GEANT4 CaFe Simulation: simulating proton-Ca48 target interactions

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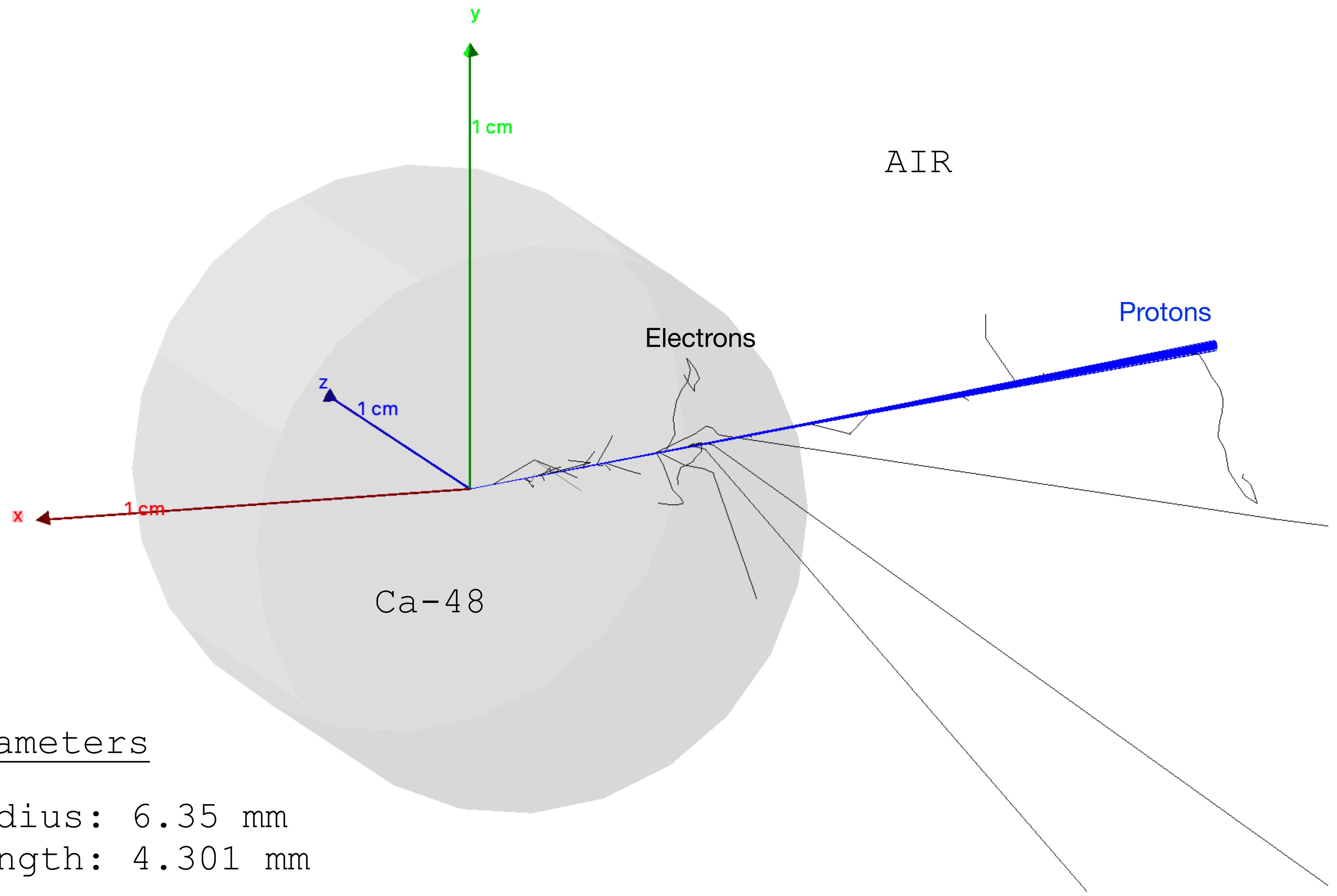
March 23, 2022

GEANT4 Simulation: Side View



Input Parameters

Target Radius: 6.35 mm
Target Length: 4.301 mm

Density: 1.86 g/cm3
Molar Mass: 47.952 g/mole

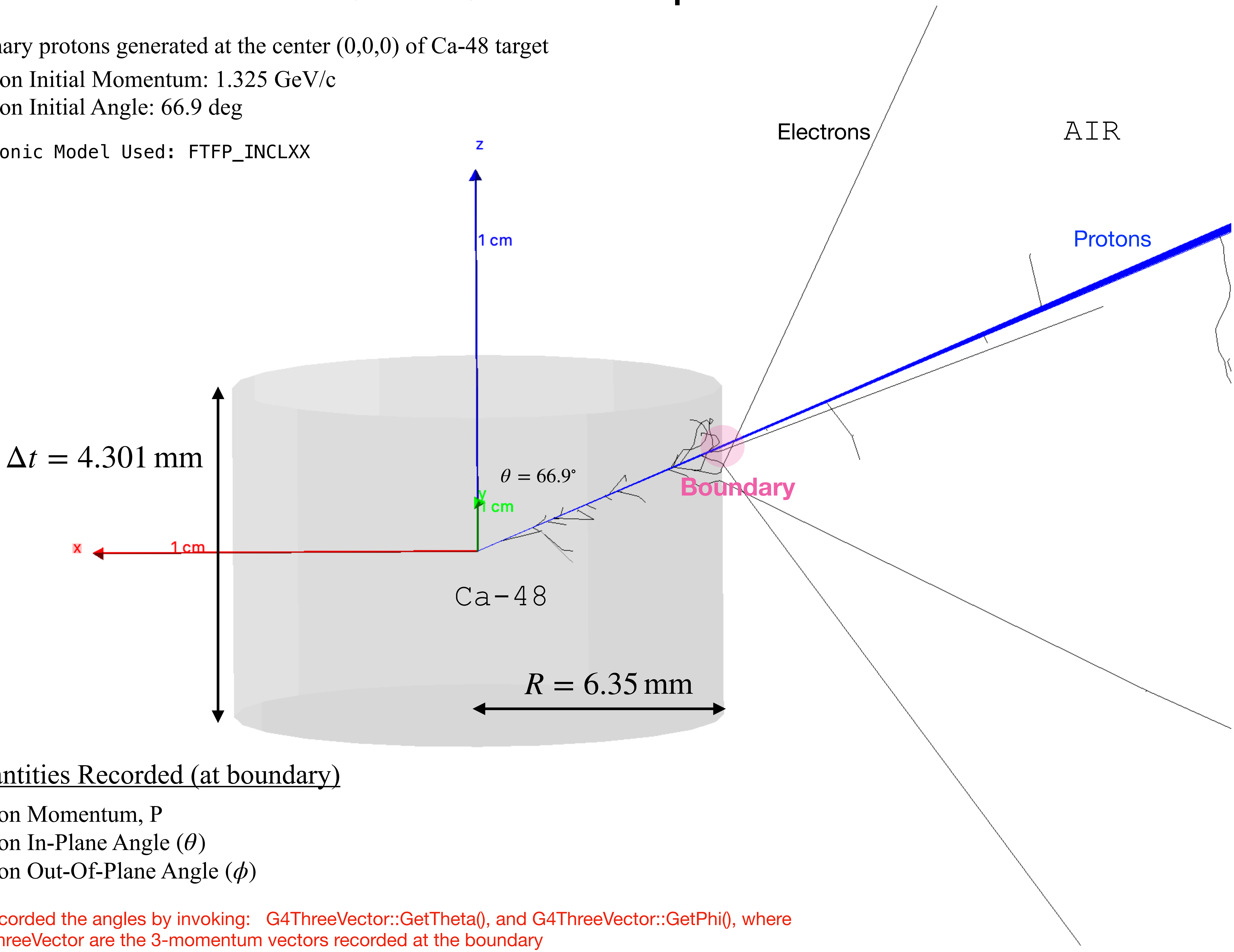
Geant4 Simulation: Top View

Primary protons generated at the center (0,0,0) of Ca-48 target

Proton Initial Momentum: 1.325 GeV/c

Proton Initial Angle: 66.9 deg

`Hadronic Model Used: FTFP_INCLXX`



<u>Quantities Recorded (at boundary)</u>

Proton Momentum, P

Proton In-Plane Angle (θ)

Proton Out-Of-Plane Angle (ϕ)

* I recorded the angles by invoking: G4ThreeVector::GetTheta(), and G4ThreeVector::GetPhi(), where G4ThreeVector are the 3-momentum vectors recorded at the boundary

A few other Physics Lists

- FTFP_BERT_**HP** : as FTFP_BERT, but with **NeutronHP** for neutrons of kinetic energy below 20 MeV
  - **Shielding** : similar to FTFP_BERT_HP, but with **Radioactive Decay** and **QMD** (Quantum Molecular Dynamics) for ions
    - QMD used in the range [100 MeV, 10 GeV] : below BIC, above FTFP
- FTFP_**INCLXX** : similar to FTFP_BERT, but using **INCL**XX instead of BERT for some particles
  - Protons, neutrons, charged pions below 20 GeV; FTFP above 15 GeV
- **QGSP**_FTFP_BERT : similar to FTFP_BERT, but using **QGS** (Quark Gluon String) model at high energies
  - [6, 8] GeV transition BERT − FTFP ; [12, 25] GeV transition FTFP − QGSP
- QGSP_**BIC** : similar to FTFP_BERT but using QGS and BIC (Binary Cascade) instead of FTF and BERT when possible
  - Protons, neutrons : BIC < 9.9 GeV , FTFP in [9.5, 25] GeV , QGSP > 12 GeV
    Pions & kaons : BERT < 5 GeV , FTFP in [4, 25] GeV , QGSP > 12 GeV

