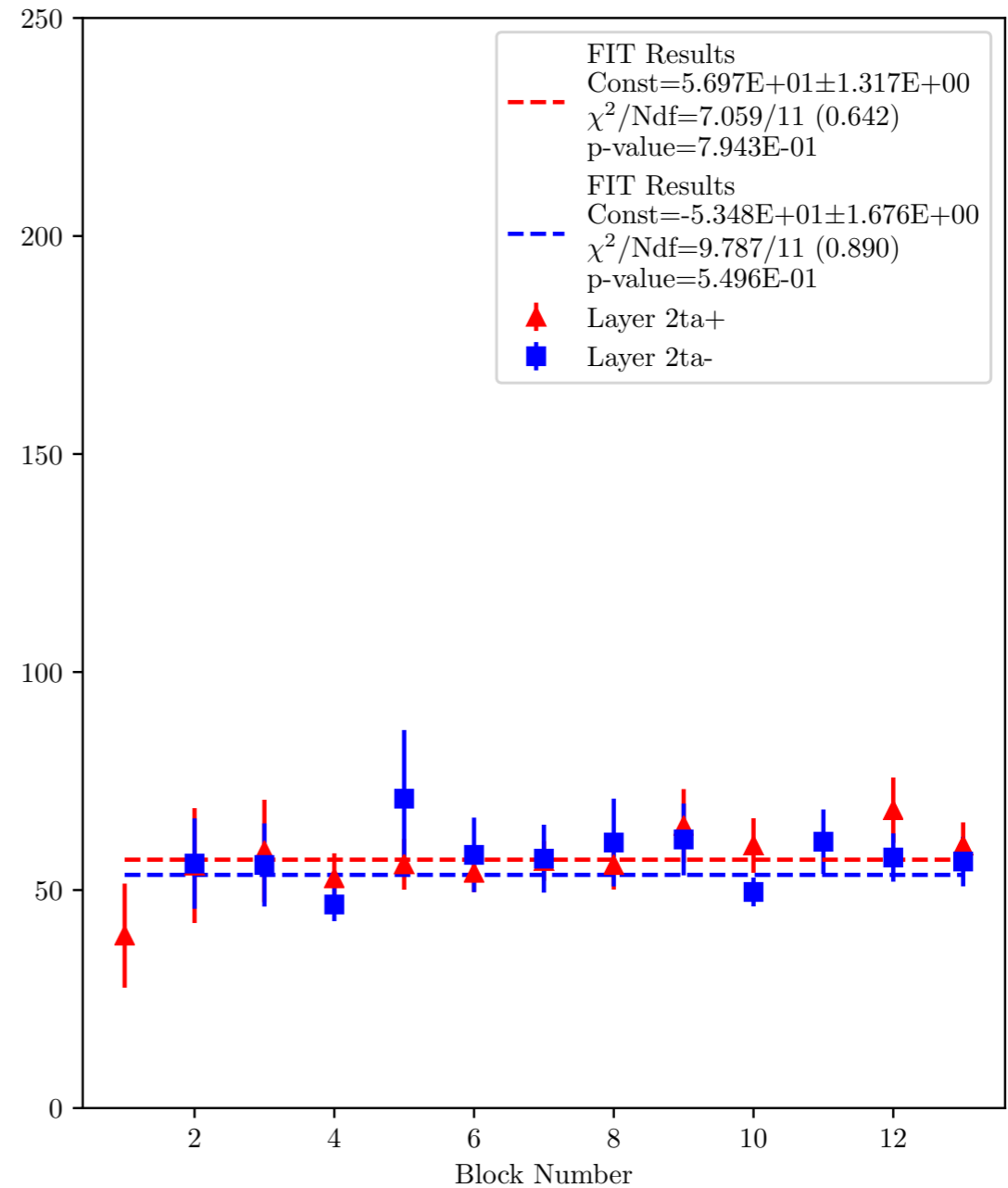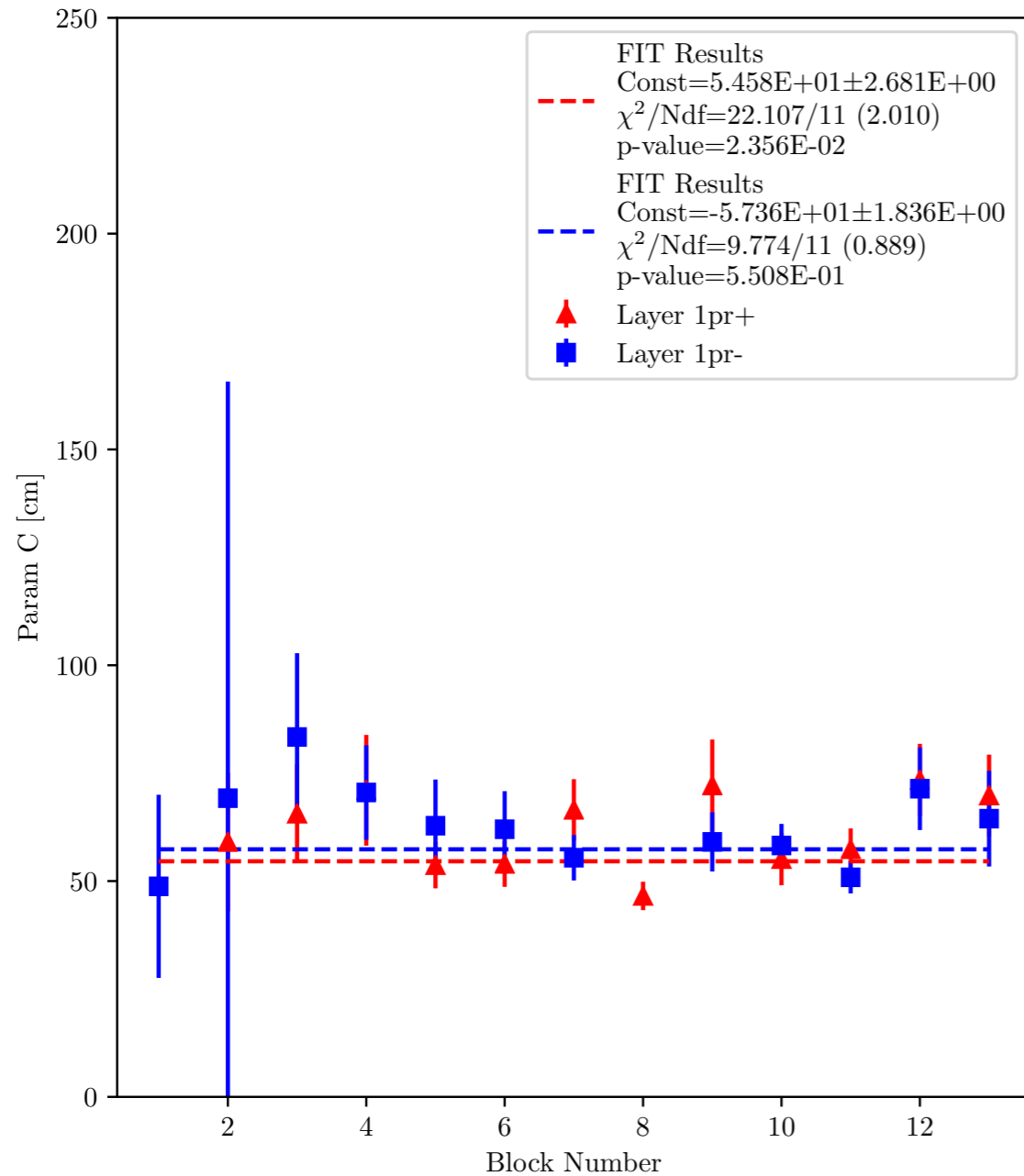Fit Residuals for Layer 2ta- have been reversed wrt. x-axis for a direct comparison of between both sides of the layer