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I used the above slide as reference, to help me choose a pre-defined hadronic physics model

The entire presentation can be found on:
https://indico.cern.ch/event/781244/contributions/3251933/attachments/1782461/2902499/HadronicPhysics.pdf

Detailed Materials Properties (output by Geant4):

```
***** Table : Nb of materials = 2 *****

 Material:    G4_AIR    density:  1.205 mg/cm3  RadL: 303.921 m    Nucl.Int.Length: 710.095 m
                        Imean:  85.700 eV   temperature: 293.15 K  pressure:   1.00 atm

   --->  Element: C (C)   Z =  6.0   N =    12   A = 12.011 g/mole
         --->  Isotope:   C12   Z =  6   N =  12   A =  12.00 g/mole   abundance: 98.930 %
         --->  Isotope:   C13   Z =  6   N =  13   A =  13.00 g/mole   abundance:  1.070 %
          ElmMassFraction:   0.01 %  ElmAbundance   0.02 %

   --->  Element: N (N)   Z =  7.0   N =    14   A = 14.007 g/mole
         --->  Isotope:   N14   Z =  7   N =  14   A =  14.00 g/mole   abundance: 99.632 %
         --->  Isotope:   N15   Z =  7   N =  15   A =  15.00 g/mole   abundance:  0.368 %
          ElmMassFraction:  75.53 %  ElmAbundance  78.44 %

   --->  Element: O (O)   Z =  8.0   N =    16   A = 15.999 g/mole
         --->  Isotope:   O16   Z =  8   N =  16   A =  15.99 g/mole   abundance: 99.757 %
         --->  Isotope:   O17   Z =  8   N =  17   A =  17.00 g/mole   abundance:  0.038 %
         --->  Isotope:   O18   Z =  8   N =  18   A =  18.00 g/mole   abundance:  0.205 %
          ElmMassFraction:  23.18 %  ElmAbundance  21.07 %

   --->  Element: Ar (Ar)   Z = 18.0   N =    40   A = 39.948 g/mole
         --->  Isotope:  Ar36   Z = 18   N =  36   A =  35.97 g/mole   abundance:  0.337 %
         --->  Isotope:  Ar38   Z = 18   N =  38   A =  37.96 g/mole   abundance:  0.063 %
         --->  Isotope:  Ar40   Z = 18   N =  40   A =  39.96 g/mole   abundance: 99.600 %
          ElmMassFraction:   1.28 %  ElmAbundance   0.47 %


 Material:   Calcium    density:  1.860 g/cm3   RadL:  10.385 cm   Nucl.Int.Length:  68.364 cm
                        Imean: 191.000 eV   temperature: 293.15 K  pressure:   1.00 atm

   --->  Element: Calcium (Ca)   Z = 20.0   N =    48   A = 47.953 g/mole
         --->  Isotope:  Ca48   Z = 20   N =  48   A =  47.95 g/mole   abundance: 100.000 %
          ElmMassFraction: 100.00 %  ElmAbundance 100.00 %
```

PLOTS

In the following 2D Histograms:

1. 100,000 protons were generated with momentum and angle (1.325 GeV/c, 66.9 deg)

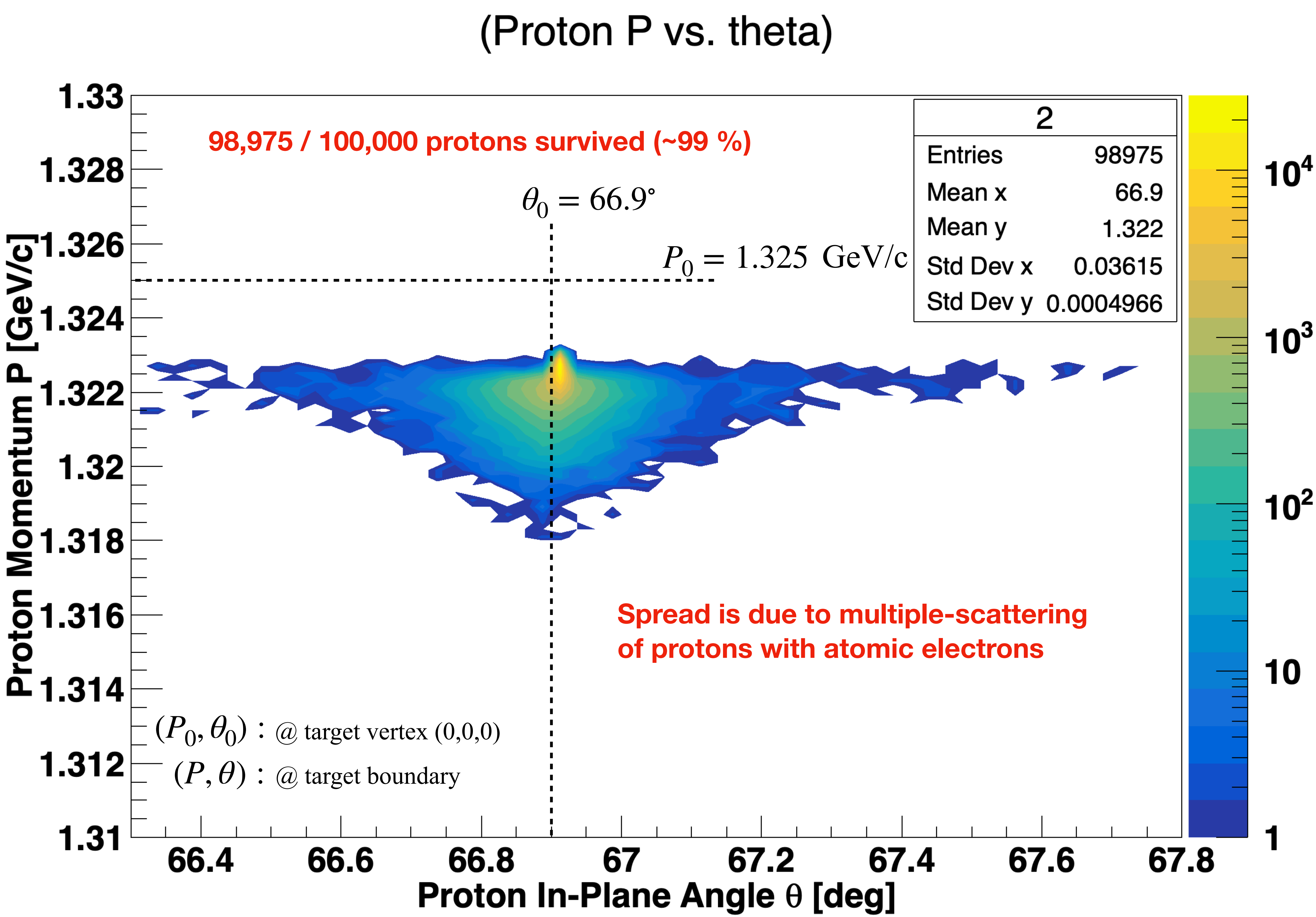
2. The momentum and angle were recorded at the boundary between Ca-48 and AIR

3. The protons at the boundary were required to be the primary protons (via trackID requirement)

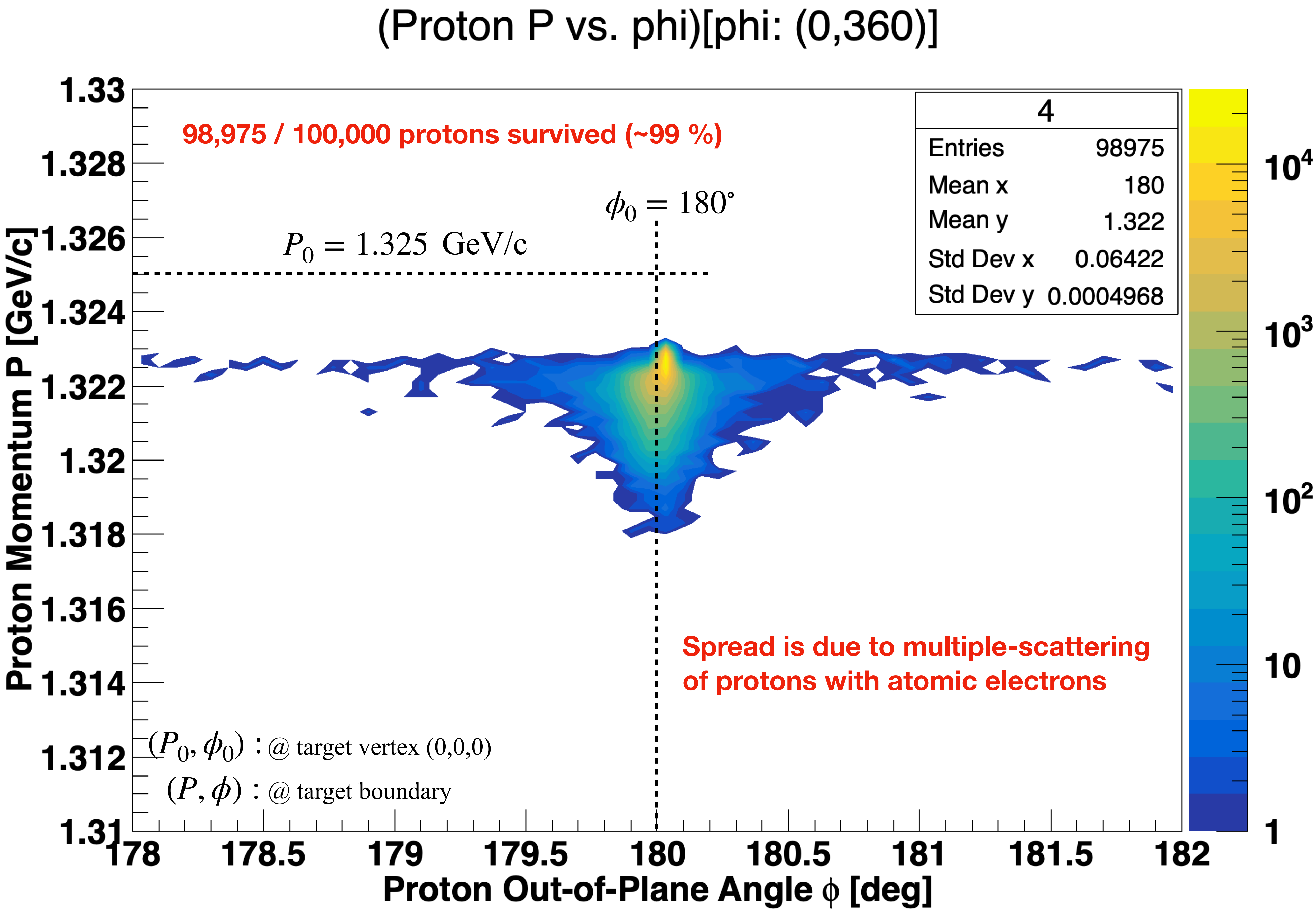
4. The number of protons recorded at the boundary represents the number of protons out of 100,000 protons that actually made it to the target boundary

Geant4 Result:

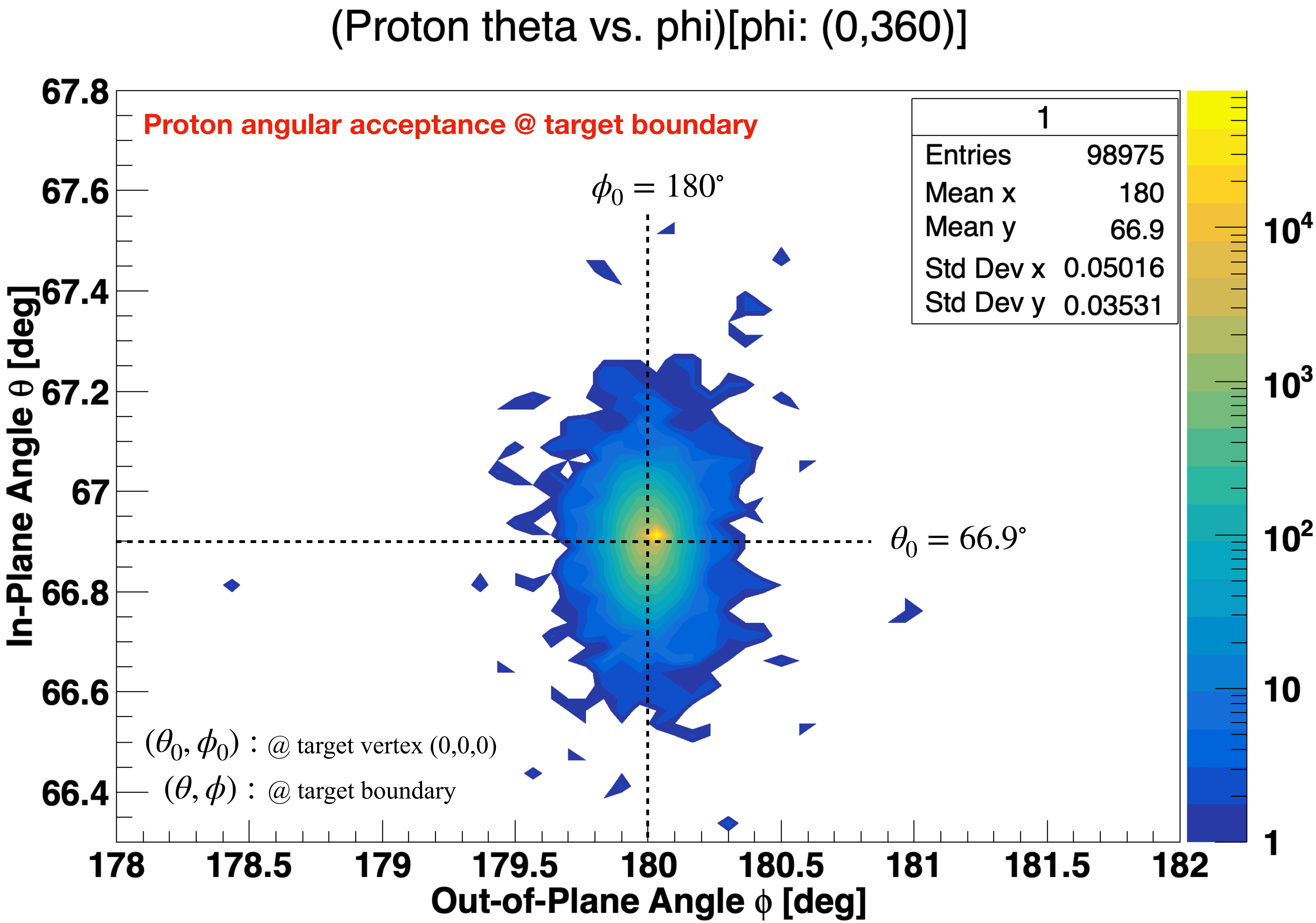
~ 1 % of protons are lost in the Ca-48 target



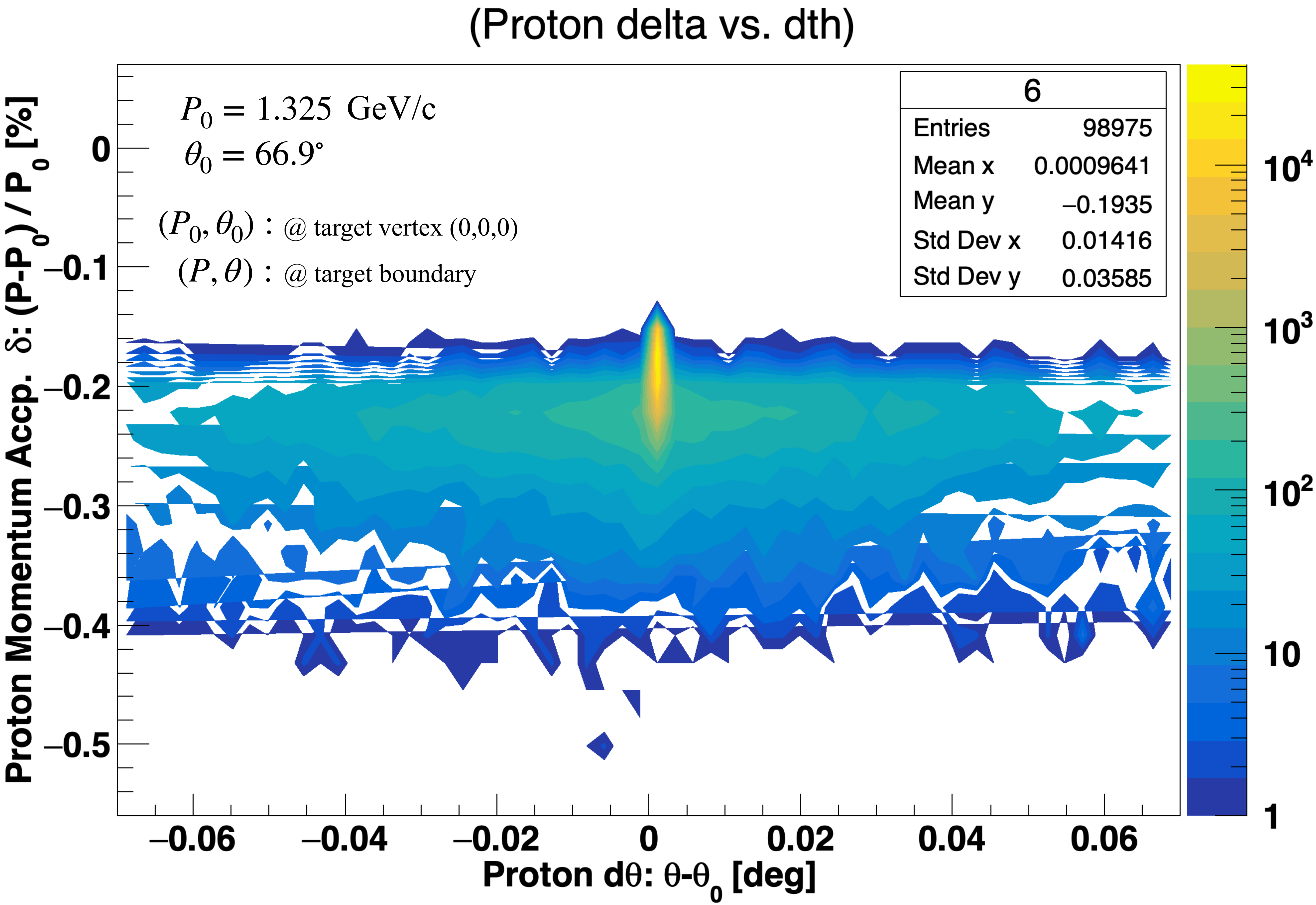
(Proton P vs. theta)
98,975 / 100,000 protons survived (~99 %)
$\theta_0 = 66.9^\circ$
$P_0 = 1.325$ GeV/c
2
Entries 98975
Mean x 66.9
Mean y 1.322
Std Dev x 0.03615
Std Dev y 0.0004966
Spread is due to multiple-scattering
of protons with atomic electrons
(P_0, θ_0) : @ target vertex (0,0,0)
(P, θ) : @ target boundary
Proton Momentum P [GeV/c]
Proton In-Plane Angle θ [deg]
1.33
1.328
1.326
1.324
1.322
1.32
1.318
1.316
1.314
1.312
1.31
66.4
66.6
66.8
67
67.2
67.4
67.6
67.8
10^4
10^3
10^2
10
1