BACK-UP SLIDES

Good Pulse Integral vs. Y track: Offset Param Fit Results

First FIT Iteration (using new function)

Good Pulse Integral vs. Y track: Param C Fit Results

Left panel legend:
FIT Results
Const=5.458E+01±2.681E+00
χ^2/Ndf=22.107/11 (2.010)
p-value=2.356E-02
FIT Results
Const=-5.736E+01±1.836E+00
χ^2/Ndf=9.774/11 (0.889)
p-value=5.508E-01
Layer 1pr+
Layer 1pr-

Right panel legend:
FIT Results
Const=5.697E+01±1.317E+00
χ^2/Ndf=7.059/11 (0.642)
p-value=7.943E-01
FIT Results
Const=-5.348E+01±1.676E+00
χ^2/Ndf=9.787/11 (0.890)
p-value=5.496E-01
Layer 2ta+
Layer 2ta-

Axis labels: Param C [cm], Block Number

○ First FIT Iteration (using new function)

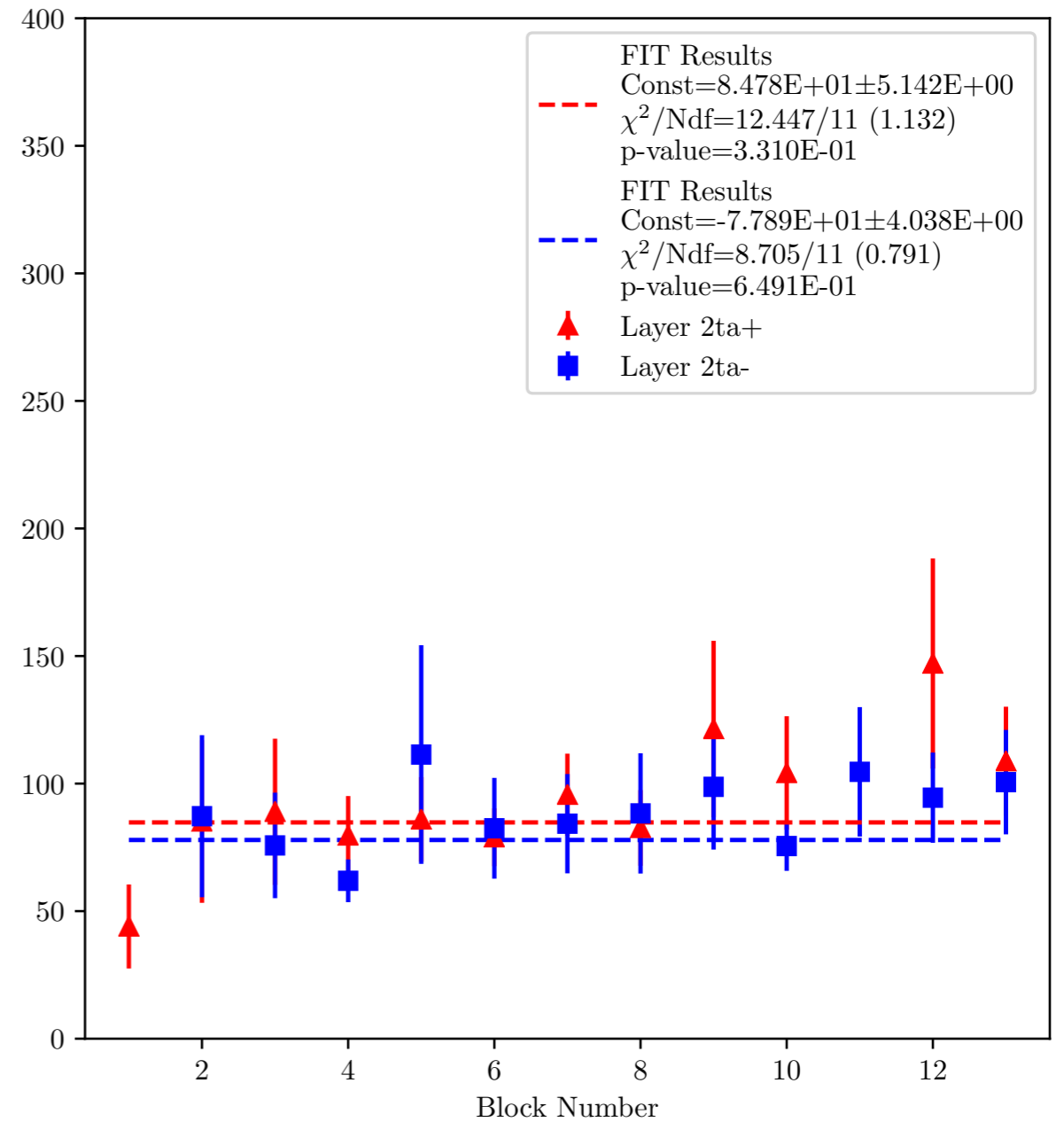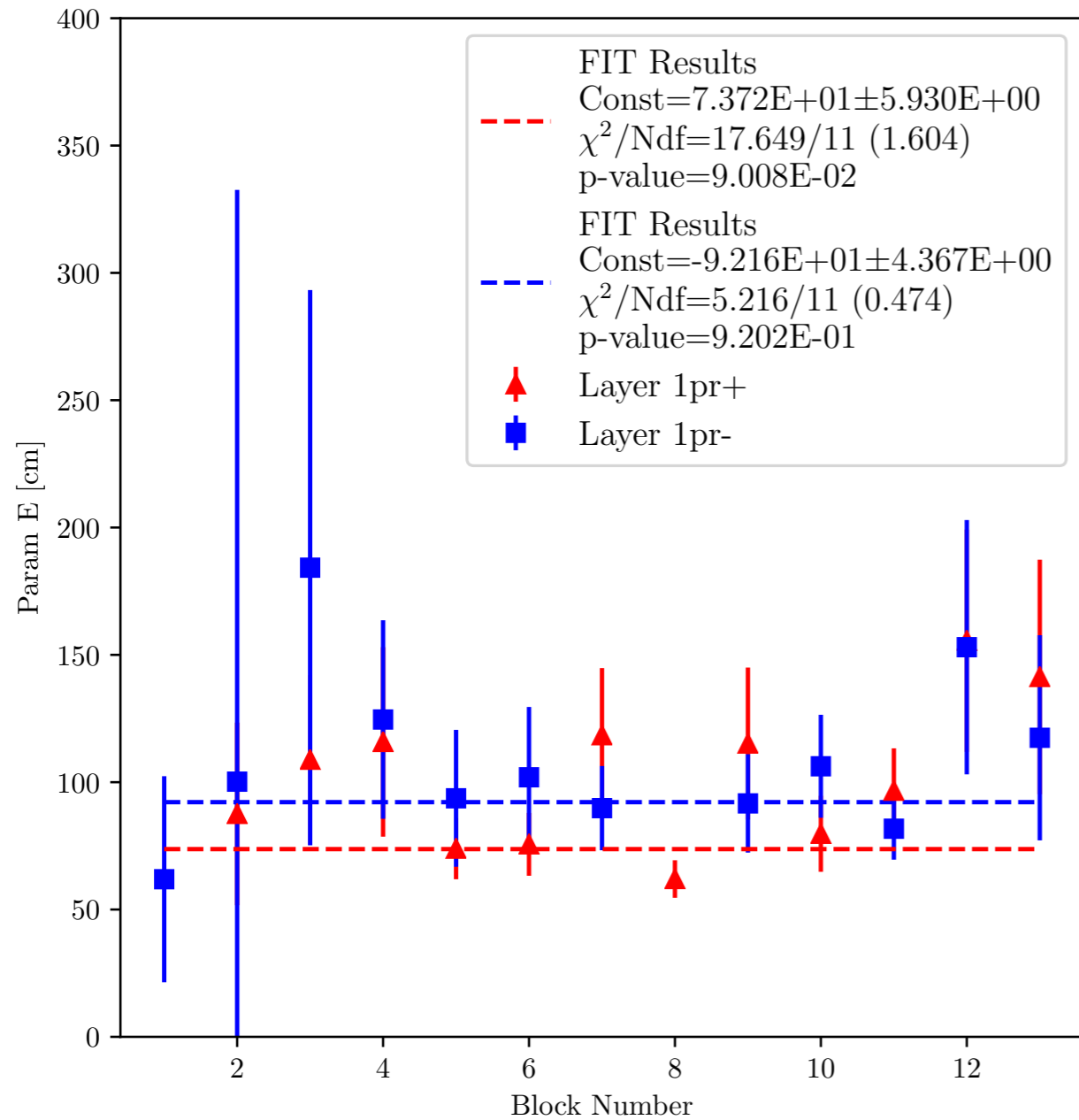◆ (negative layer sign is flipped for ease of comparison)

Good Pulse Integral vs. Y track: Param E Fit Results

First FIT Iteration (using new function)

◆ (negative layer sign is flipped for ease of comparison)