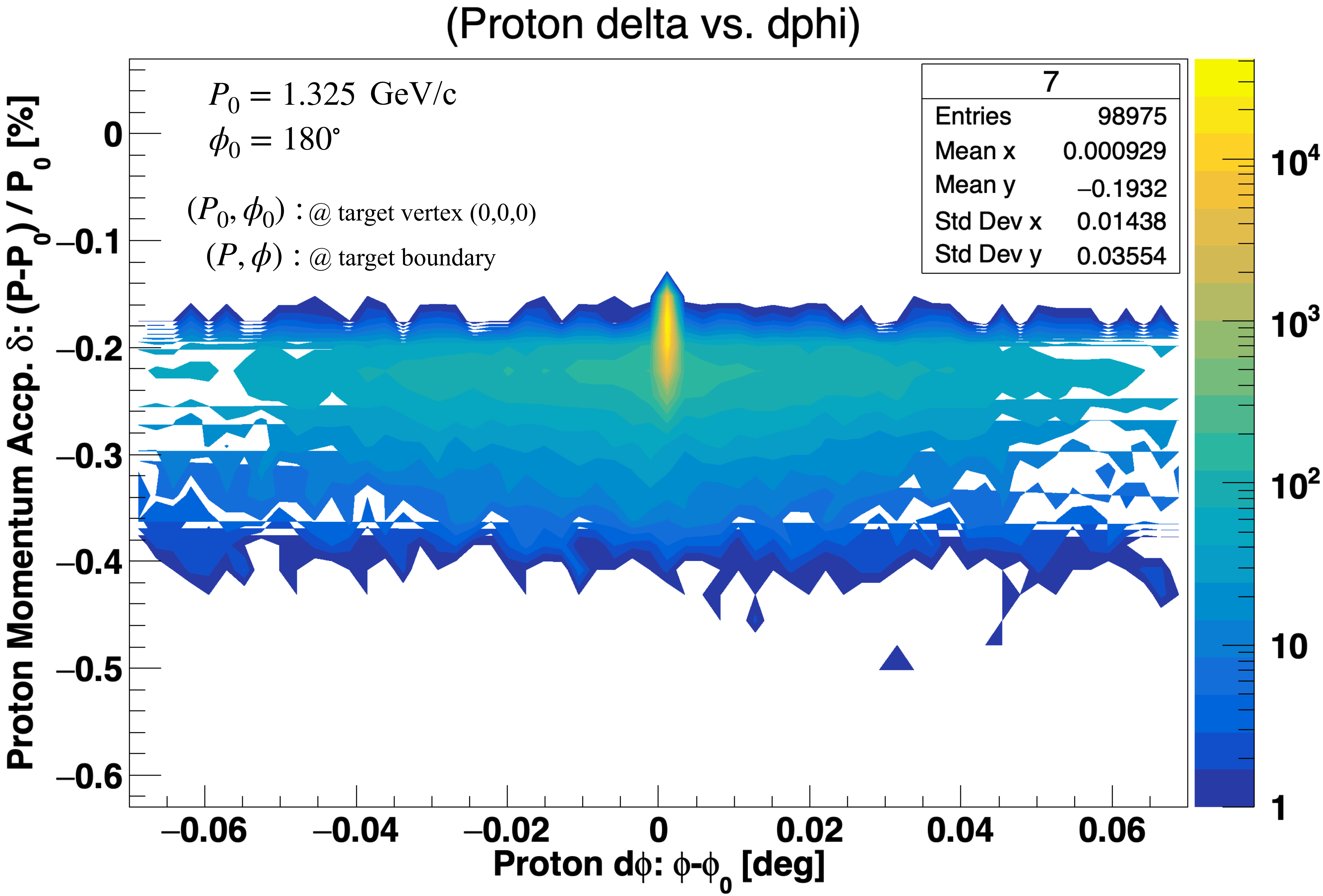


(Proton P vs. phi)[phi: (0,360)]
98,975 / 100,000 protons survived (~99 %)
$\phi_0 = 180^\circ$
$P_0 = 1.325$ GeV/c
4
Entries 98975
Mean x 180
Mean y 1.322
Std Dev x 0.06422
Std Dev y 0.0004968
Spread is due to multiple-scattering of protons with atomic electrons
(P_0, ϕ_0) : @ target vertex (0,0,0)
(P, ϕ) : @ target boundary
Proton Momentum P [GeV/c]
Proton Out-of-Plane Angle ϕ [deg]
1.33
1.328
1.326
1.324
1.322
1.32
1.318
1.316
1.314
1.312
1.31
178
178.5
179
179.5
180
180.5
181
181.5
182
10^4
10^3
10^2
10
1



(Proton theta vs. phi)[phi: (0,360)]
Proton angular acceptance @ target boundary
1
Entries 98975
Mean x 180
Mean y 66.9
Std Dev x 0.05016
Std Dev y 0.03531
$\phi_0 = 180^\circ$
$\theta_0 = 66.9^\circ$
(θ_0, ϕ_0) : @ target vertex (0,0,0)
(θ, ϕ) : @ target boundary
In-Plane Angle θ [deg]
Out-of-Plane Angle ϕ [deg]
67.8
67.6
67.4
67.2
67
66.8
66.6
66.4
178
178.5
179
179.5
180
180.5
181
181.5
182
10^4
10^3
10^2
10
1


(Proton delta vs. dth)
$P_0 = 1.325$ GeV/c
$\theta_0 = 66.9°$
(P_0, θ_0) : @ target vertex (0,0,0)
(P, θ) : @ target boundary
6
Entries 98975
Mean x 0.0009641
Mean y −0.1935
Std Dev x 0.01416
Std Dev y 0.03585
Proton Momentum Accp. δ: (P-P0) / P0 [%]
Proton dθ: θ-θ0 [deg]
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.06
−0.04
−0.02
0.02
0.04
0.06
10^4
10^3
10^2
10
1

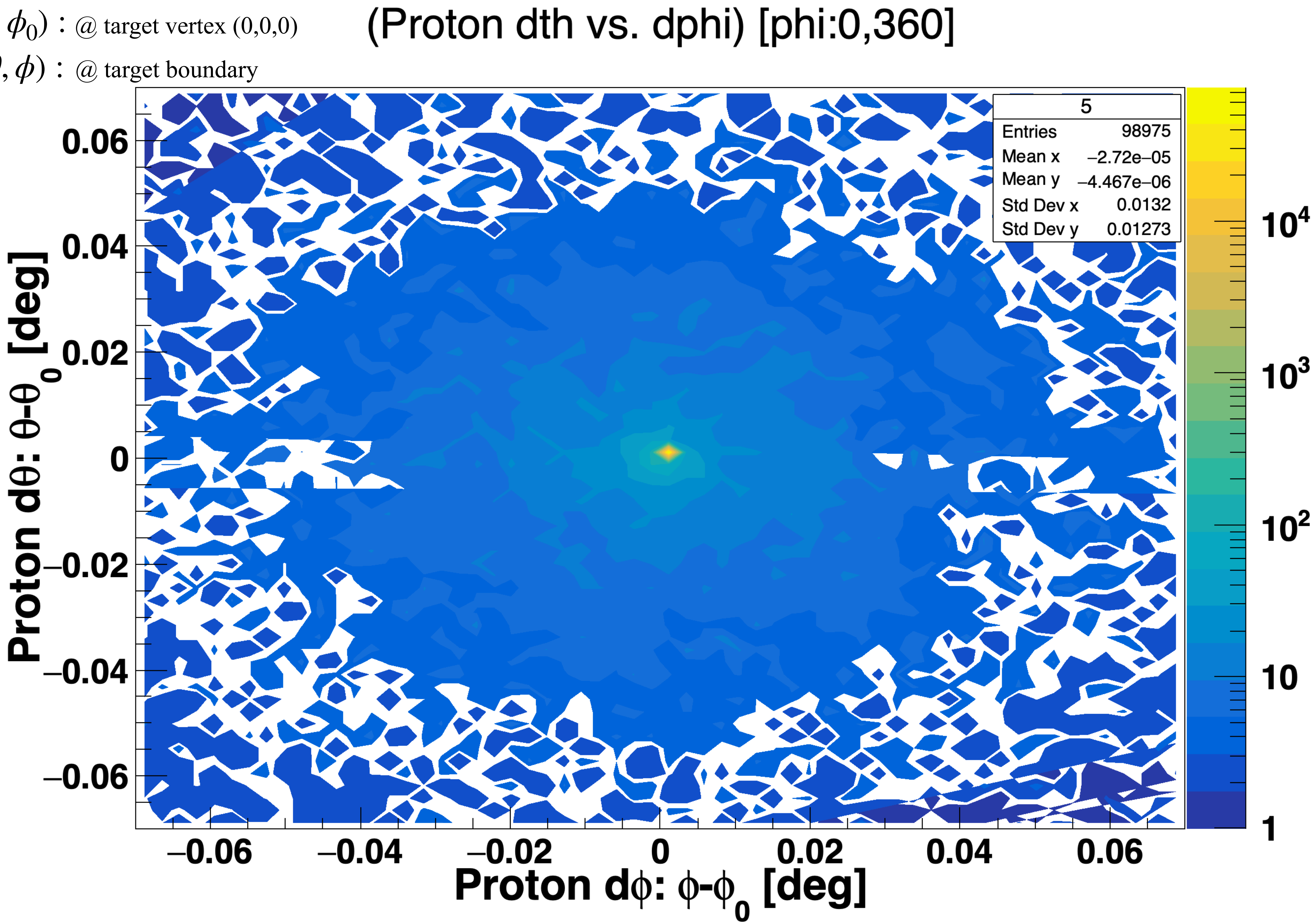


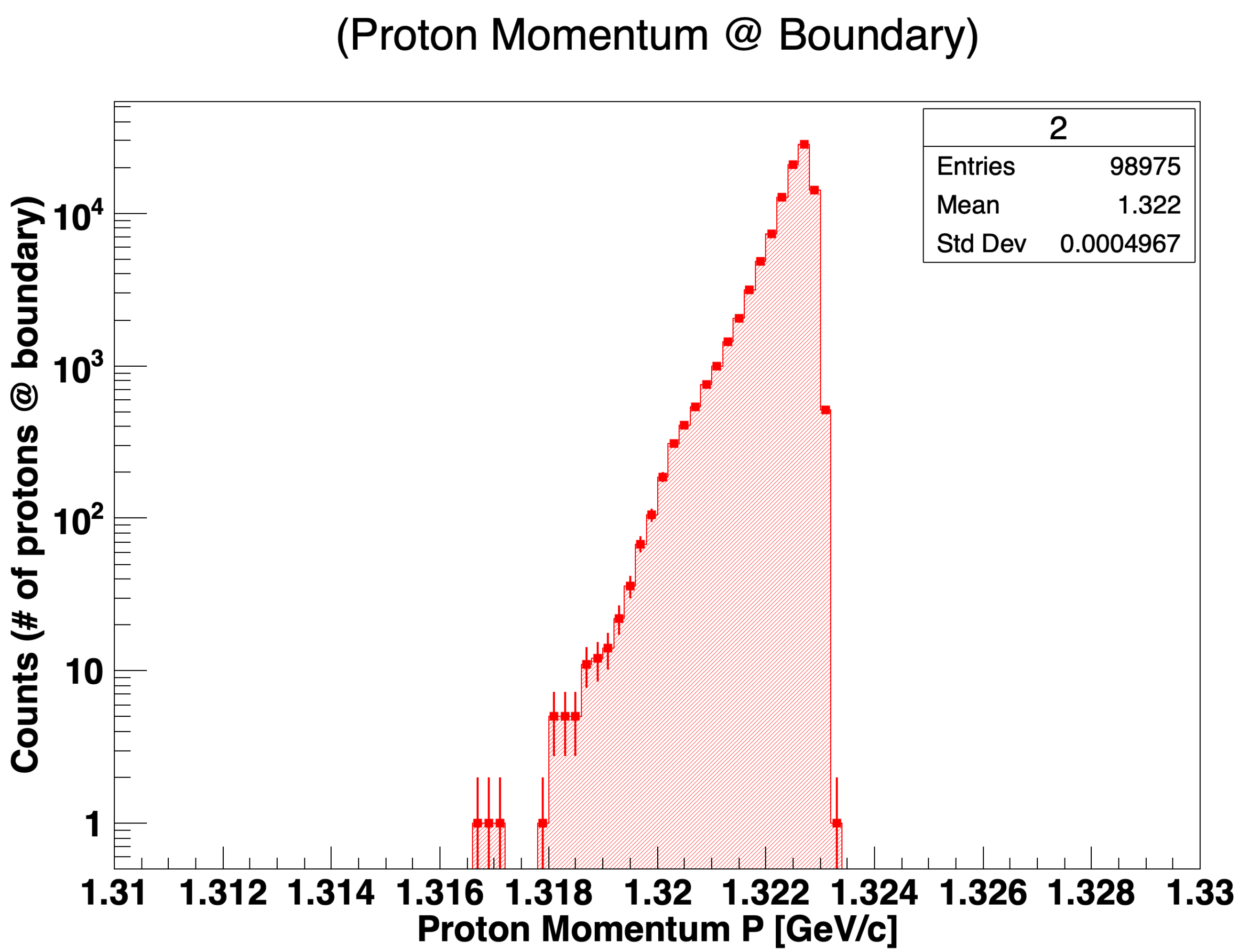
(Proton delta vs. dphi)
$P_0 = 1.325$ GeV/c
$\phi_0 = 180°$
(P_0, ϕ_0) : @ target vertex (0,0,0)
(P, ϕ) : @ target boundary
7
Entries 98975
Mean x 0.000929
Mean y −0.1932
Std Dev x 0.01438
Std Dev y 0.03554
Proton Momentum Accp. δ: (P-P0) / P0 [%]
Proton dϕ: ϕ-ϕ0 [deg]
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
1
10
10^2
10^3
10^4

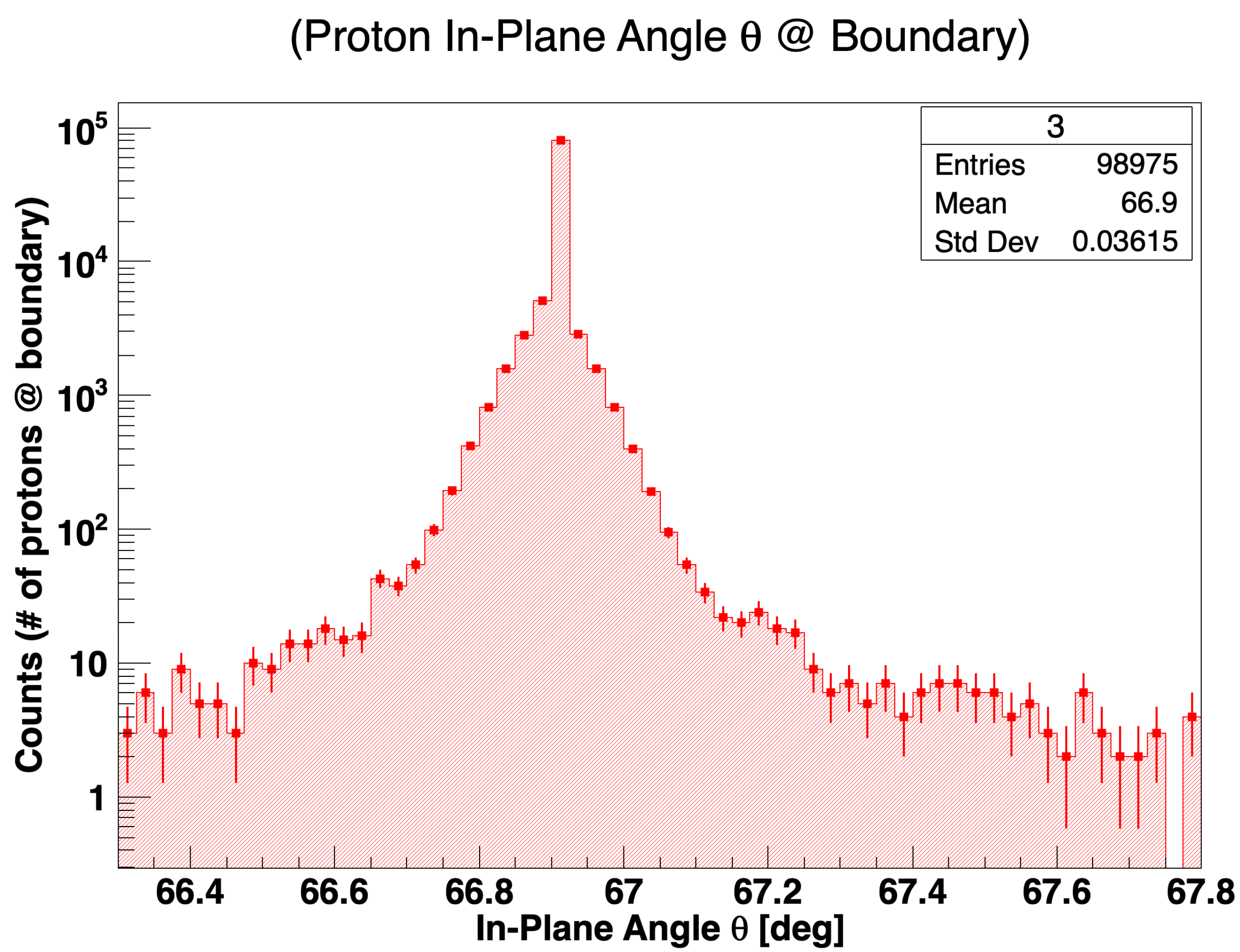
$\theta_0 = 66.9^\circ$
$\phi_0 = 180^\circ$

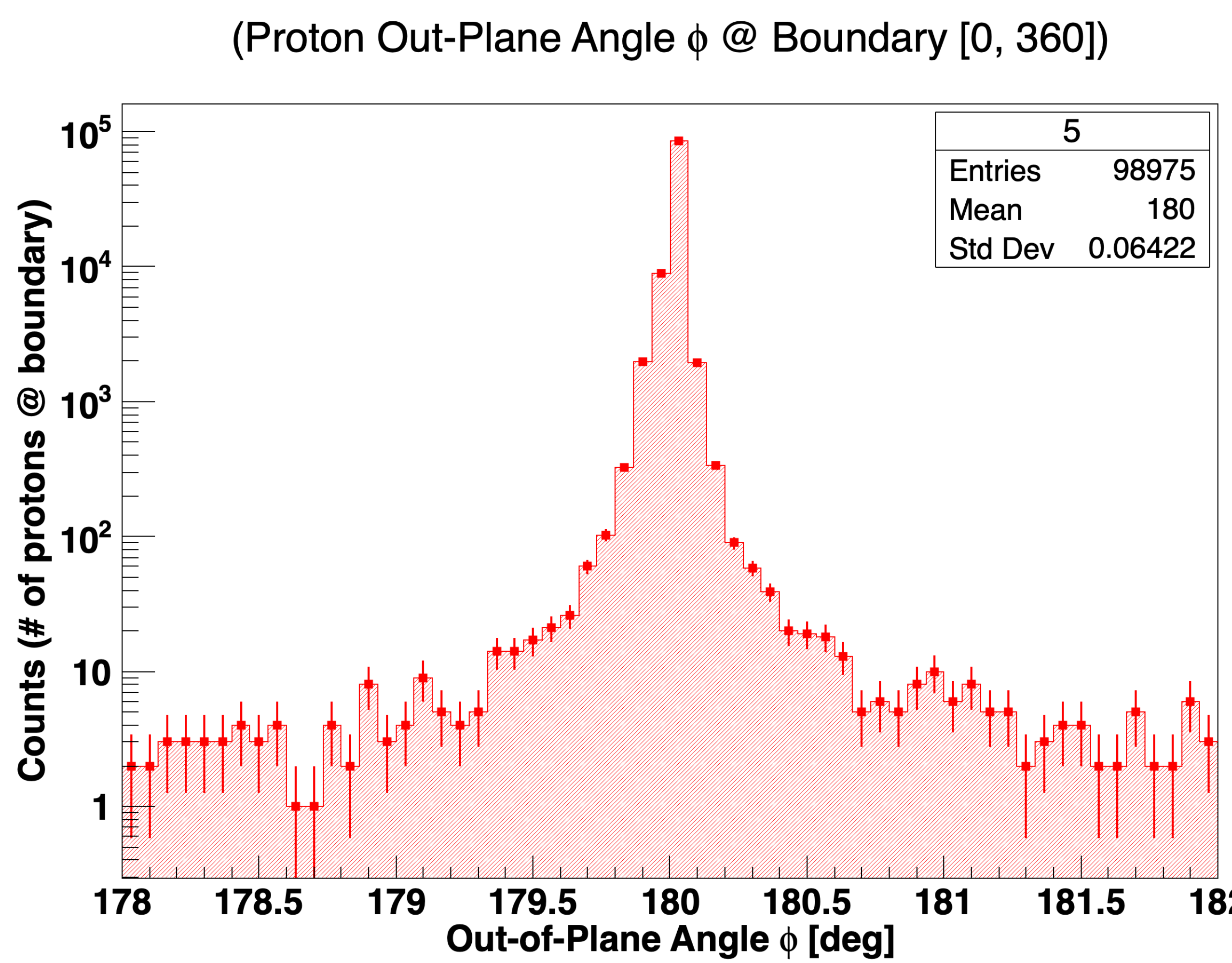
(θ_0, ϕ_0) : @ target vertex (0,0,0)

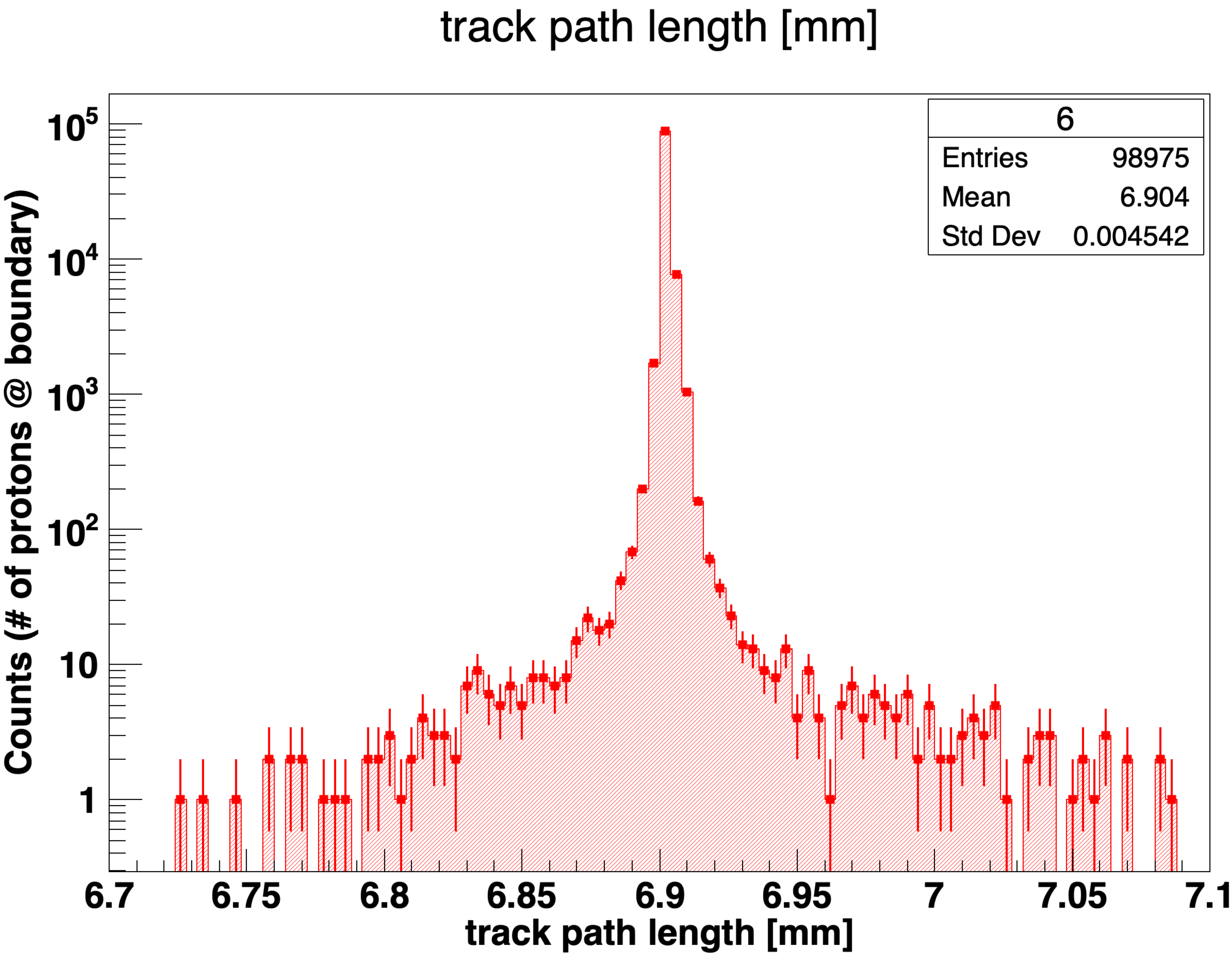
(θ, ϕ) : @ target boundary

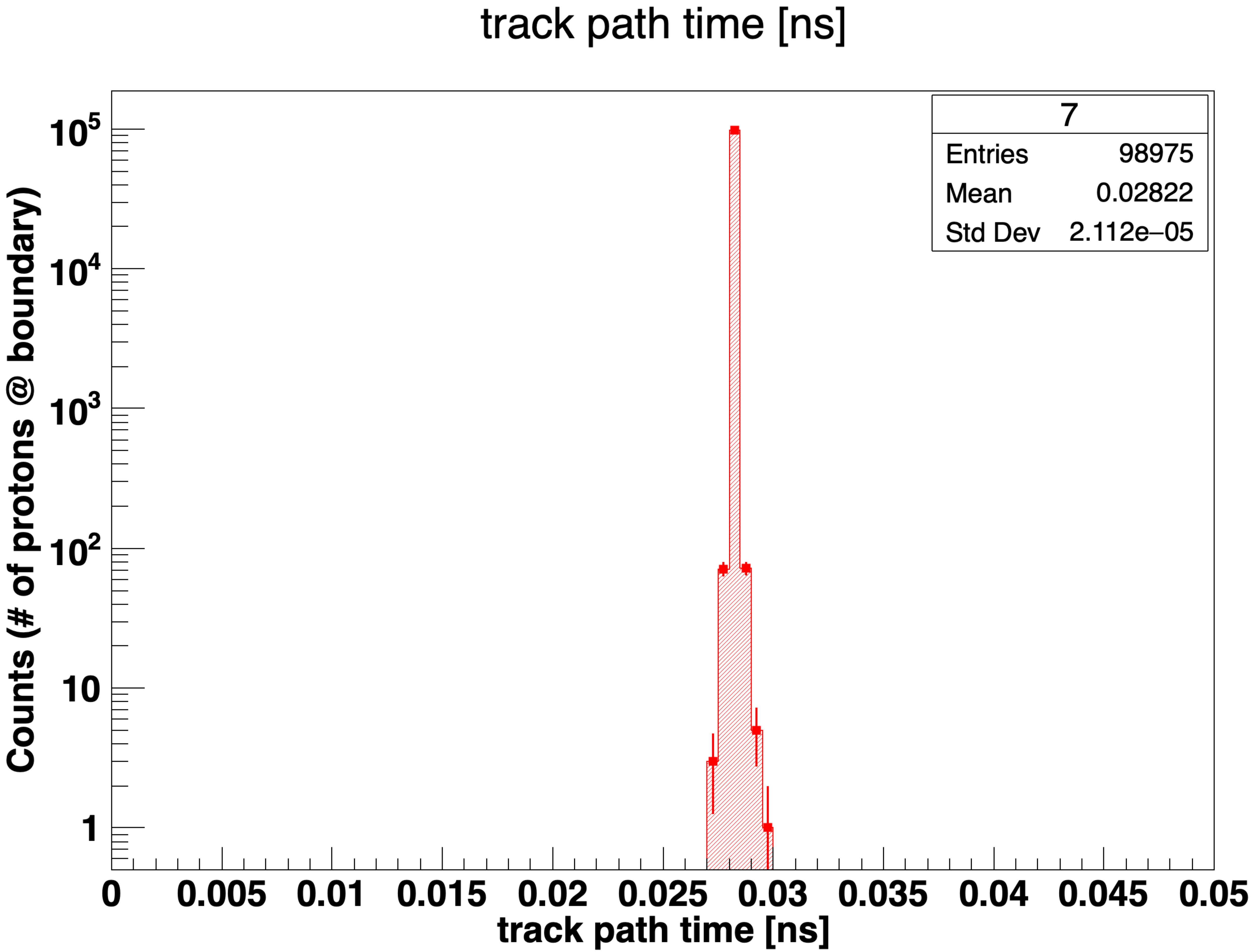



(Proton Momentum @ Boundary)
2
Entries 98975
Mean 1.322
Std Dev 0.0004967
Counts (# of protons @ boundary)
1
10
10^2
10^3
10^4
1.31 1.312 1.314 1.316 1.318 1.32 1.322 1.324 1.326 1.328 1.33
Proton Momentum P [GeV/c]


(Proton In-Plane Angle θ @ Boundary)
3
Entries 98975
Mean 66.9
Std Dev 0.03615
Counts (# of protons @ boundary)
1
10
10^2
10^3
10^4
10^5
66.4 66.6 66.8 67 67.2 67.4 67.6 67.8
In-Plane Angle θ [deg]

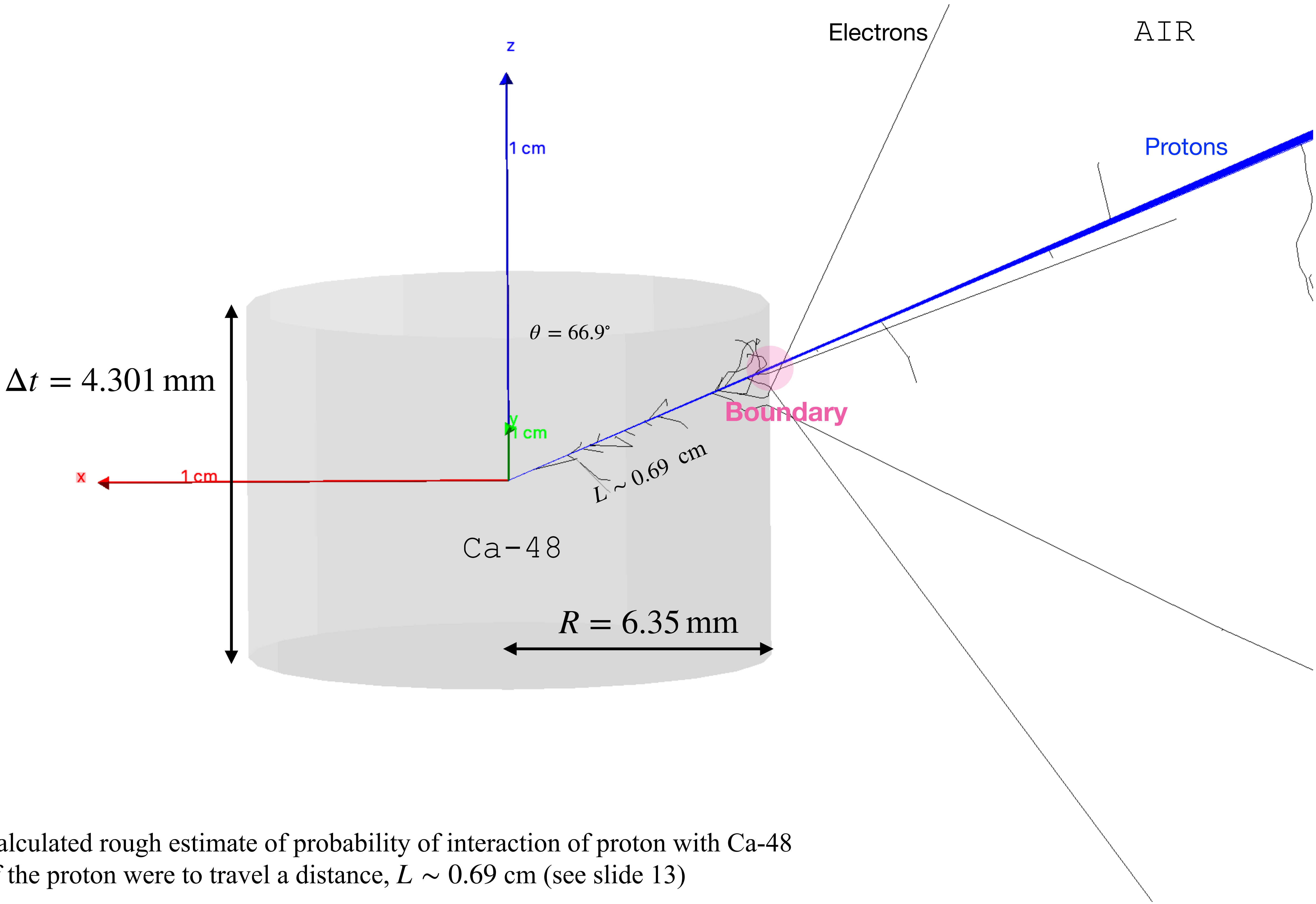

(Proton Out-Plane Angle ϕ @ Boundary [0, 360])
5
Entries 98975
Mean 180
Std Dev 0.06422
Counts (# of protons @ boundary)
1
10
10^2
10^3
10^4
10^5
178 178.5 179 179.5 180 180.5 181 181.5 182
Out-of-Plane Angle ϕ [deg]


track path length [mm]
6
Entries 98975
Mean 6.904
Std Dev 0.004542
Counts (# of protons @ boundary)
10^5
10^4
10^3
10^2
10
1
6.7
6.75
6.8
6.85
6.9
6.95
7
7.05
7.1
track path length [mm]



track path time [ns]
7
Entries 98975
Mean 0.02822
Std Dev 2.112e−05
Counts (# of protons @ boundary)
10^5
10^4
10^3
10^2
10
1
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
track path time [ns]

proton-Ca48 target interaction probability sanity checks



- calculated rough estimate of probability of interaction of proton with Ca-48 if the proton were to travel a distance, $L \sim 0.69$ cm (see slide 13)

- used the mean-free-path formula (see next 17) to calculate the interaction probability
    - for the microscopic proton-nucleus (p-A) cross-section used inelastic proton-hadron cross sections from J. Carvalho / Nuclear Physics A 725 (2003) 269–275 (see slide 18)

Cross section and mean free path

- **Cross section per atom/nucleus σ (microscopic cross section)** [area]

For a given particle with energy E, on an atom with atomic/mass number Z/A

$$\sigma = \sigma(E, Z, A)$$

(common unit: 1 barn (b) = 10^{-24} cm^2)

- **Mean free path λ** [length]

$$\lambda = \frac{1}{N\sigma} = \frac{M}{\rho N_A \sigma}$$

- M: **Molar mass [mass/mol]**
- N: **Atom density [1/volume]**
- σ: **Microscopic cross section [area]**
- ρ: **Material density [mass/volume]**
- N_A: **Avogadro's constant** [6.022 x 10^{23} mol^{-1}]

= average distance travelled by a particle between two successive collisions

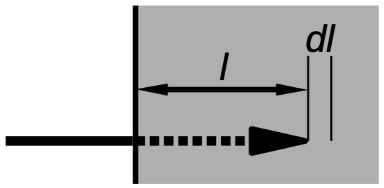
Interaction probability

Assume that particles are normally incident on a homogeneous material and that they are subject to a process with a mean free path λ between collisions:

- **Path length distribution**

Note : $\int_0^\infty p(l')dl' = 1$

$$p(l)\,dl = \frac{1}{\lambda}\exp\left(-\frac{l}{\lambda}\right)dl$$



$p(l)\,dl$ = probability that a particle has an interaction between l and $l + dl$

- **Cumulative interaction probability**

P(λ) = 63.2%
P(2λ) = 86.5%
P(3λ) = 95.0%
P(4λ) = 98.2%

$$P(l) = \int_0^l p(l')\,\mathrm{d}l' = 1 - \exp\left(-\frac{l}{\lambda}\right)$$

Survival probability:

$$P_s(l) = 1 - P(l) = \exp(-l/\lambda)$$

$P(l)$ = probability that a particle interacts before reaching a path length l

Reference: “Particles Interaction with Matter”, A. Lechner (CERN 2017)

Compilation of cross sections for proton–nucleus interactions at the HERA energy

J. Carvalho / Nuclear Physics A 725 (2003) 269–275

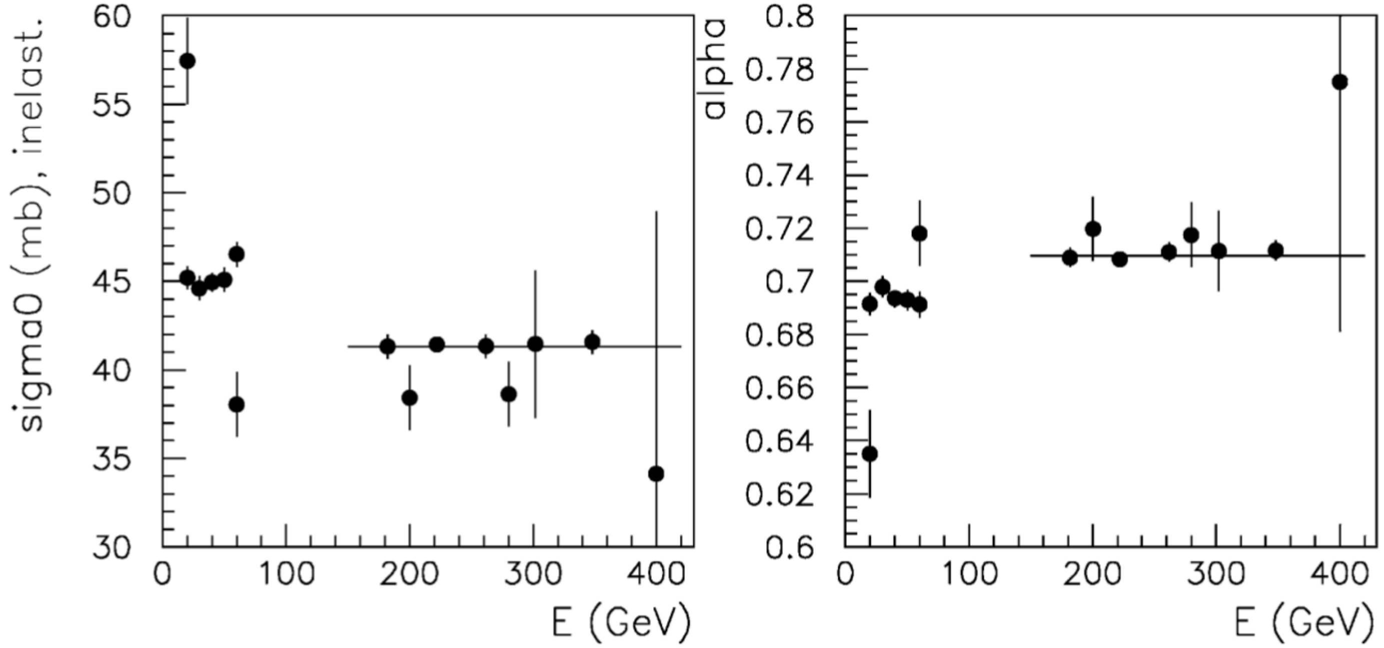
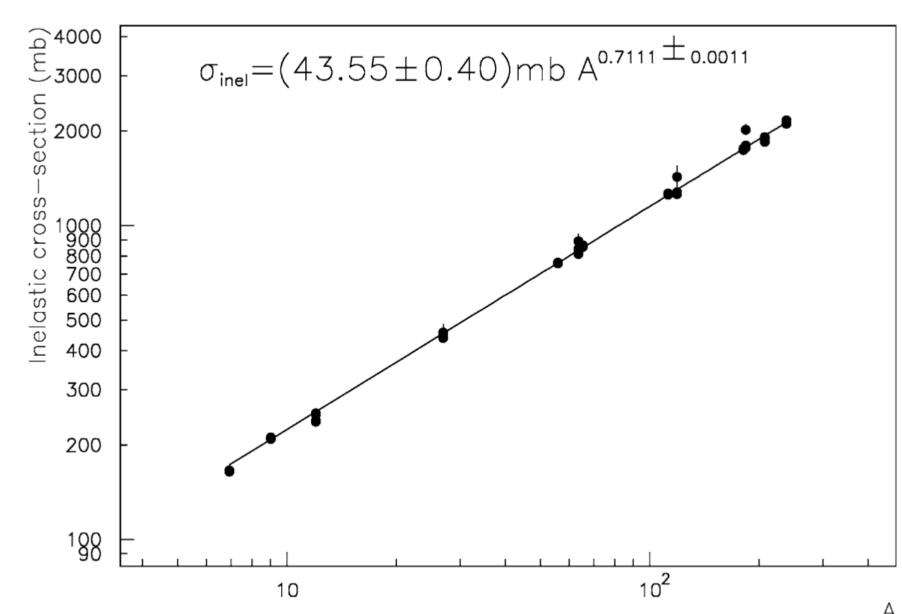


Fig. 2. Top row: fit parameters of the total cross section as a function of A, by different experiments, σ_0 (left) and α (right), as a function of the beam energy. Bottom row: the same for the inelastic cross section. The line is the result of a fit with a constant, for a beam energy greater than 150 GeV.

- for CaFe, the very low proton energies ~ 1.6 GeV (proton momentum ~1.3 GeV), parameters used: $\sigma_0 \sim 45$ mb, $\alpha \sim 0.69$ from the data point cluster at small energies

$$\sigma_{\text{inel}} = \sigma_0 A^{\alpha}$$

- as an example, right plot shows linear relation, but for proton beam > 150 GeV/c
    - forCaFe, since E < 150 GeV, then parameters are slightly difference (practically speaking, it does not make much difference in the σ_{inel})



Putting everything together . . .

$$\lambda^{\mathrm{Ca48}} = \frac{M}{\rho N_A \sigma_{\mathrm{inel}}} = \frac{47.952\mathrm{g/mole}}{(1.86\mathrm{g/cm^3}) N_A (\sigma_0 A^{\alpha})}$$

$N_A = 6.022 \times 1023$ $\sigma_0 \sim 45$ mb $\alpha \sim 0.69$ $A = 48$

$l \sim 0.69$ cm (proton path length travelled)

$$P_{\mathrm{survival}} = e^{\left(-\frac{l}{\lambda}\right)}$$

Conclusion

```
A = 48
 p-A elastic cross section (mb):  388.6077955653359
 p-A inelastic cross section (mb):  650.5324538279483
 p-A total cross section (mb):  1039.1402493932842

mean free path (cm):  65.80949998018123
path length travelled (cm):  0.69
1. interaction prob. with material : 0.010430034422386103
2. survival prob. in material :  0.9895699655776139
nuclear@deuteron:~/Geant4/CaFe_docs$
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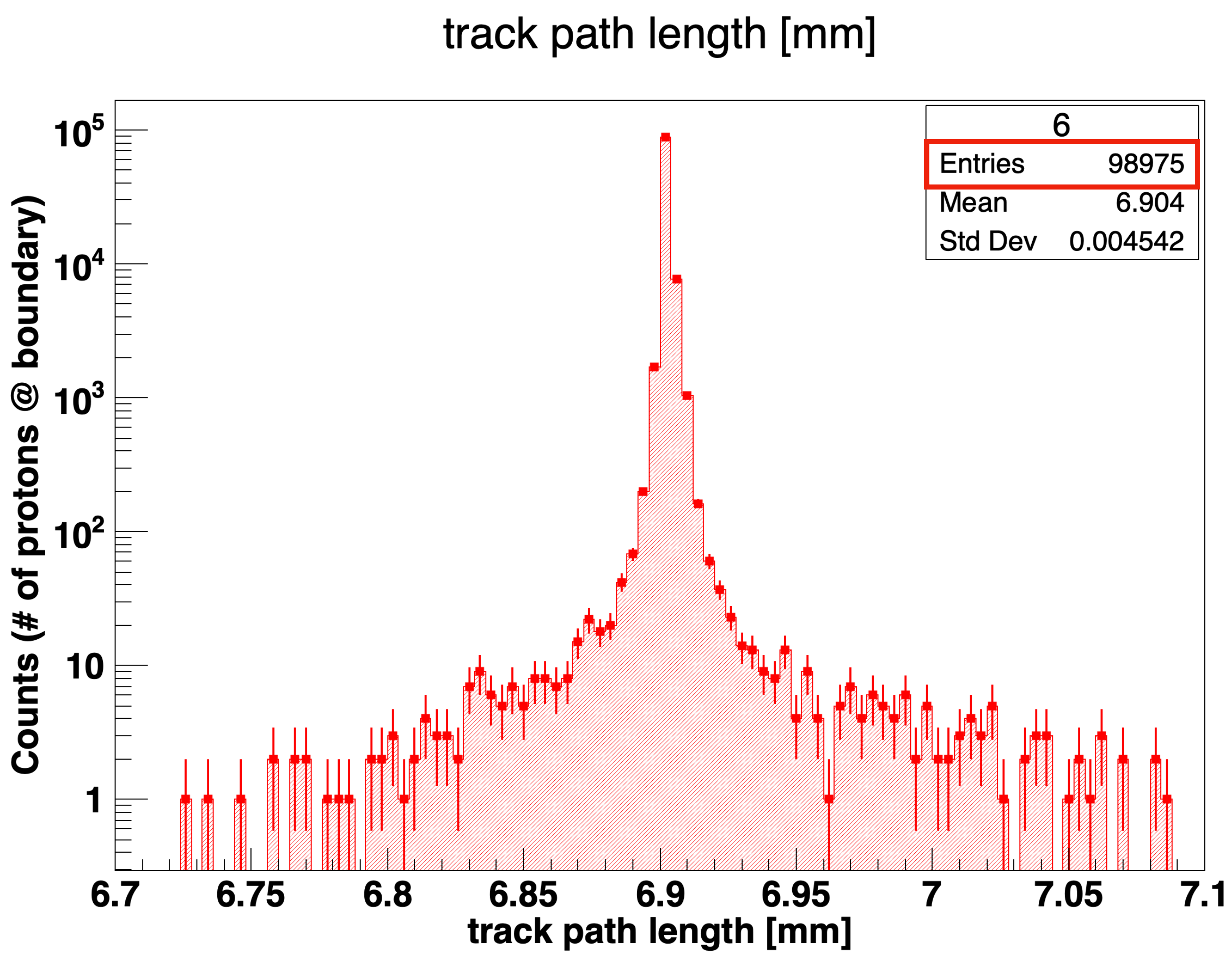
My rough calculations

Probability of Survival: ~98.95 %



Recall from GEANT4 simulation:

98,975 out of 100,000 protons reached the target boundary

Probability of Survival: ~98.97 %